10.3 Hypothesis Test for a Population Mean **MATH 241** **THOMPSON**

1) To test H_0: $\mu = 40$ versus H_1: $\mu < 40$, a random sample of size $n = 26$ is obtained from a population that is known to be normally distributed. Complete parts (a) through (d) below.

Click here to view the t-Distribution Area in Right Tail.

STATS- TSTATS-ONE SAMPLE WITH SUMMARY

(a) If $\bar{x} = 37.9$ and $s = 12.4$, compute the test statistic.

$t_0 =$ $-.864$ (Round to three decimal places as needed.)

One Sample T Summary

Sample mean: 37.9
Sample std. dev.: 12.4
Sample size: 26

Perform:
Hypothesis test for μ
H_0: μ = 40
H_A: μ < 40
Confidence interval for μ
Level: 0.95

(b) If the researcher decides to test this hypothesis at the $\alpha = 0.1$ level of significance, determine the critical value(s). Although technology or a t-distribution table can be used to find the critical value(s), for this problem use the t-distribution table given.

critical value

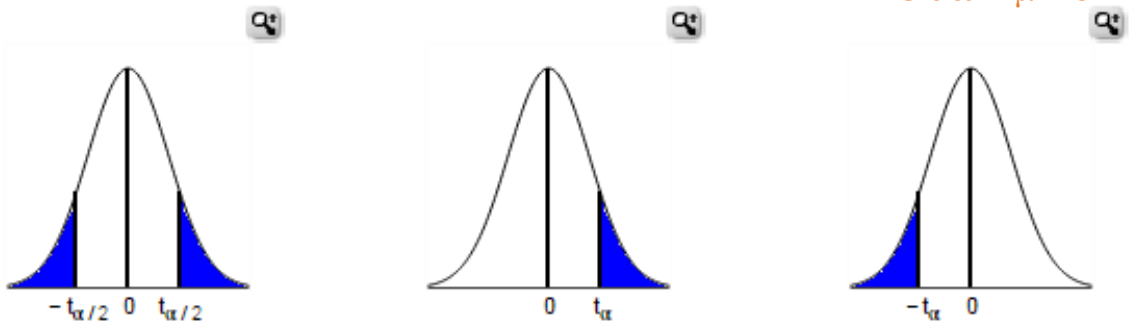
$t_\alpha =$ -1.316

USE TABLE df=n-1 df = 25 $\alpha = 0.1$ -1.316 bc its left tail

(Round to three decimal places. Use a comma to separate answers as needed.)

(c) Draw a t-distribution that depicts the critical region. Choose the correct answer below.

A. B. C. Left tail $\mu<40$



(d) Will the researcher reject the null hypothesis?

A. Yes, because the test statistic does not fall in the critical region.
B. No, because the test statistic does not fall in the critical region.

The test statistic doesn't fall in the shaded region (critical region)

2) To test H_0: $\mu = 100$ versus H_1: $\mu \neq 100$, a simple random sample size of $n = 22$ is obtained from a population that is known to be normally distributed. Answer parts (a)-(d).

Click here to view the t-Distribution Area in Right Tail.

STATS- TSTATS-ONE SAMPLE WITH SUMMARY

(a) If $\bar{x} = 105.6$ and $s = 9.3$, compute the test statistic.

$t =$ 2.824 (Round to three decimal places as needed.)

(b) If the researcher decides to test this hypothesis at the $\alpha = 0.01$ level of significance, determine the critical values.

$\alpha = \frac{.01}{2} = .005$ USE TABLE df = 21 $\alpha = 0.005$ -,831,2.831

The critical values are $-2.831, 2.831$.

(Use a comma to separate answers as needed. Round to three decimal places as needed.)

One Sample T Summary

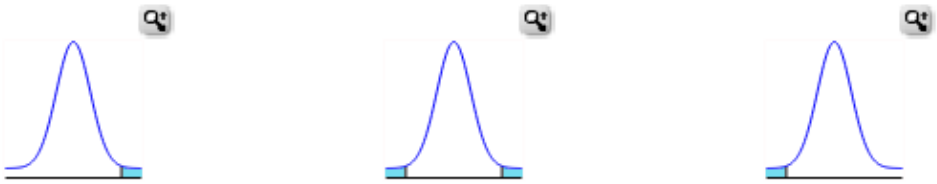
Sample mean: 105.6
Sample std. dev.: 9.3
Sample size: 22

Perform:
Hypothesis test for μ
H_0: μ = 100
H_A: μ ≠ 100

(c) Draw a t-distribution that depicts the critical region(s). Which of the following graphs shows the critical region(s) in the t-distribution?

bc two tailed $\alpha = 0.01/2 = .005$

A. B. C.



(d) Will the researcher reject the null hypothesis?

A. There is not sufficient evidence for the researcher to reject the null hypothesis since the test statistic is not between the critical values.

