

3.4 Measures of Position and Outliers

MATH 241

THOMPSON

1. Suppose babies born after a gestation period of 32 to 35 weeks have a mean weight of 2500 grams and a standard deviation of 700 grams while babies born after a gestation period of 40 weeks have a mean weight of 2900 grams and a standard deviation of 400 grams. If a 32-week gestation period baby weighs 2275 grams and a 40-week gestation period baby weighs 2675 grams, find the corresponding z-scores. Which baby weighs less relative to the gestation period?

Find the corresponding z-scores. Which baby weighs relatively less? Select the correct choice below and fill in the answer boxes to complete your choice.
(Round to two decimal places as needed.)

$$Z = \frac{x - \mu}{\sigma}$$

X = WEIGHT OF BABY WEEK 35 then $x = 2275$

μ = MEAN WEIGHT OF BABY WEEK 35 then $\mu = 2500$

σ = STANDARD DEVIATION OF BABY WEEK 35 then $\sigma = 700$

$$z = \frac{x - \mu}{\sigma} \text{ then } z = \frac{2275 - 2500}{700} = -0.32$$

X = WEIGHT OF BABY WEEK 40 then $x = 2675$

μ = MEAN WEIGHT OF BABY WEEK 40 then $\mu = 2900$

σ = STANDARD DEVIATION OF BABY WEEK 40 then $\sigma = 400$

$$z = \frac{x - \mu}{\sigma} \text{ then } z = \frac{2675 - 2900}{400} = -0.56$$

The baby born in week 40 weighs relatively less since its z-score -0.56 is smaller than the z-score of -0.32 for the baby born in week 32.

Z-scores measure the number of standard deviations an observation is above or below the mean. As the magnitude of the z-score increases, the relative difference of the observation from the mean increases.

Therefore, compare the z-scores of the two babies. The baby with the larger z-score weighs relatively more, and the baby with the smaller z-score weighs relatively less.

2. In a certain city, the average 20- to 29-year old man is 69.6 inches tall, with a standard deviation of 3.1 inches, while the average 20- to 29-year old woman is 64.3 inches tall, with a standard deviation of 3.9 inches. Who is relatively taller, a 75-inch man or a 70-inch woman?

Find the corresponding z-scores. Who is relatively taller, a 75-inch man or a 70-inch woman? Select the correct choice below and fill in the answer boxes to complete your choice.
(Round to two decimal places as needed.)

$$Z = \frac{x - \mu}{\sigma}$$

$$\text{man } \frac{75 - 69.6}{3.1} = 1.74$$

$$\text{woman } \frac{70 - 64.3}{3.9} = 1.46$$

The z-score for the man, 1.74, is larger than the z-score for the woman, 1.46, so he is relatively taller.

3. One year Todd had the lowest ERA (earned-run average, mean number of runs yielded per nine innings pitched) of any male pitcher at his school, with an ERA of 3.34. Also, Nina had the lowest ERA of any female pitcher at the school with an ERA of 3.14. For the males, the mean ERA was 4.844 and the standard deviation was 0.687. For the females, the mean ERA was 3.793 and the standard deviation was 0.998. Find their respective z-scores. Which player had the better year relative to their peers, Todd or Nina? (Note: In general, the lower the ERA, the better the pitcher.)

$$\mathbf{Z} = \frac{\mathbf{X} - \mu}{\sigma}$$

$X = \text{Todd's ERA}$ then $x = 3.34$

μ = mean ERA then $\mu = 4.844$

σ = standard deviation ERA then $\sigma = 0.687$

$$Z = \frac{x - \mu}{\sigma} \text{ then } z = \frac{3.34 - 4.844}{.687} = -2.19$$

X = Nina's ERA then $x = 3.14$

$\mu = \text{man ERA then } \mu = 3.793$

σ = standard deviation of men then $\sigma = 0.998$

$$Z = \frac{x - \mu}{\sigma} \text{ then } z = \frac{3.14 - 3.793}{.998} = -0.65$$

Todd had a better year because of a lower z-score.

4. A highly selective boarding school will only admit students who place at least 2.5 standard deviations above the mean on a standardized test that has a mean of 300 and a standard deviation of 26. What is the minimum score that an applicant must make on the test to be accepted?

The minimum score that an applicant must make on the test to be accepted is 365.

Mean + 2.5(st dev) $300 + 2.5(26) = 365$

5. A manufacturer of bolts has a quality-control policy that requires it to destroy any bolts that are more than 2 standard deviations from the mean. The quality-control engineer knows that the bolts coming off the assembly line have mean length of 12 cm with a standard deviation of 0.05 cm. For what lengths will a bolt be destroyed?

