5.2 The Addition Rule and Complements MATH 241 THOMPSON

1) If E and F are disjoint events, then P(E or F) = P(E) + P(F).

2) If E and F are not disjoint events, then P(E or F) = P(E) + P(F) − P(E and F).

3) A probability experiment is conducted in which the sample space of the experiment is S = {4,5,6,7,8,9,10,11,12,13,14,15}. Let event E = {5,6,7,8,9,10} and event F = {9,10,11,12}. List the outcomes in E and F. Are E and F mutually exclusive?

List the outcomes in E and F. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ✔ A. {9,10} (Use a comma to separate answers as needed.) #s in both E and F
- B. { }

Are E and F mutually exclusive?

- A. Yes. E and F have no outcomes in common.
- B. Yes. E and F have outcomes in common.
- ✔ C. No. E and F have outcomes in common.
- D. No. E and F have no outcomes in common.

4) A probability experiment is conducted in which the sample space of the experiment is S = {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}, F = {6, 7, 8, 9, 10}, and event G = {10, 11, 12, 13}. Assume that each outcome is equally likely. List the outcomes in F or G. Find P(F or G) by counting the number of outcomes in F or G. Determine P(F or G) using the general addition rule.

List the outcomes in F or G. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ✔ A. F or G = {6,7,8,9,10,11,12,13} Contains both F and G
  (Use a comma to separate answers as needed.)
- B. F or G = { }

Find P(F or G) by counting the number of outcomes in F or G.

P(F or G) = .667 $\frac{8}{12} = .667$

(Type an integer or a decimal rounded to three decimal places as needed.) P(f or G) is in common

Determine P(F or G) using the general addition rule. Select the correct choice below and fill in any answer boxes within your choice.
(Type the terms of your expression in the same order as they appear in the original expression. Round to three decimal places as needed.)
P(f and G) contains only numbers in both

- A. P(F or G) = ☐ + ☐ = ☐
- ✔ B. P(F or G) = .417 + .333 − .083 = .667

P(F or G) = P(F) + P(G) – P(F and G)

$$\frac{5}{12} + \frac{4}{12} - \frac{1}{12} = .667$$

5) A probability experiment is conducted in which the sample space of the experiment is S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, event E = {1, 2, 3, 4, 5} and event G = {6, 7, 8, 9}. Assume that each outcome is equally likely. List the outcomes in E and G. Are E and G mutually exclusive?

List the outcomes in E and G. Choose the correct answer below.

- A. E and G = { }
  (Use a comma to separate answers as needed.)
- ✔ B. E and G = { }

Are E and G mutually exclusive?

- A. No, because the events E and G have at least one outcome in common.
- B. Yes, because the events E and G have at least one outcome in common.
- C. No, because the events E and G have outcomes in common.
- ✔ D. Yes, because the events E and G have no outcomes in common.

6) A probability experiment is conducted in which the sample space of the experiment is S = {10,11,12,13,14,15,16,17,18,19,20,21}. Let event E = {12,13,14,15,16,17,18,19}. Assume each outcome is equally likely. List the outcomes in E^c. Find $P(E^c)$.

List the outcomes in E^c. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ✔ A. E^c = {10,11,20,21}
  (Use a comma to separate answers as needed.)
- B. E^c = { }

#s that are in S but not in E

$\frac{4}{12} = .333$

$P(E^c)$ = .333 (Type an integer or a decimal rounded to three decimal places as needed.)

7) Find the probability of the indicated event if P(E) = 0.40 and P(F) = 0.40.

Find P(E or F) if P(E and F) = 0.05.

P(E or F) = .75 (Simplify your answer.)

P(E or F) = P(E) + P(F) – P(E and F)

0.40 + 0.40 – 0.05 = 0.75

8) Find the probability of the indicated event if P(E) = 0.35 and P(F) = 0.50.

Find P(E and F) if P(E or F) = 0.70

P(E and F) = .15 (Simplify your answer.)

P(E and F) = P(E) + P(F) – P(E or F)

0.35 + 0.50 – 0.70 = 0.15

9) Find the probability P(E or F) if E and F are mutually exclusive, P(E) = 0.41, and P(F) = 0.45.

The probability P(E or F) is .86. (Simplify your answer.)

P(E or F) = P(E) + P(F) because they are disjoint events

0.41 + 0.45 = 0.86

10) Find the probability $P(E^c)$ if P(E) = 0.42.

The probability $P(E^c)$ is .58. (Simplify your answer.)

#s that are in S but not in E therefore 1 – 0.42 = 0.58

11) If P(E) = 0.40, P(E or F) = 0.50, and P(E and F) = 0.15, find P(F).

