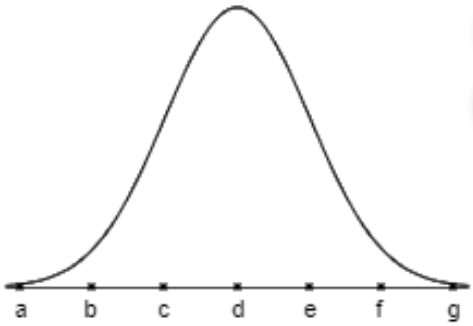
7.1 Properties of Normal Distribution **MATH 241** **THOMPSON**

1) Dr. Lorenzo finds that his patients' heart rates are normally distributed, with an average of 82 beats per minute and with a standard deviation of 7 beats per minute. He then graphs the normal curve as seen to the right. Label the tick marks on the x-axis of the normal curve.

a = 61
b = 68
c = 75
d = 82
e = 89
f = 96
g = 103

MEAN IS MIDDLE
left side: subtract standard deviation from the mean
(start in middle and move left)
right side: add standard deviation to the mean
(start in middle and move right)



82

-7 -7 -7 +7 +7 +7

61 68 75 82 89 96 103

2) Which of the following are properties of the normal curve?

Select all that apply.

- A. The graph of a normal curve is skewed right.
- B. The graph of a normal curve is symmetric.
- C. The high point is located at the value of the mean.
- D. The area under the normal curve to the right of the mean is 0.5.

3) Fill in the blank to complete the statement.

The area under the normal curve to the right of μ equals _____.

The area under the normal curve to the right of μ equals $\frac{1}{2}$.

The area under the normal curve is 1 and the curve is symmetric about μ, so half of the area under the curve lies to the right of μ.

4) Complete the statement below.

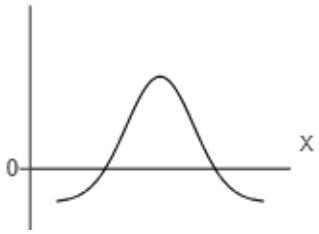
The points at x = _____ and x = _____ are the inflection points on the normal curve.

What are the two points?

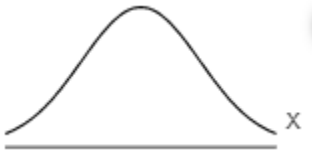
- A. The points are $x = \mu - \sigma$ and $x = \mu + \sigma$.

Determine whether the following graph can represent a normal curve.

5)

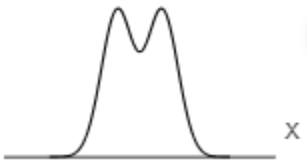


No, because the graph is not always greater than or equal to zero.

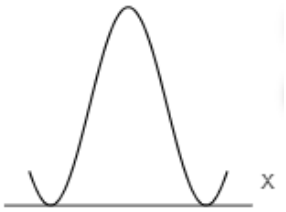


Yes, because the graph has the shape of a normal curve.

6)



No, because the graph has multiple peaks.



No, because the graph does not approach the X-axis as X increases or decreases without bound.

7) Discuss the validity of the following statement. If the statement is always true, explain why. If not, give a counterexample.

If two normal distributions have the same mean and standard deviation, then they have the same shape.

Choose the correct answer below.

- A. The statement is false because the normal distribution with a mean equal to -1 and a standard deviation equal to 2 can change shape based on the value of x.
- B. The statement is true because a normal distribution is completely defined by its mean and standard deviation.

