

5.4 CONDITIONAL PROBABILITY AND THE GENERAL MULTI STATS THOMPSON

- 1) The notation $P(F|E)$ means the probability of event F given event E .
- 2) The probability that a randomly selected individual in a country earns more than \$75,000 per year is 10.5%. The probability that a randomly selected individual in the country earns more than \$75,000 per year, given that the individual has earned a bachelor's degree, is 10.5%. Are the events "earn more than \$75,000 per year" and "earned a bachelor's degree" independent?

Are these events independent?

 Yes

- 3) This past semester, a professor had a small business calculus section. The students in the class were Neta, William, Al, Allison, and Jinita. Suppose the professor randomly selects two people to go to the board to work problems. What is the probability that Jinita is the first person chosen to go to the board and Allison is the second?

First one is out of 5 but second is out of 4: $\frac{1}{5} \cdot \frac{1}{4}$

$P(\text{Jinita is chosen first and Allison is second}) = \frac{1}{20}$ (Type an integer or a simplified fraction.)

- 4) A bag of 32 tulip bulbs contains 14 red tulip bulbs, 11 yellow tulip bulbs, and 7 purple tulip bulbs. Suppose two tulip bulbs are randomly selected without replacement from the bag.
- What is the probability that the two randomly selected tulip bulbs are both red?
 - What is the probability that the first bulb selected is red and the second yellow?
 - What is the probability that the first bulb selected is yellow and the second red?
 - What is the probability that one bulb is red and the other yellow?

(a) The probability that both bulbs are red is $.183$. $\frac{14}{32} \cdot \frac{13}{31}$
(Round to three decimal places as needed.)

(b) The probability that the first bulb is red and the second is yellow is $.155$. $\frac{14}{32} \cdot \frac{11}{31}$
(Round to three decimal places as needed.)

(c) The probability that the first bulb is yellow and the second is red is $.155$. $\frac{11}{32} \cdot \frac{14}{31}$
(Round to three decimal places as needed.)

(d) The probability that one bulb is red and one is yellow is $.310$. $\frac{14}{32} \cdot \frac{11}{31} + \frac{11}{32} \cdot \frac{14}{31}$
(Round to three decimal places as needed.)

- 5) Suppose that two cards are randomly selected from a standard 52-card deck.
- What is the probability that the first card is a king and the second card is a king if the sampling is done without replacement?
 - What is the probability that the first card is a king and the second card is a king if the sampling is done with replacement?

(a) If the sampling is done without replacement, the probability that the first card is a king and the second card is a king is $.005$. (Round to three decimal places as needed.) $\frac{4}{52} \cdot \frac{3}{51}$

(b) If the sampling is done with replacement, the probability that the first card is a king and the second card is a king is $.006$. (Round to three decimal places as needed.) $\frac{4}{52} \cdot \frac{4}{52}$

- 6) Suppose you just received a shipment of fifteen televisions. Three of the televisions are defective. If two televisions are randomly selected, compute the probability that both televisions work. What is the probability at least one of the two televisions does not work?

The probability that both televisions work is $.629$. $\frac{12}{15} \cdot \frac{11}{14}$ *second chance take one out of both*
(Round to three decimal places as needed.)

The probability that at least one of the two televisions does not work is $.371$. $1 - 0.629$
(Round to three decimal places as needed.)

- 7) The data represent the number of driving fatalities for a certain area by age for male and female drivers.

	Male	Female
under 16	190	145
16-20	6156	2057
21-34	13,250	4522
35-54	10,504	4855
55-69	5184	2438
70 and over	2861	1788

Sum of males: 38145

Sum of male and female 55-69: 15359

- (a) What is the probability that a randomly selected driver fatality who was male was 35 to 54 years old?

The probability that a randomly selected driver fatality who was male was 35 to 54 years old is approximately $.275$.
(Round to three decimal places as needed.)

- (b) What is the probability that a randomly selected driver fatality who was 35 to 54 was male?

$$\frac{\text{male } 35-54}{\text{total male}} = \frac{10504}{38145}$$

The probability that a randomly selected driver fatality who was 35 to 54 was male is approximately $.684$. (Round to three decimal places as needed.)

- (c) Is a victim of a fatal accident aged 35 to 54 more likely to be male or female? Choose the correct statement below.

☒ A. The driver is more likely to be male because the probability is greater than 0.5. $\frac{\text{male } 35-54}{(\text{male} + \text{female } 35-54)} = \frac{10504}{10504 + 4855} = \frac{10504}{15359}$

** If male was less than .5 then you answer is driver is more likely a female because probability is greater than 0.5*

- 8) In a recent poll, a random sample of adults in some country (18 years and older) was asked, "When you see an ad emphasizing that a product is "Made in our country," are you more likely to buy it, less likely to buy it, or neither more nor less likely to buy it?" The results of the survey, by age group, are presented in the following contingency table. Complete parts (a) through (c).