B. There is not sufficient evidence for the researcher to reject the null hypothesis since the test statistic is between the critical values.

The test stat falls between the critical values: t = 2.824 is between -2.831, 2.831

If is doe not in the critical values then the researcher will reject

3) To test H_0: $\mu = 20$ versus H_1: $\mu < 20$, a simple random sample of size $n = 20$ is obtained from a population that is known to be normally distributed. Answer parts (a)-(d).

Click here to view the t-Distribution Area in Right Tail.

STATS- TSTATS-ONE SAMPLE WITH SUMMARY

(a) If $\bar{x} = 18.4$ and $s = 3.9$, compute the test statistic.

t = −1.835 (Round to two decimal places as needed.)

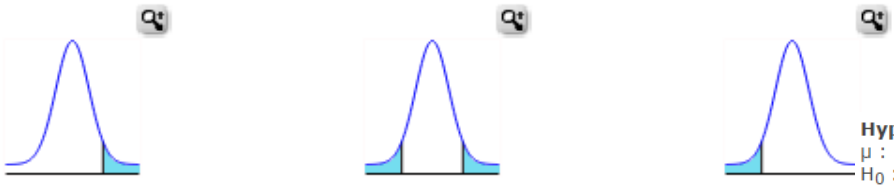
One Sample T Summary

Sample mean: 18.4
Sample std. dev.: 3.9
Sample size: 20

Perform:
Hypothesis test for μ
H_0: μ = 20
H_A: μ < 20

(b) Draw a t-distribution with the area that represents the P-value shaded. Which of the following graphs shows the correct shaded region?

A. B. C.



Hypothesis test results:
μ : Mean of population
H_0 : μ = 20
H_A : μ < 20

Mean	Sample Mean	Std. Err.	DF	T-Stat	P-value
μ	18.4	0.87206651	19	-1.8347224	0.0411

(c) Approximate the P-value. Choose the correct range for the P-value below.

A. 0.10 < P-value < 0.15
B. 0.025 < P-value < 0.05
C. 0.05 < P-value < 0.10
D. 0.15 < P-value < 0.20

(d) If the researcher decides to test this hypothesis at the $\alpha = 0.05$ level of significance, will the researcher reject the null hypothesis?

P = 0.0411

If P-value < α, reject the null hypothesis.

C. The researcher will reject the null hypothesis since the P-value is less than α.

If P is > than 0.05 then you will not reject since it is greater than α

4) Suppose the mean IQ score of people in a certain country is 100. Suppose the director of a college obtains a simple random sample of 42 students from that country and finds the mean IQ is 104.3 with a standard deviation of 13.4. Complete parts (a) through (d) below.

(a) Consider the hypotheses H_0: $\mu = 100$ versus H_1: $\mu > 100$. Explain what the director is testing. Perform the test at the $\alpha = 0.05$ level of significance. Write a conclusion for the test.

Explain what the director is testing. Choose the correct answer below.

D. The director is testing if the sample provided sufficient evidence that the population mean IQ score is actually greater than 100.

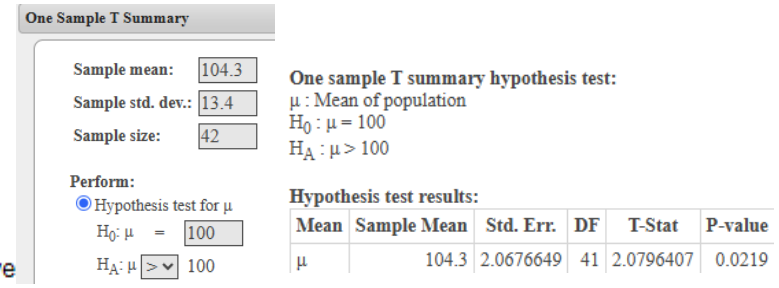
STATS- TSTATS-ONE SAMPLE WITH SUMMARY

Find the test statistic for this hypothesis test.

2.08 (Round to two decimal places as needed.)

Find the P-value for this hypothesis test.

.022 (Round to three decimal places as needed.)



Write a conclusion for the test. Choose the correct answe

A. Reject H_0. There is sufficient evidence to conclude that the mean IQ score of people in the country is greater than 100 at the $\alpha = 0.05$ level of significance.

(b) Consider the hypotheses H_0: $\mu = 101$ versus H_1: $\mu > 101$. Explain what the director is testing. Perform the test at the $\alpha = 0.05$ level of significance. Write a conclusion for the test.

Explain what the director is testing. Choose the correct answer below.

C. The director is testing if the sample provided sufficient evidence that the population mean IQ score is actually greater than 101.

Find the test statistic for this hypothesis test.

1.6 (Round to two decimal places as needed.)