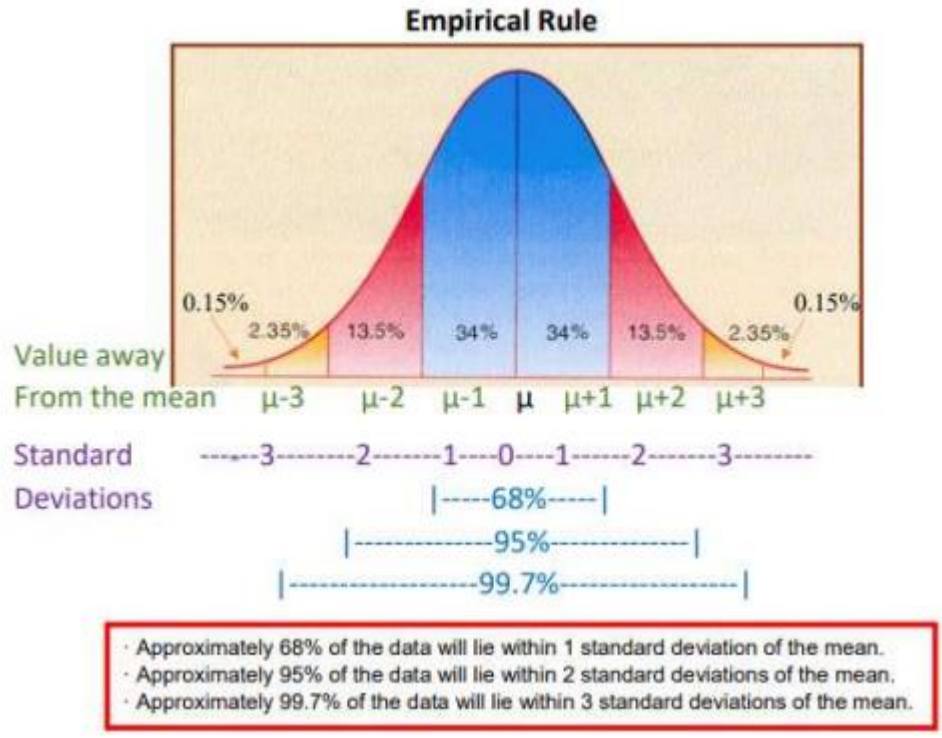
8) Discuss the validity of the following statement. If the statement is always true, explain why. If not, give a counterexample.

The area under a normal distribution and above the horizontal axis is equal to 1.

Choose the correct answer below.

- A. The statement is false because the area under the normal curve with a mean equal to 1 and a standard deviation equal to 2 does not equal 1.
- B. The statement is true because the graph of a normal distribution is a normal curve, and irrespective of the shape, the area between the normal curve and the x-axis is always 1.

For any symmetric distribution, the median equals the mean. The graph of the distribution must be symmetric about the median because it is the point where 50% of the area under the distribution lies on either side. The graph must also be symmetric about the mean, because it is the balancing point of a graph. For a symmetric graph, the balancing point will always be at the middle. The normal curve also only has one peak, and so for it to be symmetric, it must be symmetric at the highest point on the peak, which is the mode. Therefore, for the normal curve, mean = median = mode.



- Approximately 68% of the data will lie within 1 standard deviation of the mean.
- Approximately 95% of the data will lie within 2 standard deviations of the mean.
- Approximately 99.7% of the data will lie within 3 standard deviations of the mean.

9) If 120 scores are chosen from a normal distribution with mean 80 and standard deviation 5, how many scores x would be expected to satisfy $75 \le x \le 85$?

80-5=75 80+5 = 85 one standard deviation is 68%

82 scores

120 · 0.68 = 82

(Round to the nearest integer as needed.)

10) Determine whether the statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

The mean, median, and mode of a normal distribution are all equal.

Choose the correct answer below.

A. The statement is true.

11) The scores on a test are normally distributed with a mean of 150 and a standard deviation of 30. What is the score that is $1\frac{1}{2}$ standard deviations above the mean?

130 + 1 ½(30)

A score of 195 is $1\frac{1}{2}$ standard deviations above the mean.