P(F) = .25 (Simplify your answer.)

P(E or F) = P(E) + P(F) – P(E and F)

0.50 = 0.40 + P(F) – 0.15

0.50 – 0.40 + 0.15 = .25

12) A golf ball is selected at random from a golf bag. If the golf bag contains 6 red balls, 9 yellow balls, and 5 black balls, find the probability the following event.

The golf ball is red or yellow.

The probability that the golf ball is red or yellow is .75 .
(Type an integer or a decimal rounded to three decimal places as needed.)

$$\frac{\textit{possible outcomes}}{\textit{total}} \quad \frac{6+9}{20} = \frac{15}{20} = .333$$

13) A golf ball is selected at random from a golf bag. If the golf bag contains 8 type A balls, 5 type B balls, and 7 type C balls, find the probability that the golf ball is not a type A ball.

The probability that the golf ball is not a type A ball is 0.6 .
(Type an integer or decimal rounded to three decimal places as needed.)

$$\frac{\textit{possible outcomes}}{\textit{total}} \quad \frac{5+7}{20} = \frac{12}{20} = .0.6$$

14) The following table shows the distribution of murders by type of weapon for murder cases in a particular country over the past 12 years. Complete parts (a) through (e).

Weapon	Probability
Handgun	0.475
Rifle	0.029
Shotgun	0.034
Unknown firearm	0.149
Knives	0.133
Hands, fists, etc.	0.055
Other	0.125

(a) Is the given table a probability model? Why or why not?

- A. No; the sum of the probabilities of all outcomes does not equal 1.
- B. No; the probability of all events in the table is not greater than or equal to 0 and less than or equal to 1.
- C. No; the probability of all events in the table is not greater than or equal to 0 and less than or equal to 1, and the sum probabilities of all outcomes does not equal 1.
- ✔ D. Yes; the rules required for a probability model are both met.

(b) What is the probability that a randomly selected murder resulted from a rifle or shotgun?

P(rifle or shotgun) = .063 (Type a decimal rounded to three decimal places as needed.)

$0.029 + 0.034 = 0.063$

Interpret this probability. Select the correct choice below and fill in the answer box to complete your choice.

- A. If 1000 murders were randomly selected, exactly ☐ of them would have resulted from a rifle or shotgun.
- ✔ B. If 1000 murders were randomly selected, we would expect about 63 of them to have resulted from a rifle or shotgu

(c) What is the probability that a randomly selected murder resulted from a handgun, rifle, or shotgun?

P(handgun, rifle, or shotgun) = .538

$0.475 + 0.029 + 0.034 = 0.538$

(Type a decimal rounded to three decimal places as needed.)

Interpret this probability. Select the correct choice below and fill in the answer box to complete your choice.

A. If 1000 murders were randomly selected, we would expect about 538 of them to have resulted from a handgun, rifle, or shotgun.

B. If 1000 murders were randomly selected, exactly ___ of them would have resulted from a handgun, rifle, or shotgun.

(d) What is the probability that a randomly selected murder resulted from a weapon other than a gun?

P(weapon other than a gun) = 0.313 No guns or rifle

(Type a decimal rounded to three decimal places as needed.)

$$P(\text{weapon other than a gun}) = P(\text{knives}) + P(\text{hands, fists, etc.}) + P(\text{other})$$
$$= 0.133 + 0.055 + 0.125$$

P(weapon other than a gun) = .313 (Simplify your answer.)

Interpret this probability. Select the correct choice below and fill in the answer box to complete your choice.

A. If 1000 murders were randomly selected, we would expect about 313 of them to be have resulted from a weapon other th a gun.

B. If 1000 murders were randomly selected, exactly ___ of them would have resulted from a weapon other than a gun.

(e) Are murders with a shotgun unusual?

- No
- Yes

15)

The data on the right represent the number of live multiple-delivery births (three or more babies) in a particular year for women 15 to 54 years old. Use the data to complete parts (a) through (d) below.

Age	Number of Multiple Births
15-19	89
20-24	514
25-29	1620
30-34	2827
35-39	1842
40-44	372
45-54	120

(a) Determine the probability that a randomly selected multiple birth for women 15-54 years old involved a mother 30 to 39 years old.

P(30 to 39) = .632 $2827 + 1842 = 4669$ $\frac{4669}{7384} = 0.632$

(Type an integer or decimal rounded to three decimal places as needed.)

(b) Determine the probability that a randomly selected multiple birth for women 15-54 years old involved a mother who was not 30 to 39 years old.