Purchase likelihood	18-34	35-44	45-54	55 +	Total
More likely	202	329	373	406	1310
Less likely	29	8	21	19	77
Neither more nor less likely	297	214	176	112	799
Total	528	551	570	537	2186

- (a) What is the probability that a randomly selected individual is 45 to 54 years of age, given the individual is less likely to buy a product emphasized as "Made in our country"?

The probability is approximately $.273$. $\frac{21}{77}$
(Round to three decimal places as needed.)

- (b) What is the probability that a randomly selected individual is less likely to buy a product emphasized as "Made in our country," given the individual is 45 to 54 years of age?

The probability is approximately $.037$. $\frac{21}{570}$
(Round to three decimal places as needed.)

- (c) Are 18- to 34-year-olds more likely to buy a product emphasized as "Made in our country" than individuals in general?

- ☐ Yes, more likely
☒ No, less likely

- 9) Suppose you just purchased a digital music player and have put 15 tracks on it. After listening to them you decide that you like 3 of the songs. With the random feature on your player, each of the 15 songs is played once in random order. Find the probability that among the first two songs played
- You like both of them. Would this be unusual?
 - You like neither of them.
 - You like exactly one of them.
 - Redo (a)-(c) if a song can be replayed before all 15 songs are played.

(a) The probability that you like both songs is $.029$.
(Round to three decimal places as needed.)

Would it be unusual for you to like both of the songs?

$$\frac{3}{15} \cdot \frac{2}{14} \text{ the second is when one is taken out}$$

- ☐ No
☒ Yes

Because it is less than 5%

(b) The probability that you like neither song is $.629$.

$$\frac{12}{15} \cdot \frac{11}{14}$$

(c) The probability that you like exactly one song is $.342$. $1 - 0.029 - .629$ subtract the two previous from 1
(Round to three decimal places as needed.)

(d) The probability that you like both songs is 0.040 .
(Round to three decimal places as needed.)

$$\frac{3}{15} \cdot \frac{3}{15}$$

The probability that you like neither song is $.64$.
(Round to three decimal places as needed.)

$$\frac{12}{15} \cdot \frac{12}{15}$$

The probability that you like exactly one song is $.32$.
(Round to three decimal places as needed.)

$$1 - 0.04 - .64 \text{ subtract the two previous from 1}$$

- 10) Due to a manufacturing error, ten cans of regular soda were accidentally filled with diet soda and placed into a 36-pack. Suppose that two cans are randomly selected from the 36-pack. Complete parts (a) through (c).

(a) Determine the probability that both contain diet soda.

$$\frac{10}{36} \cdot \frac{9}{35}$$

P(both diet) = $.0714$ (Round to four decimal places as needed.)

(b) Determine the probability that both contain regular soda.

$$\frac{26}{36} \cdot \frac{25}{35}$$

P(both regular) = $.5159$ (Round to four decimal places as needed.)

Would this be unusual?

- ☒ No

(c) Determine the probability that exactly one is diet and exactly one is regular.

$$\frac{10}{36} \cdot \frac{26}{35} + \frac{26}{36} \cdot \frac{10}{35}$$

P(one diet and one regular) = 0.4127 (Round to four decimal places as needed.)

11)

Suppose there is a 13.7% probability that a randomly selected person aged 25 years or older is a jogger. In addition, there is a 29.1% probability that a randomly selected person aged 25 years or older is male, given that he or she jogs. What is the probability that a randomly selected person aged 25 years or older is male and jogs? Would it be unusual to randomly select a person aged 25 years or older who is male and jogs?

The probability that a randomly selected person aged 25 years or older is male and jogs is **.04**
(Round to three decimal places as needed.)

Would it be unusual?

$$0.291 \cdot 0.285$$

yes, since .04 is less than .05

If it was less greater .05 then it is not unusual

☐ No

☒ Yes

12)

Determine the probability that at least 2 people in a room of 6 people share the same birthday, ignoring leap years and assuming each birthday is equally likely, by answering the following questions:

(a) Compute the probability that 6 people have different birthdays.

(b) The complement of "6 people have different birthdays" is "at least 2 share a birthday". Use this information to compute the probability that at least 2 people out of 6 share the same birthday.

6 fractions

(a) The probability that 6 people have different birthdays is **.9595**.
(Round to four decimal places as needed.)

