5.3 INDEPENDENCE AND THE MULITPLICATION RULE STATS THOMPSON

1) Two events E and F are independent if the occurrence of event E in a probability experiment does not affect the probability of event F.

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2) The multiplication rule states that if events E, F, G, ... are independent, then P(E and F and G and ...) = P(E) • P(F) • P(G) .... Accordingly, *and* probabilities use the multiplication rule.

The word *and* in probability implies that we use the multiplication rule.

3) The word *or* in probability implies that we use the Addition Rule.

4) When two events are disjoint, they are also independent.

Choose the correct answer below.

- ✔ False
- True

Two events are disjoint if they have no outcomes in common. In other words, the events are disjoint if, knowing that one of the events occurs, we know the other event did not occur. Independence means that one event occurring does not affect the probability of the other event occurring. Therefore, knowing two events are disjoint means that the events are not independent.

5) Determine whether the events E and F are independent or dependent. Justify your answer.

(a) E: A person having a high GPA.
F: The same person being a heavy reader of assigned course materials.

- A. E and F are dependent because having a high GPA has no effect on the probability of a person being a heavy reader of assigned course materials.
- B. E and F are independent because being a heavy reader of assigned course materials has no effect on the probability of a person having a high GPA.
- ✔ C. E and F are dependent because being a heavy reader of assigned course materials can affect the probability of a person having a high GPA.
- D. E and F are independent because having a high GPA has no effect on the probability of a person being a heavy reader of assigned course materials.

Determine whether the events E and F are independent or dependent. Justify your answer.

(b) E: A randomly selected person accidentally killing a spider.
F: A different randomly selected person accidentally swallowing a spider.

- A. E can affect the probability of F, even if the two people are randomly selected, so the events are dependent.
- ✔ B. E cannot affect F and vice versa because the people were randomly selected, so the events are independent.
- C. E cannot affect F because "person 1 accidentally killing a spider" could never occur, so the events are neither dependent nor independent.
- D. E can affect the probability of F because the people were randomly selected, so the events are dependent.

(c) E: The consumer demand for synthetic diamonds.
F: The amount of research funding for diamond synthesis.

- ✔ A. The consumer demand for synthetic diamonds could affect the amount of research funding for diamond synthesis so E and F are dependent.
- ○ B. The consumer demand for synthetic diamonds could not affect the amount of research funding for diamond synthesis, so E and F are independent.
- ○ C. The amount of research funding for diamond synthesis could affect the consumer demand for synthetic diamonds so E and F are dependent.

6) Suppose that events E and F are independent, P(E) = 0.7, and P(F) = 0.9. What is the P(E and F)?

The probability P(E and F) is .63. $0.7 \cdot 0.9 = 0.63$

(Type an integer or a decimal.)

$$P(A) = \frac{\text{Number of favorable to A}}{\text{Total number of possible outcomes}}$$

7) What is the probability of obtaining seven heads in a row when flipping a coin? Interpret this probability.

The probability of obtaining seven heads in a row when flipping a coin is .00781.

(Round to five decimal places as needed.) 1 favorable outcome with 2 possible outcomes

*Exponent is number of times it will occur $\left(\frac{1}{2}\right)^7$

Interpret this probability. 0.00781 times 10,000

Consider the event of a coin being flipped seven times. If that event is repeated ten thousand different times, it is expected that the event would result in seven heads about 78 time(s).

(Round to the nearest whole number as needed.)

8) Players in sports are said to have "hot streaks" and "cold streaks." For example, a batter in baseball might be considered to be in a slump, or cold streak, if that player has made 10 outs in 10 consecutive at-bats. Suppose that a hitter successfully reaches base 29% of the time he comes to the plate. Complete parts (a) through (c) below.

(a) Find the probability that the hitter makes 10 outs in 10 consecutive at-bats, assuming at-bats are independent events. Hint: The hitter makes an out 71% of the time.

P(hitter makes 10 consecutive outs) = .03255 *Exponent is number of times it will occur

(Round to five decimal places as needed.) 0.71^{10}

(b) Are cold streaks unusual?

The probability of a cold streak is less than 0.05, so cold streaks are unusual.

(c) Interpret the probability from part (a). **P < 0.05 is UNUSUAL EVENT**

In repeated sets of 10 consecutive at-bats, the hitter is expected to make an out in all 10 at-bats about 33 times out of 1000.

(Type a whole number.)

9) For the fiscal year 2007, a tax authority audited 1.61% of individual tax returns with income of $100,000 or more. Suppose this percentage stays the same for the current tax year. What is the probability that two randomly selected returns with income of $100,000 or more will be audited?

The probability is .000259.
(Round to six decimal places as needed.)