Select the correct choice below and fill in the answer box(es) to complete your choice.
(Round to one decimal place as needed.)

- ☐ A. A bolt will be destroyed if the length is between cm and cm.
- ☒ B. A bolt will be destroyed if the length is less than 11.9 cm or greater than 12.1 cm.
- ☐ C. A bolt will be destroyed if the length is less than cm.
- 12-2(0.05) 12+2(0.05)
 subtract for the left add for the right

6. Last year, the verbal portion of the Scholastic Aptitude Test (SAT) has a mean score of $\mu = 500$ and a standard deviation of $\sigma = 100$, while in the same year, the verbal exam from the American College Testing Program (known as ACT) has a mean of $\mu = 20$ and a standard deviation of $\sigma = 5$. Assume that the scores from both exams are approximately normally distributed in any given year.

Two friends applying for college took the tests, the first of the two scoring 650 on the verbal portion of the SAT and the second scoring 30 on the verbal portion of the ACT. Which of these students scored higher among the population of students taking the relevant test, and why? Choose the best answer.

- ⓐ. The student with a score of 30 on the ACT did better because it is 2 standard deviations above the mean. **the more the st dev is above the mean, the better the score*

$$Z = \frac{x - \mu}{\sigma}$$

$$\text{SAT} \quad \frac{650 - 500}{100} = 1.5$$

$$\text{ACT} \quad \frac{30 - 20}{5} = 2$$

7. The **z-score** represents the number of standard deviations an observation is from the mean.

8. The prices of the 19 top-rated all-season tires for a specific tire size, are as follows.
Find the answers to parts (a) - (c) by hand.

\$87 \$117 \$98 \$78 \$82 \$90 \$91 \$89 \$91 \$81
\$109 \$115 \$104 \$96 \$86 \$90 \$77 \$99 \$90

Open statcrunch in a new window, use mathxl login
enter the list of values in var1

GRAPH – BOX PLOT – Draw boxes horizontally

- a) Determine Q_2 .

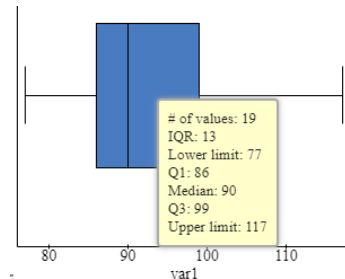
$$Q_2 = 90$$

- b) Determine Q_1 .

$$Q_1 = 86$$

- c) Determine Q_3 .

$$Q_3 = 99$$



hover over the box to get scores

The most common percentiles are quartiles. Quartiles divide data sets into fourths, or four equal parts. The first quartile, denoted Q_1 , divides the bottom 25% of the data from the top 75%. Therefore, the first quartile is equivalent to the 25th percentile. The second quartile, Q_2 , divides the bottom 50% of the data from the top 50%; it is equivalent to the 50th percentile or the median. Finally, the third quartile, Q_3 , divides the bottom 75% of the data from the top 25%; it is equivalent to the 75th percentile.

9. **Quartiles** divide data sets in fourths.

10. Consider the following data set.

29 29 13 27 14 24 20 17 14 26 13 27 13

a) Determine the quartiles for this data using your calculator.

b) Determine the interquartile range for this data. [Open statcrunch in a new window, use mathxl login](#)

a) The first quartile is 14 , the second quartile is 20 , and the third quartile is 27 .

(Type integers or decimals.)

enter the list of values in var1

b) The interquartile range is 13 .

(Type an integer or a decimal.)

GRAPH – BOX PLOT – Draw boxes horizontally

Interpret the quartiles. Choose the correct answer below.

- ☒ A. Of the monthly returns, 25% are less than or equal to the first quartile, 50% are less than or equal to the second quartile, and 75% are less than or equal to the third quartile.

11. The accompanying data represent the monthly rate of return of a certain company's common stock for the past few years.

 Click the icon to view the data table.

Determine and interpret the quartiles.

The first quartile is $Q_1 = -0.0200$.

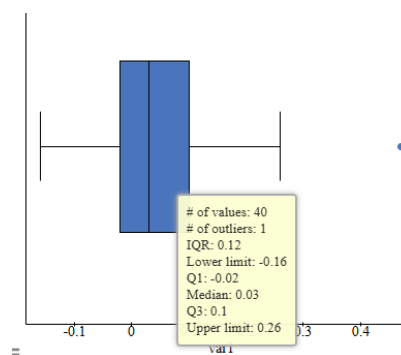
(Round to four decimal places as needed.)

The second quartile is $Q_2 = 0.0300$.

(Round to four decimal places as needed.)

The third quartile is $Q_3 = 0.1000$.

(Round to four decimal places as needed.)