P(not 30 to 39) = .368 $1 - 0.632 = 0.368$

(Type an integer or decimal rounded to three decimal places as needed.)

(c) Determine the probability that a randomly selected multiple birth for women 15-54 years old involved a mother who was less than 45 years old.

P(less than 45) = .984 $7384 - 120 = 7264$ $\frac{7264}{7384} = 0.984$

(Type an integer or decimal rounded to three decimal places as needed.)

(d) Determine the probability that a randomly selected multiple birth for women 15-54 years old involved a mother who was at least 40 years old. Interpret this result. Is it unusual?

Find the probability that a randomly selected multiple birth for women 15-54 years old involved a mother who was at least 40 years o

P(at least 40) = .067   372 + 120 = 492 $\frac{492}{7384} = 0.067$

(Type an integer or decimal rounded to three decimal places as needed.)

Interpret this result. Select the correct choice below and fill in the answer box to complete your choice.
(Type a whole number.)

A. If 1000 multiple births for women 15-54 years old were randomly selected, we would expect about 67 of them to involve a mother who was at least 40 years old.

B. If 1000 multiple births for women 15-54 years old were randomly selected, exactly ___ of them would involve a mother who was at least 40 years old.

Is a multiple birth involving a mother who was at least 40 years old unusual?

D. No, because the probability of a multiple birth involving a mother who was at least 40 years old is greater than 0.05.

16) A standard deck of cards contains 52 cards. One card is selected from the deck.

(a) Compute the probability of randomly selecting a queen or ten.

(b) Compute the probability of randomly selecting a queen or ten or jack.

(c) Compute the probability of randomly selecting a seven or club.

52 cards in a deck
13 of each suit
4 suits of each card

(a) P(queen or ten) = .154   P(queen or ten) = P(queen) + P(ten). $\frac{4}{52} + \frac{4}{52}$

(Type an integer or a decimal rounded to three decimal places as needed.)

(b) P(queen or ten or jack) = .231   P(queen or ten or jack) = P(queen) + P(ten) + P(jack) $\frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{52}$

(Type an integer or a decimal rounded to three decimal places as needed.)

You can have a seven of club so we use the rule

(c) P(seven or club) = .308

(Type an integer or a decimal rou[nded to three decimal places as needed.)]

P(seven or club) = P(seven) + P(club) − P(seven and club)

$\frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

17) Exclude leap years from the following calculations.

(a) Compute the probability that a randomly selected person does not have a birthday on November 12.

(b) Compute the probability that a randomly selected person does not have a birthday on the 2nd day of a month.

(c) Compute the probability that a randomly selected person does not have a birthday on the 30th day of a month.

(d) Compute the probability that a randomly selected person was not born in February.

$P(E^c) = 1 - P(E)$

(a) The probability that a randomly selected person does not have a birthday on November 12 is .997.

(Type an integer or a decimal rounded to three decimal places as needed.) $1 - \frac{1}{365} = 0.997$

(b) The probability that a randomly selected person does not have a birthday on the 2nd day of a month is .967.

(Type an integer or a decimal rounded to three decimal places as needed.) $1 - \frac{12}{365} = 0.967$ 12 second days

(c) The probability that a randomly selected person does not have a birthday on the 30th day of a month is 0.970.

(Type an integer or a decimal rounded to three decimal places as needed.) $1 - \frac{11}{365} = 0.970$ *Feb has 28 days*

(d) The probability that a randomly selected person was not born in February is .923.   days in that month

(Type an integer or a decimal rounded to three decimal places as needed.) $1 - \frac{28}{365} = 0.923$

18) According to a center for disease control, the probability that a randomly selected person has hearing problems is 0.145. The probability that a randomly selected person has vision problems is 0.094. Can we compute the probability of randomly selecting a person who has hearing problems or vision problems by adding these probabilities? Why or why not?

Choose the correct answer below.

- ✔ A. No, because hearing and vision problems are not mutually exclusive. So, some people have both hearing and vision problems. These people would be included twice in the probability.
- B. Yes, because hearing and vision are two different senses, and therefore, they are two unique problems.
- C. Yes, because this is an application of the Addition Rule for Disjoint Events.
- D. No, because hearing problems and vision problems are events that are too similar to one another.

19) According to a survey, the probability that a randomly selected worker primarily drives a bicycle to work is 0.712. The probability that a randomly selected worker primarily takes public transportation to work is 0.092. Complete parts (a) through (d).

(a) What is the probability that a randomly selected worker primarily drives a bicycle or takes public transportation to work?