0.0161^2

*Exponent is number of times it will occur

If you have two percentages then multiply the decimals together

10) About 16% of the population of a large country is hopelessly romantic. If two people are randomly selected, what is the probability both are hopelessly romantic? What is the probability at least one is hopelessly romantic?

(a) The probability that both will be hopelessly romantic is .0256.
(Round to four decimal places as needed.)

16^2

(b) The probability that at least one person is hopelessly romantic is 0.2944.
(Round to four decimal places as needed.)

Romantic is not hopeless

at least one is **(1-not homeless)**

*Exponent is number of times it will occur

1 – 0.16 =.84 is not homeless

$1-.84^2=.2944$

$$P(A) = \frac{\text{Number of favorable to A}}{\text{Total number of possible outcomes}}$$

11) A test to determine whether a certain antibody is present is 99.7% effective. This means that the test will accurately come back negative if the antibody is not present (in the test subject) 99.7% of the time. The probability of a test coming back positive when the antibody is not present (a false positive) is 0.003. Suppose the test is given to seven randomly selected people who do not have the antibody.
(a) What is the probability that the test comes back negative for all seven people?
(b) What is the probability that the test comes back positive for at least one of the seven people?

(a) P(all 7 tests are negative) = .9792 (Round to four decimal places as needed.)

$0.997^7 = 0.9792$

(b) P(at least one positive) = .0208 (Round to four decimal places as needed.)

$1 - 0.9792 = 0.208$

12) The probability that a randomly selected 3-year-old male feral cat will live to be 4 years old is 0.99082.
(a) What is the probability that two randomly selected 3-year-old male feral cats will live to be 4 years old?
(b) What is the probability that seven randomly selected 3-year-old male feral cats will live to be 4 years old?
(c) What is the probability that at least one of seven randomly selected 3-year-old male feral cats will not live to be 4 years ol
Would it be unusual if at least one of seven randomly selected 3-year-old male feral cats did not live to be 4 years old?

(a) The probability that two randomly selected 3-year-old male feral cats will live to be 4 years old is .98172.
(Round to five decimal places as needed.)

$0.99082^2 = 0.98175$

(b) The probability that seven randomly selected 3-year-old male feral cats will live to be 4 years old is .93748.
(Round to five decimal places as needed.)

$0.99082^7 = 0.93748$

(c) The probability that at least one of seven randomly selected 3-year-old male feral cats will not live to be 4 years old is .06252.
(Round to five decimal places as needed.)

1-0.93748 = 0.6252”

Would it be unusual if at least one of six randomly selected 5-year-old male chipmunks did not live to be 6 years old?

$1 - 0.99082^1 = 0.002$

Yes, because the probability of this happening is less than 0.05.

$P < 0.05$ is UNUSUAL EVENT

$$P(A) = \frac{\text{Number of favorable to A}}{\text{Total number of possible outcomes}}$$

13) A computer can be classified as either cutting-edge or ancient. Suppose that 97% of computers are classified as ancient.

97% is 0.97

(a) Two computers are chosen at random. What is the probability that both computers are ancient?
(b) Eight computers are chosen at random. What is the probability that all eight computers are ancient?
(c) What is the probability that at least one of eight randomly selected computers is cutting-edge? Would it be unusual that at least one of eight randomly selected computers is

(a) Two computers are chosen at random. What is the probability that both computers are ancient?

*Exponent is number of times it will occur

The probability is .9409.
(Round to four decimal places as needed.) $.97^2$

(b) Eight computers are chosen at random. What is the probability that all eight computers

The probability is .7837.
(Round to four decimal places as needed.) $.97^8$

(c) What is the probability that at least one of eight randomly selected computers is cutting-edge?

The probability is .2163. $1 - 0.97^8$

No, it is not unusual

since .2162 is > .05

$P < 0.05$ is UNUSUAL EVENT

14)

In finance, one example of a derivative is a financial asset whose value is determined (derived) from a bundle of various assets, such as mortgages. Suppose a randomly selected mortgage in a certain bundle has a probability of 0.01 of default.
(a) What is the probability that a randomly selected mortgage will not default?
(b) What is the probability that five randomly selected mortgages will not default assuming the likelihood any one mortgage bein paid off is independent of the others? Note: A derivative might be an investment that only pays when all five mortgages do not default.
(c) What is the probability that the derivative from part (b) becomes worthless? That is, at least one of the mortgages defaults.

(a) The probability is .99.
(Type an integer or a decimal. Do not round.)

$1-0.01=0.99$

(b) The probability is .9510.
(Round to four decimal places as needed.)

$(0.99)^5= 0.9510$

(c) The probability is .049.

