11.3 Inference for Two Means: Independent Samples MATH 241 THOMPSON

$\mu_1 \neq \mu_2$ always check pool variance

Otherwise uncheck it

1) Assume that both populations are normally distributed.
(a) Test whether $\mu_1 \neq \mu_2$ at the $\alpha = 0.01$ level of significance for the given sample data.
(b) Construct a 99% confidence interval about $\mu_1 - \mu_2$.

	Population 1	Population 2
n	19	19
$\bar{x}$	15.6	13.8
s	4.6	4

(a) Test whether $\mu_1 \neq \mu_2$ at the $\alpha = 0.01$ level of significance for the given sample data.
Determine the null and alternative hypothesis for this test.

- A. $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$
- B. $H_0: \mu_1 \neq \mu_2$ $H_1: \mu_1 = \mu_2$
- ✔ C. $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$
- D. $H_0: \mu_1 \neq \mu_2$ $H_1: \mu_1 > \mu_2$

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE

WITH SUMMARY

Two Sample T Summary

Sample 1:
Sample mean: 15.6
Sample std. dev.: 4.6
Sample size: 19

Sample 2:
Sample mean: 13.8
Sample std. dev.: 4
Sample size: 19

Calculation options:
☑ Pool variances

Perform:
◉ Hypothesis test for $\mu_1 - \mu_2$
H_0: $\mu_1 - \mu_2$ = 0
H_A: $\mu_1 - \mu_2$ ≠ 0

Detemine the P-value for this hypothesis test.

P = .206 (Round to three decimal places as needed.)

Should the null hypothesis be rejected?

- A. Reject H_0, there is not sufficient evidence to conclude that the two populations have different means.
- ✔ B. Do not reject H_0, there is not sufficient evidence to conclude that the two populations have different means.

2) Use the given statistics to complete parts (a) and (b). Assume that the populations are normally distributed.
(a) Test whether $\mu_1 > \mu_2$ at the $\alpha = 0.10$ level of significance for the given sample data.
(b) Construct a 95% confidence interval about $\mu_1 - \mu_2$.

	Population 1	Population 2
n	28	19
$\bar{x}$	48.5	46.8
s	5.6	10.7

(a) Identify the null and alternative hypotheses for this test.

- A. $H_0: \mu_1 > \mu_2$ $H_1: \mu_1 = \mu_2$
- B. $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$
- C. $H_0: \mu_1 \neq \mu_2$ $H_1: \mu_1 = \mu_2$
- D. $H_0: \mu_1 < \mu_2$ $H_1: \mu_1 = \mu_2$
- E. $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$
- ✔ F. $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$

Find the test statistic for this hypothesis test.

.64 (Round to two decimal places as needed.)

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE
WITH SUMMARY

Two Sample T Summary

Sample 1:
Sample mean: 48.5
Sample std. dev.: 5.6
Sample size: 28

Sample 2:
Sample mean: 46.8
Sample std. dev.: 10.7
Sample size: 19

Calculation options:
☐ Pool variances

Perform:
◉ Hypothesis test for $\mu_1 - \mu_2$
H_0: $\mu_1 - \mu_2$ = 0
H_A: $\mu_1 - \mu_2$ > 0

Determine the P-value for this hypothesis test.

.265 (Round to three decimal places as needed.)

State the conclusion for this hypothesis test.

Do not reject H_0. There is not sufficient evidence at the $\alpha = 0.10$ level of significance to conclude that $\mu_1 > \mu_2$. If P value is < .1. then reject

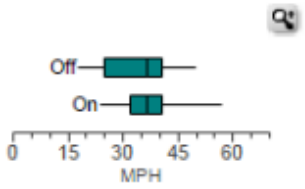
3) Ramp metering is a traffic engineering idea that requires cars entering a freeway to stop for a certain period of time before joining the traffic flow. The theory is that ramp metering controls the number of cars on the freeway and the number of cars accessing the freeway, resulting in a freer flow of cars, which ultimately results in faster travel times. To test whether ramp metering is effective in reducing travel times, engineers conducted an experiment in which a section of freeway had ramp meters installed on the on-ramps. The response variable for the study was speed of the vehicles. A random sample of 15 cars on the highway for a Monday at 6 p.m. with the ramp meters on and a second random sample of 15 cars on a different Monday at 6 p.m. with the meters off resulted in the following speeds (in miles per hour).