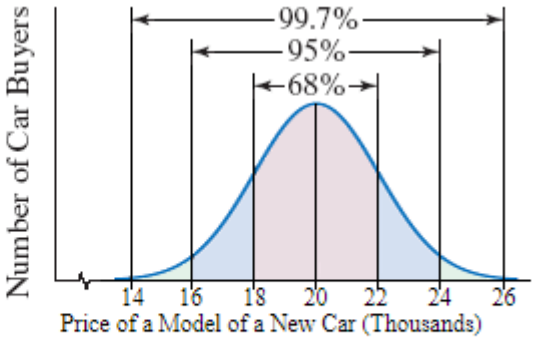
12) The scores on a test are normally distributed with a mean of 140 and a standard deviation of 28. Find the score that is one and one-half standard deviations below the mean.

A score of 98 is one and one-half standard deviations below the mean.

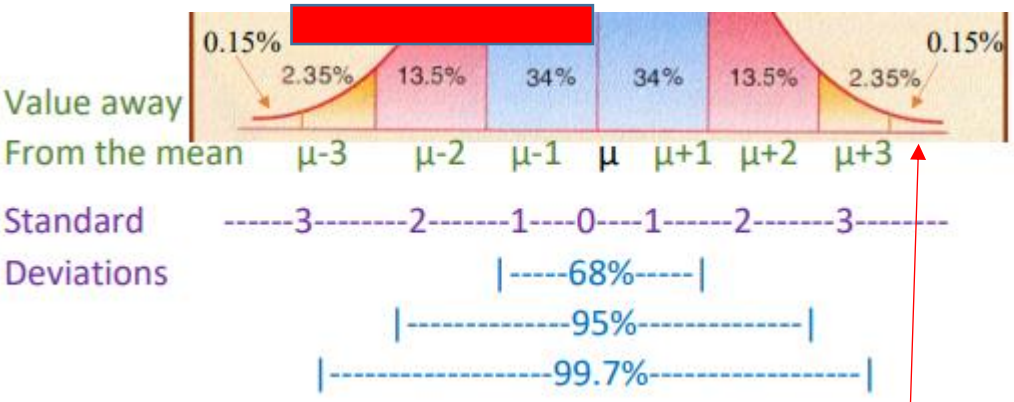
140 – 1.5(28)

13) Not everyone pays the same price for the same model of a car. The figure illustrates a normal distribution for the prices paid for a particular model of a new car. The mean is $20,000 and the standard deviation is $2000.

Use the 68-95-99.7 Rule to find what percentage of buyers paid between $14,000 and $20,000.



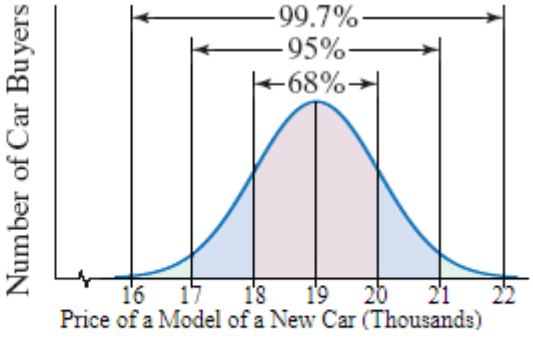
The percentage of buyers who paid between $14,000 and $20,000 is 49.85 %.
(Type an exact answer.)



2.35+13.5+34

14) Not everyone pays the same price for the same model of a car. The figure illustrates a normal distribution for the prices paid for a particular model of a new car. The mean is $19,000 and the standard deviation is $1000.

Use the 68-95-99.7 Rule to find the percentage of buyers who paid more than $22,000.



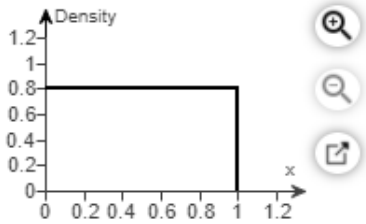
The percentage of buyers who paid more than $22,000 is .15 %.