ROTATE THE INEQUALITY TO FIND P-VALUE

H_A: μ $<$

Find the P-value for this hypothesis test.

.059 (Round to three decimal places as needed.)

Write a conclusion for the test. Choose the correct answer below.

C. Do not reject H_0. There is not sufficient evidence to conclude that the mean IQ score of people in the country is greater than 101 at the $\alpha = 0.05$ level of significance.

D. Reject H_0. There is sufficient evidence to conclude that the mean IQ score of people in the country is greater than 101 at the $\alpha = 0.05$ level of significance.

(c) Consider the hypotheses H_0: $\mu = 102$ versus H_1: $\mu > 102$. Explain what the director is testing. Perform the test at the $\alpha = 0.05$ level of significance. Write a conclusion for the test.

B. The director is testing if the sample provided sufficient evidence that the population mean IQ score is actually greater than 102.

Find the test statistic for this hypothesis test.

1.11 (Round to two decimal places as needed.)

Find the P-value for this hypothesis test.

.136 (Round to three decimal places as needed.)

C. Do not reject H_0. There is not sufficient evidence to conclude that the mean IQ score of people in the country is greater than 102 at the $\alpha = 0.05$ level of significance.

One should [not reject] rather than [accept] the null hypothesis. If one [accepts] the null hypothesis, this indicates that the [population mean] is a specific value, such as 100, 101, or 102, and so the same data have been used to conclude that the [population mean] is three different values. If one [does not reject] the null hypothesis, this indicates that the [population mean] could be 100, 101, or 102 or even some other value; we are simply not ruling them out as the value of the [population mean.] Therefore, [accepting] the null hypothesis can lead to contradictory conclusions, whereas [not rejecting] does not.

5) In a study, researchers wanted to measure the effect of alcohol on the hippocampal region, the portion of the brain responsible for long-term memory storage, in adolescents. The researchers randomly selected 22 adolescents with alcohol use disorders to determine whether the hippocampal volumes in the alcoholic adolescents were less than the normal volume of 9.02 cm^3. An analysis of the sample data revealed that the hippocampal volume is approximately normal with no outliers and $\bar{x} = 8.13$ cm^3 and $s = 0.7$ cm^3. Conduct the appropriate test at the $\alpha = 0.01$ level of significance.

State the null and alternative hypotheses.

$H_0: \mu$ [=] [9.02]

$H_1: \mu$ [<] [9.02]

(Type integers or decimals. Do not round.)

STATS- TSTATS-ONE SAMPLE

Identify the t-statistic.

$t_0 =$ [−5.96] (Round to two decimal places as needed.)

Identify the P-value.

ROTATE THE INEQUALITY TO FIND P-VALUE >

P-value = [.000] (Round to three decimal places as needed.)

Make a conclusion regarding the hypothesis.

If P-value < α, reject the null hypothesis.

[Reject] the null hypothesis. There [is] sufficient evidence to claim that the mean hippocampal volume is [less than] [9.02] cm^3.

One Sample T Summary

Sample mean: 8.13
Sample std. dev.: .7
Sample size: 22

Perform:
Hypothesis test for μ
$H_0: \mu$ = 9.02
$H_A: \mu$ < 9.02
Confidence interval for μ

6) A credit score is used by credit agencies (such as mortgage companies and banks) to assess the creditworthiness of individuals. Values range from 300 to 850, with a credit score over 700 considered to be a quality credit risk. According to a survey, the mean credit score is 708.3. A credit analyst wondered whether high-income individuals (incomes in excess of $100,000 per year) had higher credit scores. He obtained a random sample of 45 high-income individuals and found the sample mean credit score to be 719.3 with a standard deviation of 82.5. Conduct the appropriate test to determine if high-income individuals have higher credit scores at the $\alpha = 0.05$ level of significance.

State the null and alternative hypotheses.

$H_0: \mu$ [=] [708.3]

$H_1: \mu$ [>] [708.3]

(Type integers or decimals. Do not round.)

Identify the t-statistic.

STATS- TSTATS-ONE SAMPLE

$t_0 =$ [.89] (Round to two decimal places as needed.)

Identify the P-value.

One Sample T Summary

Sample mean: 719.3
Sample std. dev.: 82.5
Sample size: 45

Perform:
Hypothesis test for μ
$H_0: \mu$ = 708.3
$H_A: \mu$ > 708.3
Confidence interval for μ
Level: 0.95

Make a conclusion regarding the hypothesis.

ROTATE THE INEQUALITY TO FIND P-VALUE <

Fail to reject the null hypothesis. There is not sufficient evidence to claim that the mean credit score of high-income individuals is greater than 708.3.