Click the icon to view the data sets.

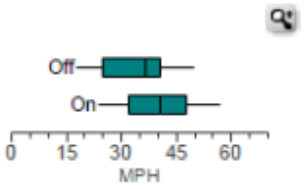
GRAPH – BOX PLOT click Draw boxes horizontally

(a) Draw side-by-side boxplots of each data set. Does there appear to be a difference in the speeds? Are there any outliers? Choose the correct box plot below.

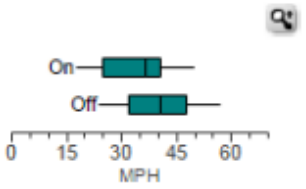
A.



B.



C.

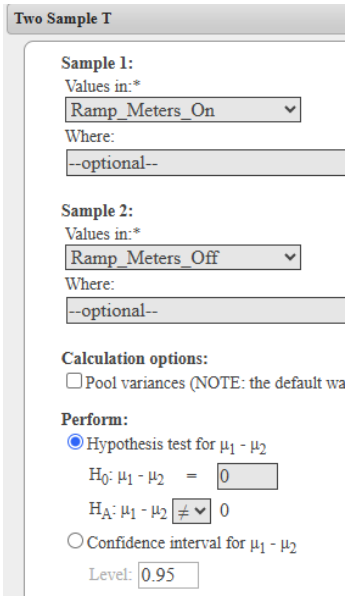


Does there appear to be a difference in the speeds?

A. Yes, the Meters On data appear to have higher speeds.

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE WITH DATA



Are there any outliers?

A. Yes, there appears to be a high outlier in the Meters On data.

B. No, there does not appear to be any outliers.

(b) Are the ramp meters effective in maintaining a higher speed on the freeway? Use the $\alpha = 0.1$ level of significance. State the null and alternative hypotheses. Choose the correct answer below.

A. $H_0: \mu_{on} \neq \mu_{off}$
$H_1: \mu_{on} > \mu_{off}$

B. $H_0: \mu_{on} < \mu_{off}$
$H_1: \mu_{on} > \mu_{off}$

C. $H_0: \mu_{on} = \mu_{off}$
$H_1: \mu_{on} \neq \mu_{off}$

D. $H_0: \mu_{on} = \mu_{off}$
$H_1: \mu_{on} > \mu_{off}$

Determine the P-value for this test.

P-value = .037 (Round to three decimal places as needed.)

Choose the correct conclusion.

A. Do not reject H_0. There is not sufficient evidence at the $\alpha = 0.1$ level of significance that the ramp meters are effective in maintaining higher speed on the freeway.

B. Reject H_0. There is not sufficient evidence at the $\alpha = 0.1$ level of significance that the ramp meters are effective in maintaining higher speed on the freeway.

C. Reject H_0. There is sufficient evidence at the $\alpha = 0.1$ level of significance that the ramp meters are effective in maintaining higher speed on the freeway.

4)

A researcher with the Department of Education followed a cohort of students who graduated from high school in a certain year, monitoring the progress the students made toward completing a bachelor's degree. One aspect of his research was to determine whether students who first attended community college took longer to attain a bachelor's degree than those who immediately attended and remained at a 4-year institution. The data in the table attached below summarize the results of his study. Complete parts a) through e) below.

Click the icon to view the sample data.

C. The response variable is the time to graduate. The explanatory variable is the use of community college or not.

b) Explain why this study can be analyzed using inference of two sample means. Determine what qualifications are met to perform the hypothesis test about the difference between two means. Select all that apply.

- [ ] A. The population is given to be normally distributed.
- [x] B. The samples are independent.
- [x] C. The samples can be reasonably assumed to be random.
- [x] D. The sample sizes are not more than 5% of the population.
- [x] E. The sample sizes are large (both greater than or equal to 30).

c) Does the evidence suggest that community college transfer students take longer to attain a bachelor's degree? Use an $\alpha = 0.01$ level of significance. Perform a hypothesis test. Determine the null and alternative hypotheses.