Interpret the quartiles. Choose the correct answer below.

- ☒ D. Of the monthly returns, 25% are less than or equal to the first quartile, 50% are less than or equal to the second quartile, and 75% are less than or equal to the third quartile.

- **First Quartile (Q1):** Represents the 25th percentile, meaning 25% of the data falls below this value.
- **Second Quartile (Q2):** Represents the 50th percentile, also known as the median.
- **Third Quartile (Q3):** Represents the 75th percentile, meaning 75% of the data falls below this value.

12. Violent crimes include rape, robbery, assault, and homicide. The following is a summary of the violent-crime rate (violent crimes per 100,000 population) for all states of a country in a certain year. Complete parts (a) through (d).

$$Q_1 = 272.8, Q_2 = 387.4, Q_3 = 529.7$$

(a) Provide an interpretation of these results. Choose the correct answer below.

- ☒ A. 25% of the states have a violent-crime rate that is 272.8 crimes per 100,000 population or less.
50% of the states have a violent-crime rate that is 387.4 crimes per 100,000 population or less.
75% of the states have a violent-crime rate that is 529.7 crimes per 100,000 population or less.

(b) Determine and interpret the interquartile range.

The interquartile range is crimes per 100,000 population.
(Type an integer or a decimal.)

$$IQR = Q_3 - Q_1$$

The middle 50% of all observations have a range of 256.9 crimes per 100,000 population.

(c) The violent-crime rate in a certain state of the country in that year was 1,459. Would this be an outlier?

The lower fence is crimes per 100,000 population.

The upper fence is crimes per 100,000 population.

(Round to two decimal places as needed.)

Lower fence	$Q_1 - 1.5(IQR)$ $272.8 - 1.5(256.9) = -112.55$
Upper fence	$Q_3 + 1.5(IQR)$ $529.7 + 1.5(256.9) = 915.05$

(d) Do you believe that the distribution of violent-crime rates is skewed or symmetric?

- ☒ A. The distribution of violent-crime rates is skewed right.

The violent-crime rate in a certain state of the country in that year was 1,459. Would this be an outlier?

Yes, because it is greater than the upper fence.

13. The accompanying data represent the miles per gallon of a random sample of cars with a three-cylinder, 1.0 liter engine.
- (a) Compute the z-score corresponding to the individual who obtained 34.3 miles per gallon. Interpret this result.
- (b) Determine the quartiles.
- (c) Compute and interpret the interquartile range, IQR.
- (d) Determine the lower and upper fences. Are there any outliers?
-  Click the icon to view the data.

(a) Compute the z-score corresponding to the individual who obtained 34.3 miles per gallon. Interpret this result.

Find mean and st. dev in SUMMARY STATS -COLUMNS

use the mean (μ) and st dev (σ) to find the z-score *DO NOT ROUND

$$z = \frac{x - \mu}{\sigma} \quad \text{where } x = 34.3$$

write positive answer

The z-score corresponding to the individual is and indicates that the data value is standard deviation(s) below the mean.

(Type integers or decimals rounded to two decimal places as needed.)

GRAPH – BOX PLOT – Draw boxes horizontally

(b) Determine the quartiles.

****hover over the box to get scores**

$$Q_1 = 36.9 \text{ mpg}$$

(Type an integer or a decimal. Do not round.)

$$Q_2 = 38.65 \text{ mpg}$$

(Type an integer or a decimal. Do not round.)

$$Q_3 = 41.05 \text{ mpg}$$

(Type an integer or a decimal. Do not round.)

(c) Compute and interpret the interquartile range, IQR. Select the correct choice below and fill in the answer box to complete your choice.

(Type an integer or a decimal. Do not round.)

$$\text{IQR} = Q_3 - Q_1$$

The interquartile range is 4.15 mpg. It is the range of the middle 50% of the observations in the data set.

(d) Determine the lower and upper fences. Are there any outliers?

The lower fence is 30.675 .

(Type an integer or a decimal. Do not round.)

The upper fence is 47.275 .

(Type an integer or a decimal. Do not round.)

$$\text{Lower fence } Q_1 - 1.5(\text{IQR})$$

$$36.9 - 1.5(4.15) = 30.675$$

$$\text{Upper fence } Q_3 + 1.5(\text{IQR})$$

$$41.05 + 1.5(4.15) = 47.275$$

Are there any outliers? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

☒ A. The outlier(s) is/are 48.9 . **Hover over the point to the right inside of the box**

(Type an integer or a decimal. Do not round. Use a comma to separate answers as needed.)

☐ B. There are no outliers.