Only portion greater than 3 sd

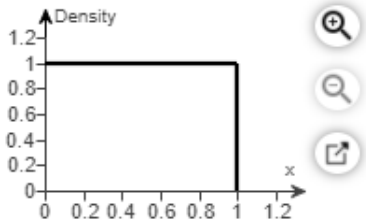
15) The random-number generator on calculators randomly generates a number between 0 and 1. The random variable X, the number ge
uniform probability distribution.
(a) Identify the graph of the uniform density function.
(b) What is the probability of generating a number between 0.12 and 0.92?
(c) What is the probability of generating a number greater than 0.89?

(a) Choose the correct graph of the uniform density function below.

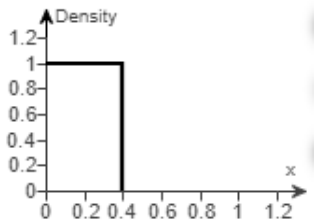
A.



B.



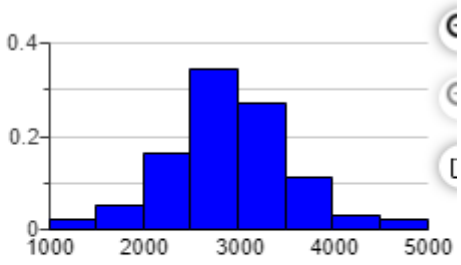
C.



(b) The probability is .8 . (Simplify your answer.) 0.92 – 0.12

(c) The probability is .11 . (Simplify your answer.) 1-.089

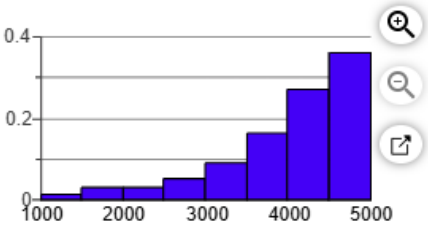
16) A study was conducted that resulted in the following relative frequency histogram. Determine whether or not the histogram indicates that a normal distribution could be used as a model for the variable.



Choose the correct answer below.

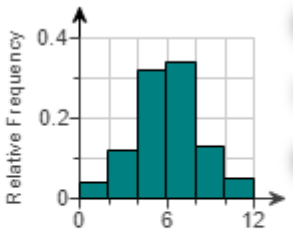
- ✔ A. The histogram is bell-shaped, so a normal distribution could be used as a model for the variable.
- B. The histogram is not bell-shaped, so a normal distribution could be used as a model for the variable.

A study was conducted that resulted in the following relative frequency histogram. Determine whether or not the histogram indicates that a normal distribution could be used as a model for the variable.



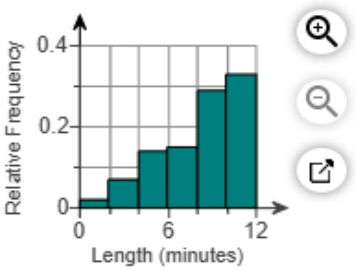
- ✔ C. The histogram is not bell-shaped, so a normal distribution could not be used as a model for the variable.

17) The relative frequency histogram represents the length of phone calls on George's cell phone during the month of September. Determine whether or not the histogram indicates that a normal distribution could be used as a model for the variable.



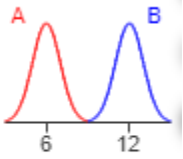
- A. No, because the histogram is symmetric about its mean.
- B. No, because the histogram does not have the shape of a normal curve.
- ✔ C. Yes, because the histogram has the shape of a normal curve.

The relative frequency histogram represents the length of phone calls on George's cell phone during the month of September. Determine whether or not the histogram indicates that a normal distribution could be used as a model for the variable.



A. No, because the histogram does not have the shape of a normal curve.

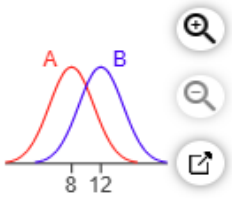
18) One graph in the figure represents a normal distribution with mean $\mu = 12$ and standard deviation $\sigma = 1$. The other graph represents a normal distribution with mean $\mu = 6$ and standard deviation $\sigma = 1$. Determine which graph is which and explain how you know.