- A. $H_0: \mu_{\text{community college}} = \mu_{\text{no transfer}}$, $H_1: \mu_{\text{community college}} < \mu_{\text{no transfer}}$
- B. $H_0: \mu_{\text{community college}} > \mu_{\text{no transfer}}$, $H_1: \mu_{\text{community college}} < \mu_{\text{no transfer}}$
- C. $H_0: \mu_{\text{community college}} < \mu_{\text{no transfer}}$, $H_1: \mu_{\text{community college}} > \mu_{\text{no transfer}}$
- D. $H_0: \mu_{\text{community college}} = \mu_{\text{no transfer}}$, $H_1: \mu_{\text{community college}} > \mu_{\text{no transfer}}$

Determine the test statistic.

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE WITH DATA

t = 12.75 (Round to two decimal places as needed.)

Determine the P-value.

P-value = .000 (Round to three decimal places as needed.)

Should the hypothesis be rejected?

Reject the null hypothesis. The evidence does suggest that community college transfer students take longer to attain a bachelor's degree at the $\alpha = 0.01$ level of significance.

d) Construct a 90% confidence interval for $\mu_{\text{community college}} - \mu_{\text{no transfer}}$ to approximate the mean additional time it takes to complete a bachelor's degree if you begin in community college.

The confidence interval is the range from .879 to 1.141.
(Round to three decimal places as needed.)

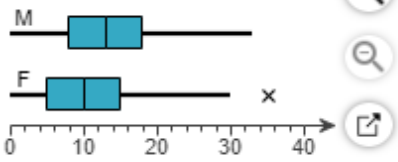
e) Do the results of parts c) and d) imply that community college causes you to take extra time to earn a bachelor's degree?

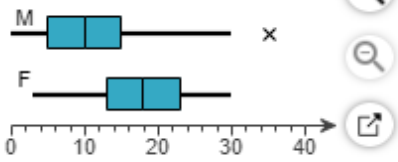
- [x] No
- [ ] Yes

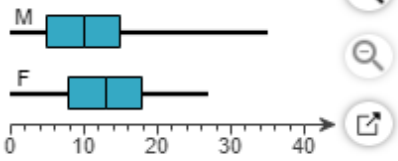
5) Do women feel differently from men when it comes to tax rates? One question on a survey of randomly selected adults asked, percent of income do you believe individuals should pay in income tax?" Complete parts a) through c).

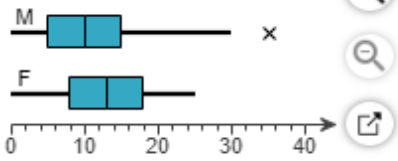
Click the icon to view the survey results.

a) Draw side-by-side boxplots of tax rates by gender. Choose the correct graph below.

A. 

B. 

C. 

D. 

Does there appear to be a difference in the income tax rates between genders?

- A. No but the maximum income tax rate for males is higher than the maximum income tax rate for females.
- B. No because the income tax rate is about the same for males and females.
- C. Yes because the income tax rate for females appears to have a higher median than the income tax rate for males.
- D. Yes because the income tax rate for males appears to have a higher median than the income tax rate for females.

b) Explain why a hypothesis test may be used to test whether the mean tax rates for the two genders differ.

Select all that apply.

- A. Each sample size is small relative to the size of its population.
- B. Each sample is obtained independently of the other.
- C. Each sample is a simple random sample.
- D. Each sample size is large.
- E. Each sample has the same sample size.

c) Test whether the mean tax rate for females differs from that of males at the $\alpha = 0.01$ level of significance.

Determine the null and alternative hypotheses for this test. Let μ_M represent the mean income tax rate for males and let μ_F represent the mean income tax rate for females.

H_0: μ_M = μ_F versus H_1: μ_M ≠ μ_F

Find t_0, the test statistic for this hypothesis test.

t_0 = −0.86 (Round to two decimal places as needed.)

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE WITH DATA

Determine the P-value for this test.

P-value = .392 (Round to three decimal places as needed.)

State the appropriate conclusion. Choose the correct answer below.

- D. Do not reject H_0. There is not sufficient evidence at the level of significance to conclude that the mean income tax rate for males is different from the mean income tax rate for females.