$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365}$$

(b) The probability that at least 2 people share a birthday is **.0405**.
(Round to four decimal places as needed.)

$$1 - .9595$$

13)

A flush in the card game of poker occurs if a player gets five cards that are all the same suit (clubs, diamonds, hearts, or spades). Complete parts (a) and (b) to obtain the probability of being dealt a flush in five cards.

clubs and fifth card is clubs).

$$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48}$$

P(five clubs) = **.000495**

(Round to six decimal places as needed.)

(b) A flush can occur if a player receives five clubs or five diamonds or five hearts or five spades. Compute P(five clubs or five diamonds or five hearts or five spades). Note that the events are mutually exclusive.

P(five clubs or five diamonds or five hearts or five spades) = **.002**

(Round to three decimal places as needed.)

$$0.000495 \times 5$$

(c) A royal flush in the game of poker occurs if the player gets the cards Ten, Jack, Queen, King, and Ace all in the same suit. Use the procedure given in parts (a) and (b) to compute the probability of being dealt a royal flush.

In a standard 52-card deck, P(royal flush) = **0.00000154**

(Round to eight decimal places as needed.)

$$\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} \cdot \frac{4}{49} \cdot \frac{4}{48}$$

- 14) Suppose the data below represent, in thousands, the type of health insurance coverage of people by age. Determine $P(< 18 \text{ years old})$ and $P(< 18 \text{ years old} \mid \text{no health insurance})$. Are the events "< 18 years old" and "no health insurance" independent?

	Age				Total
	< 18	18-44	45-64	> 64	
Private health insurance	45,502	70,830	54,118	28,385	198,835
Government health insurance	20,888	11,408	11,538	39,842	83,676
$P(< 18 \text{ years old}) = .223$					
(Round to three decimal places as needed.)					

$$P(< 18 \text{ years old} \mid \text{no health insurance}) = .157$$

(Round to three decimal places as needed.)

Are the events "< 18 years old" and "no health insurance" independent?

- ☒ No
☐ Yes

- 15) The data on the right represent the number of traffic fatalities by seat location and gender. Determine $P(\text{female})$ and $P(\text{female} \mid \text{passenger})$. Are the events "female" and "passenger" independent?

	Male	Female	Total
Passenger	32,884	11,939	44,823
Driver	6,350	6,498	12,848
Total	39,234	18,437	57,671

Determine $P(\text{female})$.

$$P(\text{female}) = .32$$

(Round to three decimal places as needed.)

Determine $P(\text{female} \mid \text{passenger})$.

$$P(\text{female} \mid \text{passenger}) = .266$$

(Round to three decimal places as needed.)

Are the events "female" and "passenger" independent?

- ☐ A. No. The occurrence of the event "passenger" does not affect the probability of the event "female."
☐ B. Yes. The occurrence of the event "passenger" affects the probability of the event "passenger."
☒ C. No. The occurrence of the event "passenger" affects the probability of the event "female."
☐ D. Yes. The occurrence of the event "passenger" does not affect the probability of the event

- 16) Suppose that a single card is selected from a standard 52-card deck. What is the probability that the card drawn is a club? Now suppose that a single card is drawn from a standard 52-card deck, but it is told that the card is black. What is the probability that the card drawn is a club?

The probability that the card drawn from a standard 52-card deck is a club is $.25$. $\frac{13}{52}$
(Round to three decimal places as needed.)

The probability that the card drawn from a standard 52-card deck is a club, given that this card is black, is $.5$. $\frac{13}{26}$
(Round to three decimal places as needed.)

- 17) For the month of March in a certain city, 77% of the days are cloudy. Also in the month of March in the same city, 38% of the days are cloudy and foggy. What is the probability that a randomly selected day in March will be foggy if it is cloudy?

The probability is approximately $.494$. (Round to three decimal places as needed.) $\frac{0.38}{0.77}$

- 18) According to a recent study, 8.9% of high school dropouts are 16- to 17-year-olds. In addition, 5.9% of high school dropouts are white 16- to 17-year-olds. What is the probability that a randomly selected dropout is white, given that he or she is 16 to 17 years old?

The probability that a randomly selected dropout is white, given that he or she is 16 to 17 years old, is $.6629$. $\frac{0.059}{0.089}$
(Round to four decimal places as needed.)

According to a research agency, in 16% of marriages the woman has a bachelor's degree and the marriage lasts at least 20 years. According to a census report, 25% of women have a bachelor's degree. What is the probability a randomly selected marriage will last at least 20 years if the woman has a bachelor's degree? Note: 53% of all marriages last at least 20 years.

$$\frac{0.16}{0.25}$$

The probability that a randomly selected marriage will last at least 20 years if the woman has a bachelor's degree is 0.640 .
(Round to three decimal places as needed.)