Choose the correct answer below.

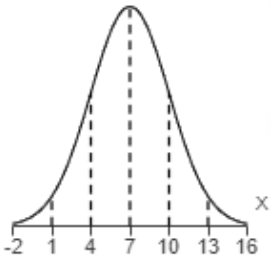
A. Graph A has a mean of $\mu = 6$ and graph B has a mean of $\mu = 12$ because a larger mean shifts the graph to the right.

One graph in the figure represents a normal distribution with mean $\mu = 12$ and standard deviation $\sigma = 3$. The other graph represents a normal distribution with mean $\mu = 8$ and standard deviation $\sigma = 3$. Determine which graph is which and explain how you know.



C. Graph A has a mean of $\mu = 8$ and graph B has a mean of $\mu = 12$ because a larger mean shifts the graph to the right.

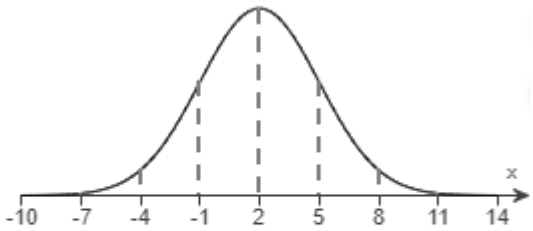
19) The graph of a normal curve is given on the right. Use the graph to identify the values of μ and σ.



$\mu =$ 7

$\sigma =$ 3

20) The graph of a normal curve is given. Use the graph to identify the value of μ and σ.



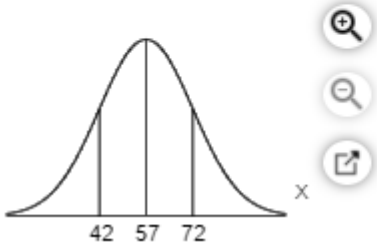
The value of μ is 2.

The value of σ is 3.

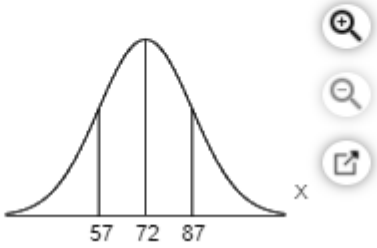
21) Draw a normal curve with $\mu = 57$ and $\sigma = 15$. Label the mean and the inflection points.

Choose the correct graph of the normal curve below.

A.



B.



C.

22) Suppose the monthly charges for cell phone plans are normally distributed with mean $\mu = \$68$ and standard deviation $\sigma = \$20$.
(a) Draw a normal curve with the parameters labeled.
(b) Shade the region that represents the proportion of plans that charge less than \$48.
(c) Suppose the area under the normal curve to the left of $X = \$48$ is 0.1587. Provide an interpretation of this result.

(a) Choose the correct graph below.

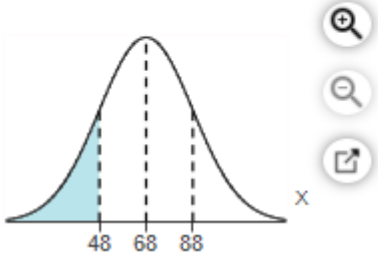
A.

B.

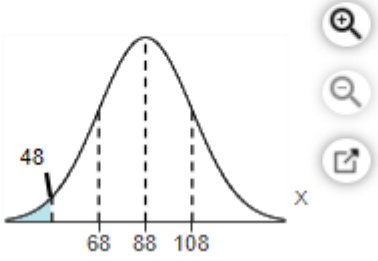
C.

(b) Choose the correct graph below.

A.



B.



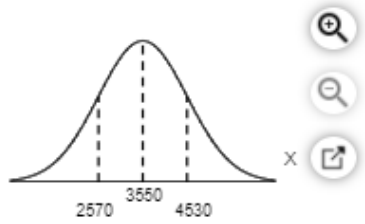
C.