6) A random sample of 40 adults with no children under the age of 18 years results in a mean daily leisure time of 5.47 hours, with a standard deviation of 2.28 hours. A random sample of 40 adults with children under the age of 18 results in a mean daily leisure time of 4.43 hours, with a standard deviation of 1.96 hours. Construct and interpret a 90% confidence interval for the mean difference in leisure time between adults with no children and adults with children $(\mu_1 - \mu_2)$.

Let μ_1 represent the mean leisure hours of adults with no children under the age of 18 and μ_2 represent the mean leisure hours of adults with children under the age of 18.

The 90% confidence interval for $(\mu_1 - \mu_2)$ is the range from .25 hours to 1.83 hours.
(Round to two decimal places as needed.)

What is the interpretation of this confidence interval?

A. There is 90% confidence that the difference of the means is in the interval. Conclude that there is a significant difference in the number of leisure hours.

Check the P value

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE WITH DATA

Two Sample T Summary

Sample 1:
Sample mean: 5.47
Sample std. dev.: 2.28
Sample size: 40

Sample 2:
Sample mean: 4.43
Sample std. dev.: 1.96
Sample size: 40

Calculation options:
☑ Pool variances

Perform:
○ Hypothesis test for $\mu_1 - \mu_2$
H_0: $\mu_1 - \mu_2$ = 0
H_A: $\mu_1 - \mu_2$ ≠ 0
◉ Confidence interval for $\mu_1 - \mu_2$
Level: 0.90

7) A physical therapist wanted to know whether the mean step pulse of men was less than the mean step pulse of women. She randomly selected 55 men and 76 women to participate in the study. Each subject was required to step up and down a 6-inch platform. The pulse of each subject was then recorded. The following results were obtained.

Two sample T for Men vs Women

	N	Mean	StDev	SE Mean
Men	55	112.5	11.5	1.6
Women	76	118.6	14.3	1.6

95% CI for mu Men − mu Women
(− 10.60, − 1.60)
T-Test mu Men = mu Women (vs <)
T = − 2.70 P = 0.0039 DF = 127

(a) State the null and alternative hypotheses. Which of the following is correct?

- A. $H_0: \mu_1 = \mu_2; H_a: \mu_1 < \mu_2$
- B. $H_0: \mu_1 = \mu_2; H_a: \mu_1 \neq \mu_2$
- C. $H_0: \mu_1 = \mu_2; H_a: \mu_1 > \mu_2$

(b) Identify the P-value and state the researcher's conclusion if the level of significance was $\alpha = 0.01$. What is the P-value?

P-value = .0039

State the researcher's conclusion. Which of the following is correct?

- D. Reject H_0, there is sufficient evidence to conclude that the mean step pulse of men was less than the mean step pulse of women

(c) What is the 95% confidence interval for the mean difference in pulse rates of men versus women?

The lower bound is − 10.62 .
The upper bound is − 1.38 .
(Round to two decimal places as needed.)

Interpret this result.

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE WITH DATA
Confidence 95%

- A. 95% percent of the time the means are in the confidence interval.
- B. We are 95% confident that the mean difference is in the confidence interval.
- C. 95% percent of the time the mean difference is in the confidence interval.
- D. We are 95% confident that the means are in the confidence interval.

8) A professor wanted to determine whether an online homework system improved scores on a final exam. In the fall semester, he taught a class using the online homework system (which meant students did their homework online and received instant feedback about their answers along with helpful guidance). In the spring semester, he taught a class without the homework system (which meant students were responsible for doing their homework the old-fashioned way – paper and pencil). The professor made sure to teach the two courses identically (same text, syllabus, tests, meeting time, meeting location, and so on). The table summarizes the results of the two classes on their final exam. Complete parts **(a)** through **(e)** below.

Click the icon to view the table.

(a) What type of experimental design is this?

- A. Completely randomized design
- B. Randomized blocked design
- C. Prospective study
- D. Retrospective study

(b) What is the response variable?

- Homework system
- Test
- Text
- Final exam scores
- Syllabus
- Weather
- Location
- Teacher

What are the treatments in the study? Select all that apply.