7) It has long been stated that the mean temperature of humans is 98.6°F. However, two researchers currently involved in the subject thought that the mean temperature of humans is less than 98.6°F. They measured the temperatures of 67 healthy adults 1 to 4 times daily for 3 days, obtaining 300 measurements. The sample data resulted in a sample mean of 98.3°F and sample standard deviation of 1°F. Use the P-value approach to conduct a hypothesis test to judge whether the mean temperature of humans is less than 98.6°F at the $\alpha = 0.01$ level of significance.

State the hypotheses.

H_0: μ = 98.6°F

H_1: μ < 98.6°F

Find the test statistic.

$t_0 = -5.2$

STATS- TSTATS-ONE SAMPLE

Sample mean: 98.3

Sample std. dev.: 1

Sample size: 300

Perform:

Hypothesis test for μ

H_0: μ = 98.6

H_A: μ < 98.6

The P-value is .000.

(Round to three decimal places as needed.)

What can be concluded?

A. Do not reject H_0 since the P-value is not less than the significance level.

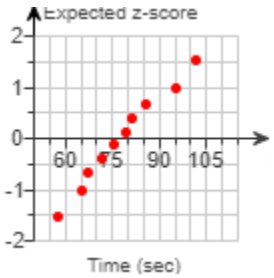
B. Reject H_0 since the P-value is less than the significance level.

8) The mean waiting time at the drive-through of a fast-food restaurant from the time an order is placed to the time the order is received is 86.7 seconds. A manager devises a new drive-through system that he believes will decrease wait time. As a test, he initiates the new system at his restaurant and measures the wait time for 10 randomly selected orders. The wait times are provided in the table to the right. Complete parts (a) and (b) below.

101.4	79.5
66.9	95.2
57.6	85.9
75.2	71.7
65.3	81.3

Click the icon to view the table of correlation coefficient critical values.

(a) Because the sample size is small, the manager must verify that the wait time is normally distributed and the sample does not contain any outliers. The normal probability plot is shown below and the sample correlation coefficient is known to be $r = 0.992$. Are the conditions for testing the hypothesis satisfied?



Yes, the conditions are satisfied. The normal probability plot is linear enough, since the correlation coefficient is greater than the critical value. In addition, a boxplot does not show any outliers.

First determine the appropriate hypotheses. Enter data in statcrunch

STATS- TSTATS-ONE SAMPLE-WITH DATA

H_0: μ = 86.7

H_1: μ < 86.7

Find the test statistic.

$t_0 = -2.02$

(Round to two decimal places as needed.)

Use the $\alpha = 0.05$ level of significance. What can be concluded from the hypothesis test?

✔ A. The P-value is less than the level of significance so there is sufficient evidence to conclude the new system is effective.

9) A golf association requires that golf balls have a diameter that is 1.68 inches. To determine if golf balls conform to the standarc a random sample of golf balls was selected. Their diameters are shown in the accompanying data table. Do the golf balls conform to the standards? Use the $\alpha = 0.05$ level of significance.

Click the icon to view the data table.

First determine the appropriate hypotheses.

H_0: μ = 1.68

H_1: μ ≠ 1.68

(Type integers or decimals. Do not round.)

STATS- TSTATS-ONE SAMPLE-WITH DATA

Find the test statistic.

.79

(Round to two decimal places as needed.)

Find the P-value.

.449

✔ D. Do not reject H_0. There is not sufficient evidence to conclude that the golf balls do not conform to the association's standards at the $\alpha = 0.05$ level of significance.

10) The head of institutional research at a university believed that the mean age of full-time students was declining. In 1995, the mean age of a full-time student was known to be 27.4 years. After looking at the enrollment records of all 4934 full-time students in the current semester, he found that the mean age was 27.1 years, with a standard deviation of 7.3 years. He conducted a hypothesis of H_0: $\mu = 27.4$ years versus H_1: $\mu < 27.4$ years and obtained a P-value of 0.0020. He concluded that the mean age of full-time students did decline. Is there anything wrong with his research?

A. Yes, the head of institutional research stated the hypotheses incorrectly; a left-tailed hypothesis test was conducted instead of a two-tailed test.

B. Yes, a P-value only indicates the likelihood of getting a result as extreme or more extreme as the one found, the head of institutional research needs to include a confidence level.

✔ C. Yes, the head of institutional research has access to the entire population, inference is unnecessary. He can say with 100% confidence that the mean age has decreased.

11) A researcher believes a new diet should improve weight gain in laboratory mice, so she runs an experiment over a 3-week period. Ten control mice staying with their standard diet gain an average of 4 ounces with a standard deviation of 0.3 ounces. Ten mice are given the new diet, and their average weight gain is 4.8 ounces with a standard deviation of 0.2 ounces. To test the researcher's theory, should a z-test or a t-test be used?

Choose the best answer from those given below.

A. The t-test should be used because the standard deviation of the population is known.

B. The z-test should be used because the standard deviation of the population is not known.

✔ C. The t-test should be used because the standard deviation of the population is not known.