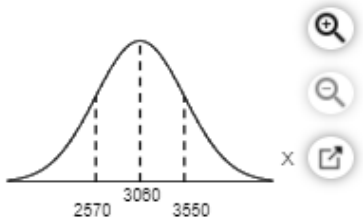
(c) Select the correct choice below and fill in the answer box to complete your choice.
(Type a whole number.)

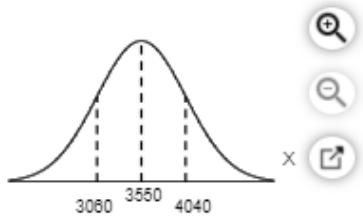
A. The probability is 0.1587 that a randomly selected cell phone plan in this population is less than \$ 48 per month.

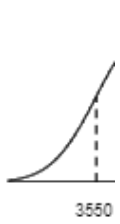
23) Suppose the birth weights of full-term babies are normally distributed with mean 3550 grams and standard deviation σ = 490 grams. (c) below.

(a) Draw a normal curve with the parameters labeled. Choose the correct graph below.

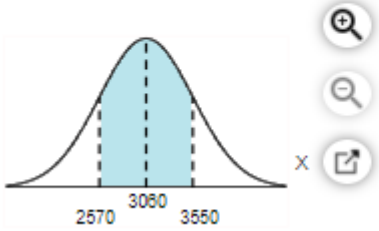
○ A. 

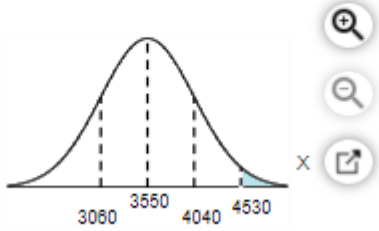
○ B. 

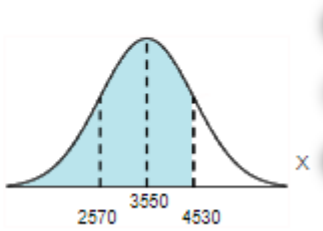
✔ C. 

○ D. 

(b) Shade the region that represents the proportion of full-term babies who weigh more than 4530 grams. Choo

○ A. 

✔ B. 

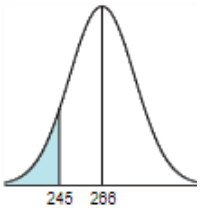
○ C. 

(c) Suppose the area under the normal curve to the right of X = 4530 is 0.0228. Provide an interpretation of this result. Select the correct choi fill in the answer box to complete your choice.
(Type a whole number.)

✔ A. The probability is 0.0228 that the birth weight of a randomly chosen full-term baby in this population is more than 4530 grams.

24) Suppose the lengths of human pregnancies are normally distributed with μ = 266 days and σ = 16 days. Complete parts (a) and (b) below.

(a) The figure to the right represents the normal curve with μ = 266 days and σ = 16 days. The area to the left of X = 245 is 0.0947. Provide two interpretations of this area.



Provide one interpretation of the area using the given values. Select the correct choice below and fill in the answer boxes to complete your choice.
(Type integers or decimals.)

○ A. The proportion of human pregnancies that last more than ____ days is ____.

✔ B. The proportion of human pregnancies that last less than 245 days is .0947.

Provide a second interpretation of the area using the given values. Select the correct choice below and fill in the answer boxes t choice.
(Type integers or decimals.)

○ A. The probability that a randomly selected human pregnancy lasts more than ____ days is ____.

✔ B. The probability that a randomly selected human pregnancy lasts less than 245 days is .0947.

Determine if the following statement is true or false.

The normal curve is symmetric about its mean, μ.

Choose the best answer below.

A. The statement is true. The normal curve is a symmetric distribution with one peak, which means the mean, median, and mode are all equal. Therefore, the normal curve is symmetric about the mean, μ.