P(worker drives a bicycle or takes public transportation to work) = .804

(Type an integer or decimal rounded to three decimal places as needed.)

$$.712 + 0.092 = 0.804$$

(b) What is the probability that a randomly selected worker primarily neither drives a bicycle nor takes public transportation to work?

P(worker neither drives a bicycle nor takes public transportation to work) = .196

(Type an integer or decimal rounded to three decimal places as needed.)

$$1 - 0.804 = 0.196$$

(c) What is the probability that a randomly selected worker primarily does not drive a bicycle to work?

P(worker does not drive a bicycle to work) = .288

(Type an integer or decimal rounded to three decimal places as needed.)

$$1 - 0.712 = 0.288$$

(d) Can the probability that a randomly selected worker primarily walks to work equal 0.30? Why or why not?

- ✔ A. No. The probability a worker primarily drives, walks, or takes public transportation would be greater than 1.
- B. Yes. The probability a worker primarily drives, walks, or takes public transportation would equal 1.
- C. Yes. If a worker did not primarily drive or take public transportation, the only other method to arrive at work would be to walk.
- D. No. The probability a worker primarily drives, walks, or takes public transportation would be less than 1.

20)

	Freshman	Sophomore	Junior	Senior	Total
Satisfied	58	49	66	60	233
Neutral	26	15	11	15	67
Not satisfied	15	16	17	27	75
Total	99	80	94	102	375

A university conducted a survey of 375 undergraduate students regarding satisfaction with student government. Results of the survey are shown in the table by class rank. Complete parts (a) through (d) below.

Click the icon to view the table.

(a) If a survey participant is selected at random, what is the probability that he or she is satisfied with student government?

P(satisfied) = .621

$$\frac{233}{375} = 0.621$$

(Round to three decimal places as needed.)

(b) If a survey participant is selected at random, what is the probability that he or she is a junior?

P(junior) = .251

(Round to three decimal places as needed.)

$\frac{94}{375} = 0.251$

(c) If a survey participant is selected at random, what is the probability that he or she is satisfied and a junior?

P(satisfied and junior) = .176

(Round to three decimal places as needed.)

$\frac{66}{375} = 0.176$

(d) If a survey participant is selected at random, what is the probability that he or she is satisfied or a junior?

P(satisfied or junior) = 0.696

(Round to three decimal places as needed.)

P(E or F) = P(E) + P(F) – P(E and F)

$\frac{233}{375} + \frac{94}{375} - \frac{66}{375}$

$.621 + .251 - .176 = 0.696$

20)

	Male	Female	Total
Sunday	4067	2275	6342
Monday	3224	1772	4996
Tuesday	3437	1692	5129
Wednesday	3264	1698	4962
Thursday	3223	1855	5078
Friday	3864	2244	6108
Saturday	4692	2439	7131
Total	25,771	13,975	39,746

The data in the table represent the number of drivers involved in fatal crashes in a certain region by day of the week and gender Complete parts (a) through (e) below.

Click the icon to view the table of fatality data.

(a) Determine the probability that a randomly selected fatal crash involved a female.

P(female) = .352 (Round to three decimal places as needed.)

$\frac{13975}{39746} = 0.352$

(b) Determine the probability that a randomly selected fatal crash occurred on a Sunday.

P(Sunday) = .16 (Round to three decimal places as needed.)

$\frac{6342}{39746} = 0.16$

(c) Determine the probability that a randomly selected fatal crash occurred on a Sunday and involved a female.

P(Sunday and female) = .057 (Round to three decimal places as needed.)

$\frac{2275}{39746} = 0.057$

(d) Determine the probability that a randomly selected fatal crash occurred on a Sunday or involved a female.

P(Sunday or female) = .454 (Round to three decimal places as needed.)

P(E or F) = P(E) + P(F) – P(E and F)

$\frac{4996}{39746} + \frac{13975}{39746} - \frac{14836}{35267}$

$.125 + .351 - .045 = 0.454$

(e) Would it be unusual for a fatality to occur on Monday and involve a female driver?

It would be unusual because the probability .045 is low.

(Round to three decimal places as needed.)

$\frac{1772}{39746} = 0.045$

21) The following data represent the number of drivers involved in a fatal crash in 2016 in various light and weather conditions. Complete parts (a) through (e) below.

Click the icon to view the table of fatal crashes in various light and weather conditions.

(a) Determine the probability that a randomly selected fatal crash in 2016 occurred in normal weather.

P(Normal) = ☐ (Round to three decimal places as needed.)