- Test
- Text
- Final exam scores
- Teacher
- Homework system
- Weather
- Syllabus
- Location

(c) What factors are controlled in the experiment? Select all that apply.

- Text
- Final exam scores
- Location
- Test
- Homework system
- Syllabus
- Teacher
- Weather

(d) In many experiments, the researcher will recruit volunteers and randomly assign the individuals to a treatment group. In what regard was this done for this experiment?

- A. The assumption is that the students "randomly" enrolled in the course.
- B. The assumption is that the students were selected for each course.
- C. The assumption is that the students knew about the experiment and choose to be in one of the two classes.
- D. No assumptions were made for this experiment.

(e) Did the students perform better on the final exam in the fall semester? Use an $\alpha = 0.10$ level of significance. State the null and alternative hypotheses.

- A. $H_0: \mu_{fall} = \mu_{spring}$ $H_1: \mu_{fall} < \mu_{spring}$
- B. $H_0: \mu_{fall} > \mu_{spring}$ $H_1: \mu_{fall} < \mu_{spring}$
- C. $H_0: \mu_{fall} = \mu_{spring}$ $H_1: \mu_{fall} \neq \mu_{spring}$
- D. $H_0: \mu_{fall} = \mu_{spring}$ $H_1: \mu_{fall} > \mu_{spring}$

Determine the P-value for this hypothesis test.

P-value = .006 (Round to three decimal places as needed.)

STAT CRUNCH

STATS – T-STATS- TWO SAMPLE WITH SUMMARY

Choose the correct conclusion.

- A. Do not reject H_0. There is not sufficient evidence that the online homework is effective in raising final exam scores.
- B. Reject H_0. There is sufficient evidence that the online homework is effective in raising final exam scores.

EXTRA PROBLEMS:

A study was conducted to determine the effectiveness of a certain treatment. A group of 102 patients were randomly divided into an experimental group and a control group. The table shows the result for their net improvement. Let the experimental group be group 1 and the control group be group 2.

Click the icon to view the net improvement results.

(a) Test whether the experimental group experienced a larger mean improvement than the control group at the $\alpha = 0.05$ level of significance.

State the null and alternative hypotheses. Choose the correct answer below.

- A. $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 < \mu_2$
- B. $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 > \mu_2$
- C. $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$

Determine the P-value for this hypothesis test.

P-value = .000 (Round to three decimal places as needed.)

Choose the correct conclusion.

- A. Reject H_0. There is not sufficient evidence at the $\alpha = 0.05$ level of significance to say that the experimental g experienced a larger mean improvement than the control group.
- B. Do not reject H_0. There is not sufficient evidence at the $\alpha = 0.05$ level of significance to say that the experim experienced a larger mean improvement than the control group.
- C. Reject H_0. There is sufficient evidence at the $\alpha = 0.05$ level of significance to say that the experimental grou a larger mean improvement than the control group.
- D. Do not reject H_0. There is sufficient evidence at the $\alpha = 0.05$ level of significance to say that the experimenta experienced a larger mean improvement than the control group.

(b) Construct a 90% confidence interval about $\mu_1 - \mu_2$ and interpret the results.

What is the interpretation of this confidence interval?

- A. There is a 90% probability that the difference of the means is in the interval.
- B. We are 90% confident that the difference of the means is in the interval.

Two Sample T Summary

Sample 1:
Sample mean: 15.5
Sample std. dev.: 7.8
Sample size: 61

Sample 2:
Sample mean: 9.1
Sample std. dev.: 4.8
Sample size: 41

Calculation options:
Pool variances

Perform:
Hypothesis test for $\mu_1 - \mu_2$
$H_0: \mu_1 - \mu_2 = 0$
$H_A: \mu_1 - \mu_2 > 0$
Confidence interval for $\mu_1 - \mu_2$
Level: 0.90

One question on a survey of adults asked, "What do you think is the ideal number of children for a family to have?" Do the results of the survey suggest there is a difference between males and females in regard to this question? Use the $\alpha = 0.01$ level of significance.

Identify the null and alternative hypotheses for this test. Let μ_1 represent the population ideal number of children for adult males and μ_2 the population ideal number of children for adult females.