$1- 0.99^5 = 0.049$

(d) No will increases

15) *Exponent is number of times it will occur

Suppose Nate loses 27% of all bingo games.
(a) What is the probability that Nate loses two bingo games in a row?
(b) What is the probability that Nate loses four bingo games in a row?
(c) When events are independent, their complements are independent as well. Use this result to determine the probability that Nate loses four bingo games in a row, but does not lose five in a row.

(a) The probability that Nate loses two bingo games in a row is .0729. 0.27^2
(Round to four decimal places as needed.)

$.027^4$

(c) The probability that Nate loses four bingo games in a row, but does not lose five in a row is .0039.
(Round to four decimal places as needed.)

$0.27^4 - 0.27^5$

16) Among 18- to 23-year-olds, 36% say they have danced in public while under the influence of alcohol. Suppose five 18- to 23-year-olds are selected at random. Complete parts (a) through (d) below.

(a) What is the probability that all five have danced in public while under the influence of alcohol?

.0060
(Round to four decimal places as needed.) 0.36^5

(b) What is the probability that at least one has not danced in public while under the influence of alcohol?

.994 $1- 0.36^5$
(Round to four decimal places as needed.)

(c) What is the probability that none of the five have danced in public while under the influence of alcohol?

.1074 $1-0.36 = 0.64$ 0.64^5
(Round to four decimal places as needed.)

(d) What is the probability that at least one has danced in public while under the influence of alcohol?

$1- 0.1074^1$

.8926

EXTRA EXAMPLES

What is the probability of obtaining three tails in a row when flipping a coin? Interpret this probability.

The probability of obtaining three tails in a row when flipping a coin is .125.
(Round to five decimal places as needed.)

Interpret this probability. $\left(\frac{1}{2}\right)^3$

Consider the event of a coin being flipped three times. If that event is repeated ten thousand different times, it is expected that the event would result in three tails about 1250 time(s).
(Round to the nearest whole number as needed.) 0.125 times 10,000

About 5% of the population of a large country is hopelessly romantic. If two people are randomly selected, what is the probability both are hopelessly romantic? What is the probability at least one is hopelessly romantic?

(a) The probability that both will be hopelessly romantic is .0025. $(0.05)(0.05) = .0025$

(Round to four decimal places as needed.)

(b) The probability that at least one person is hopelessly romantic is .0975.

(Round to four decimal places as needed.)

$1-0.05 = 0.95$

probability that two are not homeless $(0.95)(0.95) = .9025$

probability that two are homeless $1 - 0.9025 = 0.0975$

A test to determine whether a certain antibody is present is 99.8% effective. This means that the test will accurately come back negative if the antibody is not present (in the test subject) 99.8% of the time. The probability of a test coming back positive when the antibody is not present (a false positive) is 0.002. Suppose the test is given to six randomly selected people who do not have the antibody.

(a) What is the probability that the test comes back negative for all six people?

(b) What is the probability that the test comes back positive for at least one of the six people?

(a) P(all 6 tests are negative) = .9881 (Round to four decimal places as needed.) $(0.998)^6 = 0.9881$

(b) P(at least one positive) = .0119 (Round to four decimal places as needed.) $1-0.9981 = 0.0119$

The probability that a randomly selected 5-year-old male stink bug will live to be 6 years old is 0.98058.

(a) What is the probability that two randomly selected 5-year-old male stink bugs will live to be 6 years old?

(b) What is the probability that eight randomly selected 5-year-old male stink bugs will live to be 6 years old?

(c) What is the probability that at least one of eight randomly selected 5-year-old male stink bugs will not live to be 6 years old? Woul it be unusual if at least one of eight randomly selected 5-year-old male stink bugs did not live to be 6 years old?

(b) The probability that eight randomly selected 5-year-old male stink bugs will live to be 6 years old is .85480.

(Round to five decimal places as needed.) $(0.98058)^8 = 0.85480$

(c) The probability that at least one of eight randomly selected 5-year-old male stink bugs will not live to be 6 years old is .1452.

(Round to five decimal places as needed.) $1 - 0.85480 = 0.1452$

Would it be unusual if at least one of eight randomly selected 5-year-old male stink bugs did not live to be 6 years old?

No, because the probability of this happening is greater than 0.05.

A cheese can be classified as either raw-milk or pasteurized. Suppose that 99% of cheeses are classified as pasteurized.
(a) Two cheeses are chosen at random. What is the probability that both cheeses are pasteurized?
(b) Four cheeses are chosen at random. What is the probability that all four cheeses are pasteurized?
(c) What is the probability that at least one of four randomly selected cheeses is raw-milk? Would it be unusual that at least one of fou randomly selected cheeses is raw-milk?

(a) Two cheeses are chosen at random. What is the probability that both cheeses are pasteurized?