Table of number of drivers involved in fatal crashes

	Light Condition				
Weather	**Daylight**	**Dark, but Lighted**	**Dark**	**Dawn/Dusk**	**Other**
Normal	14,836	5983	8102	1249	82
Rain	865	525	695	57	4
Snow/Sleet	226	57	195	18	3
Other	143	56	249	46	9
Unknown	849	251	560	81	126

STATS – SUMMARY STATS – ROWS

Select column(s):*
Daylight
Dark, but Lighted
Dark
Dawn/Dusk
Other

Select more than one with (Shift+click) or (Ctrl+click)

Where:
--optional--

Group by:
--optional--

Statistics:*
Q3
Sum

STATS – SUMMARY STATS – COLUMNS

Select column(s):*
Daylight
Dark, but Lighted
Dark
Dawn/Dusk
Other

Select more than one with (Shift+click) or (Ctrl+click)

Where:
--optional--
Build

Statistics:*
Q3
Sum

Weather	Daylight	Dark, but Light	Dark	Dawn/Dusk	Other	var7
Normal	14836	5983	8102	1249	82	
Rain	865	525	695	57	4	
Snow/Sleet	226	57	195	18	3	
Other	143	56	249	46	9	
Unknown	849	251	560	81	126	

Options

Summary statistics:

Column	Sum
Daylight	16919
Dark, but Lighted	6872
Dark	9801
Dawn/Dusk	1451
Other	224

Options

Row statistics:

Row	Sum
1	30252
2	2146
3	499
4	503
5	1867

CALCULATE TOTALS: **35267** **35267**

(a) Determine the probability that a randomly selected fatal crash in 2016 occurred in normal weather.

P(Normal) = 0.858 (Round to three decimal places as needed.) $\frac{30252}{35267}$ .858

(b) Determine the probability that a randomly selected fatal crash in 2016 occurred in daylight.

P(Daylight) = .48 (Round to three decimal places as needed.) $\frac{16919}{35267}$ .48

(c) Determine the probability that a randomly selected fatal crash in 2016 occurred in normal weather and in daylight.

P(Normal and Daylight) = .421 (Round to three decimal places as needed.) $\frac{14836}{35267}$ .421

(d) Determine the probability that a randomly selected fatal crash in 2016 occurred in normal weather or in daylight.

P(E or F) = P(E) + P(F) – P(E and F) $\frac{30252}{35267} + \frac{16919}{35267} - \frac{14836}{35267}$

0.858 + 0.48 – 0.421 = .917

P(Normal or Daylight) = .917 (Round to three decimal places as needed.)

(e) Would it be unusual for a fatal crash in 2016 to occur while it is dark outside (without lighting) and raining? (For the purposes of this exercise, consider a probability less than 0.05 to be low.)

It would be unusual for a fatal crash in 2016 to occur while it is dark outside (without lighting) and raining because the probability P(Raining and Dark without lighting) = .02 is low.

~~(Round to three decimal places as needed.)~~ $\frac{695}{35267} = 0.02$

~~Why might this result be considered misleading? Select all that apply.~~ UNUSUAL WHEN P < 0.05

- [x] A. Common sense indicates that a dark (without lighting) road in the rain is dangerous, so it seems that the probability of a fatality should be high.
- [x] B. A better question would be "Among the drivers on the road when it is dark and raining, what proportion result in a fatality?"
- [x] C. There are likely to be fewer drivers on the road at night, especially when it is raining.
- [ ] D. The result is not misleading.

The data in the following table show the association between cigar smoking and death from cancer for 137,495 men. Note: current cigar smoker means cigar smoker at time of death.

Click the icon to view the table.

(a) If an individual is randomly selected from this study, what is the probability that he died from cancer?
(b) If an individual is randomly selected from this study, what is the probability that he was a current cigar smoker?
(c) If an individual is randomly selected from this study, what is the probability that he died from cancer and was a current cigar smoker?
(d) If an individual is randomly selected from this study, what is the probability that he died from cancer or was a current cigar smoker?

(a) P(died from cancer) = 0.007 (Round to three decimal places as needed.) $\frac{752+67+161}{137495} = 0.007$

(b) P(current cigar smoker) = 0.072 (Round to three decimal places as needed.) $\frac{161+9690}{137495} = 0.072$

(c) P(died from cancer and current cigar smoker) = 0.001 (Round to three decimal places as needed.) $\frac{161}{137495} = 0.001$

(d) P(died from cancer or current cigar smoker) = 0.078 (Round to three decimal places as needed.)

Data table

	Died from Cancer	Did Not Die from Cancer
Never smoked cigars	752	119,067
Former cigar smoker	67	7,758
Current cigar smoker	161	9,690

$$\frac{752+67+161+9690}{137495} = 0.078$$