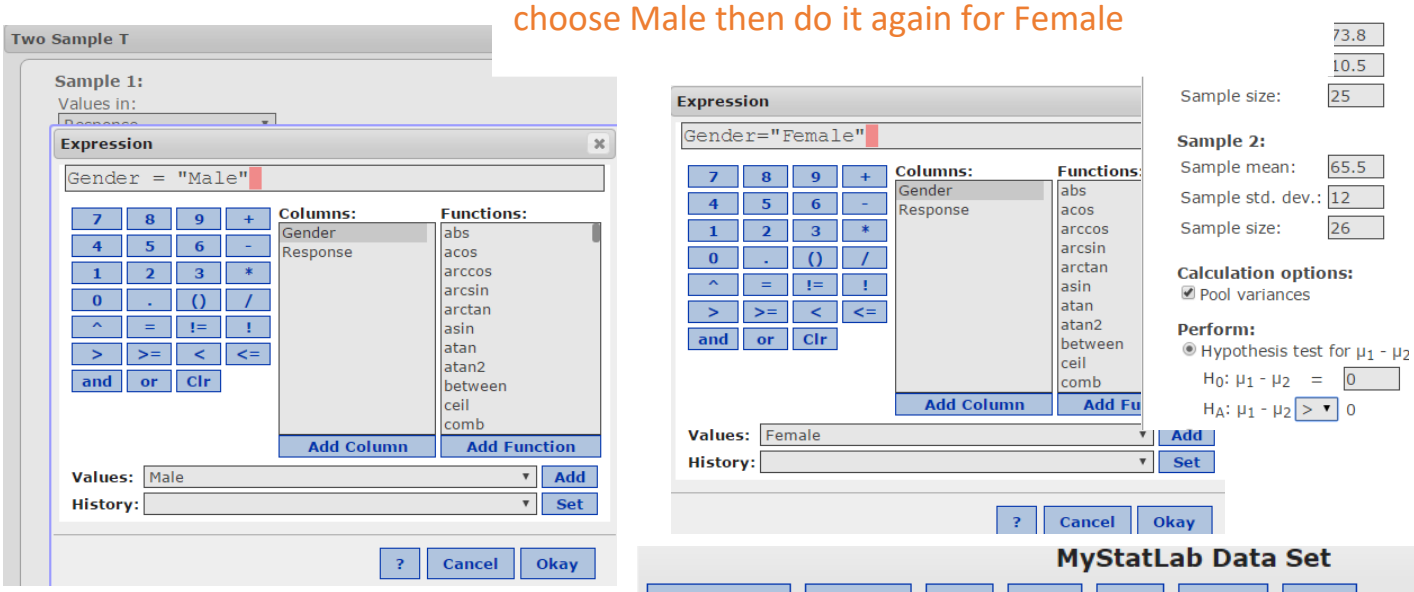
A. $H_0: \mu_1 < \mu_2$
$H_1: \mu_1 = \mu_2$

D. $H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$

T-STATS – TWO SAMPLE – WITH DATA

Then Values in: RESPONSE

Where: Double click Gender then bottom for Values: choose Male then do it again for Female



MyStatLab Data Set

StatCrunch | Applets | Edit | Data | Stat | Graph | Help

Row	Gender	Response	var3	var4	var5	var6
1	Female	3				

Options

Two sample T hypothesis test:
μ_1 : Mean of Response where Gender = "Male"
μ_2 : Mean of Response where Gender="Female"
$\mu_1 - \mu_2$: Difference between two means
$H_0 : \mu_1 - \mu_2 = 0$
$H_A : \mu_1 - \mu_2 \neq 0$
(with pooled variances)

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1 - \mu_2$	0.020768223	0.15834022	198	0.13116201	0.8958

Find the test statistic for this hypothesis test.

.13 (Round to two decimal places as needed.)

Determine the P-value for this hypothesis test.

.896 (Round to three decimal places as needed.)

State the conclusion for this hypothesis test.

A. Do not reject H_0. There is sufficient evidence at the $\alpha = 0.01$ level of significance to conclude that there is a difference in answers between males and females.

B. Do not reject H_0. There is not sufficient evidence at the $\alpha = 0.01$ level of significance to conclude that there is a difference in answers between males and females.