***Outliners are the values outside the lower and upper fence.**

14. The following data represent the monthly phone use, in minutes, of a customer enrolled in a fraud prevention program for the past 20 months. The phone company decides to use the upper fence as the cutoff point for the number of minutes at which the customer should be contacted. What is the cutoff point?

335	390	411	380
471	320	421	524
406	318	340	439
349	345	399	439
328	418	351	310

GRAPH – BOX PLOT – Draw boxes horizontally Upper fence $Q_3 + 1.5(\text{IQR})$

The cutoff point is 543 minutes.

(Round to the nearest minute.)

15. The accompanying data represent the pulse rates (beats per minute) of nine students. Treat the nine students as a population. Compute the z-scores for all the students. Compute the mean and standard deviation of these z-scores.

 Click the icon to view the data table.

Compute the z-scores for all the students. Complete the table.

Student	z score	Student	z-score
Student 1	.61	Student 6	1.01
Student 2	- 1.62	Student 7	1.01
Student 3	- 1.62	Student 8	- .57
Student 4	1.01	Student 9	.09
Student 5	.09		

Expression

(Pulse-Mean) / "Unadj. std. dev."

7	8	9	+	Columns:	Functions:
4	5	6	-	Student	abs
1	2	3	*	Pulse	acos
0	.	0	/	Column	arccos
^	=	!=	!	Mean	arcsin
>	>=	<	<=	Unadj. std. dev.	arctan
and	or	Clr			asin
					atan
					atan2

STAT – SUMMARY STATS – COLUMNS
highlight mean and unadj. Std dev

Click on (scroll to bottom)  Output:
☒ Store in data table

Compute!

close that box then...

DATA – COMPUTE-EXPRESSION

Expression box: BUILD

EXACT STEPS:

(then double click Pulse – double click mean) / double click Unadj. Std. dev

Looks like this CLICK

Okay

Compute!

Answers are populated in the 6th column

You cannot type in the expression

The mean of the z scores is always 0

The standard deviation is always 1

16. Explain the meaning of the accompanying percentiles.

- (a) The 15th percentile of the head circumference of males 3 to 5 months of age in a certain city is 41.0 cm.
(b) The 75th percentile of the waist circumference of females 2 years of age in a certain city is 49.8 cm.
(c) Anthropometry involves the measurement of the human body. One goal of these measurements is to assess how body measurements may be changing over time. The following table represents the standing height of males aged 20 years or older for various age groups in a certain city in 2015. Based on the percentile measurements of the different age groups, what might you conclude?

 Click the icon to view the data table for part c.

- (a) Explain the meaning of "The 15th percentile of the head circumference of males 3 to 5 months of age in a certain city is 41.0 cm." Choose the correct answer below.

☒ D. 15% of 3- to 5-month-old males have a head circumference that is 41.0 cm or less.

- (b) Explain the meaning of "The 75th percentile of the waist circumference of females 2 years of age in a certain city is 49.8 cm." Choose the correct answer below.

☒ C. 75% of 2-year-old females have a waist circumference that is 49.8 cm or less.

- (c) Anthropometry involves the measurement of the human body. One goal of these measurements is to assess how body measurements may be changing over time. The included table represents the standing height of males aged 20 years or older for various age groups in a certain city in 2015. Based on the percentile measurements of the different age groups, what might you conclude?

- At each percentile, the heights generally decrease as the age increases. Assuming that an adult
17. A mutual fund rating agency ranks a fund's performance by using one to five stars. A one-star mutual fund is in the bottom 20% of its investment class; a five-star mutual fund is in the top 20% of its investment class. Interpret the meaning of a four-star mutual fund.

Choose the correct answer below.

- ☒ A. A four-star fund is in the 4th quintile of the funds. That is, it is above the bottom 60%, but below the top 20% of the ranked funds.

18. Explain the circumstances for which the interquartile range is the preferred measure of dispersion. What is an advantage that the standard deviation has over the interquartile range?

The interquartile range is preferred when the data are skewed or have outliers. An advantage of the standard deviation is that it uses all the observations in its computation.

19. The following statistics represent weekly salaries at a construction company.

Mean	\$675	First quartile	\$430
Median	\$505	Third quartile	\$735
Mode	\$470	93rd percentile	\$899

...

The most common salary is \$ 470 . Mode

The salary that half the employees' salaries surpass is \$ 505 . Half is the median

The percent of employees' salaries that surpassed \$735 is 25 %.

Q_3 is to the left of 75%
surpassed that is 25%

The percent of employees' salaries that were less than \$430 is 25 %.

Q_1 is 25%

The percent of employees' salaries that surpassed \$899 is 7 %.

93rd percentile is 93%
surpassed that is 7%

If the company has 100 employees, the total weekly salary of all employees is \$ 67,500 .

mean x 100