The probability is .9801. $0.99^2 = 0.9801$
(Round to four decimal places as needed.)

(b) Four cheeses are chosen at random. What is the probability that all four cheeses are pasteurized?

The probability is .9606. $0.99^4 = 0.9606$
(Round to four decimal places as needed.)

(c) What is the probability that at least one of four randomly selected cheeses is raw-milk?

The probability is .0394. $1 - 0.9606 = 0.0394$
(Round to four decimal places as needed.)

Would it be unusual that at least one of four randomly selected cheeses is raw-milk?

It would be unusual that at least one of four randomly selected cheeses is raw-milk.

In finance, one example of a derivative is a financial asset whose value is determined (derived) from a bundle of various assets, such as mortgages. Suppose a randomly selected mortgage in a certain bundle has a probability of 0.07 of default.
(a) What is the probability that a randomly selected mortgage will not default?
(b) What is the probability that nine randomly selected mortgages will not default assuming the likelihood any one mortgage being paid off is independent of the others? Note: A derivative might be an investment that only pays when all nine mortgages do not default.

(a) The probability is .93. $1 - 0.07$
(Type an integer or a decimal. Do not round.)

(b) The probability is .5204. 0.93^9
(Round to four decimal places as needed.)

(c) The probability is .4796. $1 - 0.5204$ NO WILL INCREASE
(Round to four decimal places as needed.)

Players in sports are said to have "hot streaks" and "cold streaks." For example, a batter in baseball might be considered to be in a slump, or cold streak, if that player has made 10 outs in 10 consecutive at-bats. Suppose that a hitter successfully reaches base 32% of the time he comes to the plate. Complete parts (a) through (c) below.

(a) Find the probability that the hitter makes 10 outs in 10 consecutive at-bats, assuming at-bats are independent events. Hint: The hitter makes an out 68% of the time.

P(hitter makes 10 consecutive outs) = .02114
(Round to five decimal places as needed.)

(b) Are cold streaks unusual?

The probability of a cold streak is less than 0.05, so cold streaks are unusual.

(c) Interpret the probability from part (a).

In repeated sets of 10 consecutive at-bats, the hitter is expected to make an out in all 10 at-bats about 21 times out of 1000.
(Type a whole number.)

For the fiscal year 2007, a tax authority audited 1.74% of individual tax returns with income of $100,000 or more. Suppose this percentage stays the same for the current tax year. What is the probability that two randomly selected returns with income of $100,000 or more will be audited?

The probability is .000303. 0.0174^2

In airline applications, failure of a component can result in catastrophe. As a result, many airline components utilize something called triple modular redundancy. This means that a critical component has two backup components that may be utilized should the initial component fail. Suppose a certain critical airline component has a probability of failure of 0.0061 and the system that utilizes the component is part of a triple modular redundancy.
(a) Assuming each component's failure/success is independent of the others, what is the probability all three components fail, resulting in disaster for the flight?
(b) What is the probability at least one of the components does not fail?

(a) The probability is .00000023. $.0061^3$
(Round to eight decimal places as needed.)

(b) The probability is .99999977. 1 – ANS
(Round to eight decimal places as needed.)

In airline applications, failure of a component can result in catastrophe. As a result, many airline components utilize something called triple modular redundancy. This means that a critical component has two backup

Failure 0.026 triple modular redundancy

(a) Probability is does not fail $= 1 - .026^3$ = .99995334

(b) The minimum number of components = 6 *always 6

*Use trial and error with the exponents to get 0.99999999

$0.026^{⑤}$ then 1 – ANS = .99999988

almost; add 1

A cheese can be classified as either raw-milk or pasteurized. Suppose that 83% of cheeses are classified as pasteurized.
(a) Two cheeses are chosen at random. What is the probability that both cheeses are pasteurized?
(b) Five cheeses are chosen at random. What is the probability that all five cheeses are pasteurized?
(c) What is the probability that at least one of five randomly selected cheeses is raw-milk? Would it be unusual that at least one of five randomly selected cheeses is raw-milk?

(a) Two cheeses are chosen at random. What is the probability that both cheeses are pasteurized?

The probability is .6889.
(Round to four decimal places as needed.)

$(0.83)^2 = 0.6889$

(b) Five cheeses are chosen at random. What is the probability that all five cheeses are pasteurized?

The probability is .3939.
(Round to four decimal places as needed.)

$(0.83)^5 = 0.3939$

(c) What is the probability that at least one of five randomly selected cheeses is raw-milk?

The probability is .6061.
(Round to four decimal places as needed.)

$1-0.3939=0.6061$

Would it be unusual that at least one of five randomly selected cheeses is raw-milk?

It would not be unusual that at least one of five randomly selected cheeses is raw-milk.