7.2 Applications of Normal Distribution

MATH 241

.00

0.0808

0.0968

0.1151

0.1357

0.1587

-1.4

-1.3

-1.2

-1.1

.01

0.0778

0.0934

0.1112

0.1314

0.1539

0.0793

0.0951

0.1131

0.1335

0.1562

THOMPSON

- Determine the area under the standard normal curve that lies to the left of (a) Z = -1.17, (b) Z = -0.68, (c) Z = -0.27, and (d) Z = 1.18.
 - Click the icon to view a table of areas under the normal curve.
 - (a) The area to the left of Z = -1.17 is .1210 (Round to four decimal places as needed.)

(b) The area to the left of $Z = -0.68$ is	.2483				
(Round to four decimal places as needed.)					

(c) The area to the left of $Z = -0.27$ is	.3936 .
(Round to four decimal places as neede	ed.)

(d) The area to the left of Z = 1.18 is .8810.
(Round to four decimal places as needed.)

Table or statcrunch

Standard Normal Distribution

.04

0.0749

0.0901

0.1075

0.1271

0.1492

.05

0.0735

0.0885

0.1056

0.1251

0.1469

0.0721

0.0869

0.1038

.07

0.0708

0.0853

0.1210

.08

0.0694

0.0838

0.1003

0.1190

0.1401

.09

0.0681

0.0823

0.0985

0.1170

.03

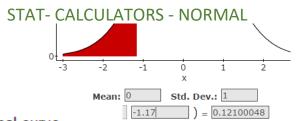
0.0764

0.0918

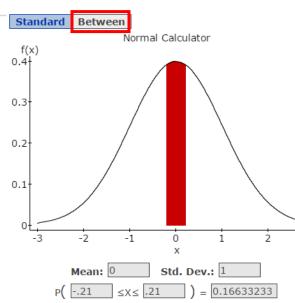
0.1093

0.1292

0.1515



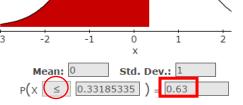
- 2) Click the icon to view a table of areas under the normal curve.
 - (a) The area that lies between Z = -0.21 and Z = 0.21 is .1663 . (Round to four decimal places as needed.)
 - (b) The area that lies between Z = -0.34 and Z = 0 is .1330 . (Round to four decimal places as needed.)
 - (c) The area that lies between Z = -0.51 and Z = 0.99 is .5339 . (Round to four decimal places as needed.)



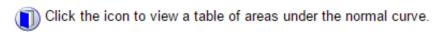
3)
Find the Z-score such that the area under the standard normal curve to the left is 0.63.

Click the icon to view a table of areas under the normal curve.

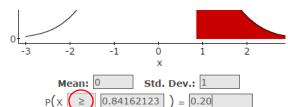
.33 is the Z-score such that the area under the curve to the left is 0.63. (Round to two decimal places as needed.)



Find the Z-score such that the area under the standard normal curve to the right is 0.20.



The approximate Z-score that corresponds to a right tail area of 0.20 is .84



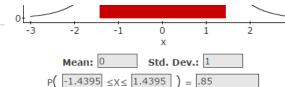
5)

Find the Z-scores that separate the middle 85% of the distribution from the area in the tails of the standard normal distribution.

STAT CRUNCH –CALCULATORS – NORMAL-BETWEEN

Click the icon to view a table of areas under the normal curve.

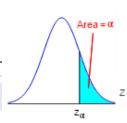
The Z-scores are - 1.44,1.44.



6)

The notation z_{lpha} is the z-score that the area under the standard normal curve to the right of z_{lpha} is _____

The notation z_{α} is the z-score that the area under the standard normal curve to the right of z_{α} is α .

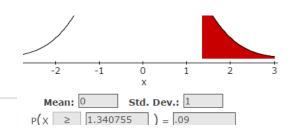


7) Find the value of z_{α} .

 $z_{0.09}$

Click the icon to view a table of areas under the normal curve.

 $z_{0.09} = 1.34$ (Round to two decimal places as needed.)



8) Find the value of z_{α} .

 $\alpha = 0.07$

Click the icon to view a table of areas under the normal curve.



The value of z_{0.07} is 1.48. (Round to two decimal places as needed.)

Assume the random variable X is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 7$. Compute the probability. Be sure to draw a normal curve with the area corresponding to the probability shaded.

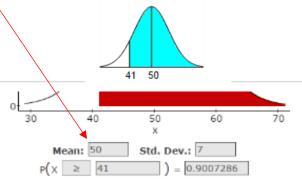
P(X > 41)

Click the icon to view a table of areas under the normal curve.

Which of the following normal curves corresponds to P(X > 41)?

$$P(X > 41) = .9007$$

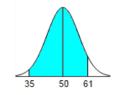
(Round to four decimal places as needed.)



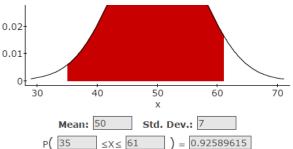
10)

Assume the random variable X is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 7$. Compute the probability. Be sure to draw a normal curve with the area corresponding to the probability shaded.

P(35 < X < 61)

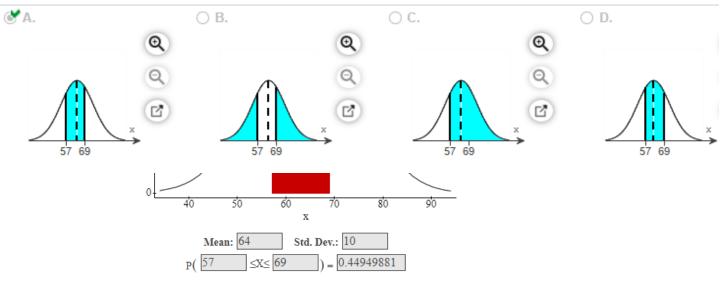


P(35 < X < 61) = .9259 (Round to four decimal places



Assume that the random variable X is normally distributed, with mean $\mu = 64$ and standard deviation $\sigma = 10$. Compute the proba 11) P(57 < X≤69). Be sure to draw a normal curve with the area corresponding to the probability shaded.

Click here to view the standard normal distribution table (page 1). Click here to view the standard normal distribution table (page 2).

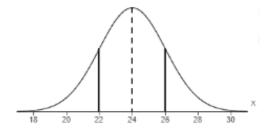


The probability P(57 < X ≤ 69) is .4495. (Round to four decimal places as needed.)

- 12) The mean incubation time for a type of fertilized egg kept at a certain temperature is 24 days. Suppose that the incubation times are approximately normally distributed with a standard deviation of 2 days. Complete parts (a) through (e) below. Click here to view the standard normal distribution table (page 1).
 Click here to view the standard normal distribution table (page 2).
 - (a) Draw a normal model that describes egg incubation times of these fertilized eggs.

Choose the correct graph below.

- Click here to view graph d.
- Olick here to view graph b.
- Click here to view graph a.
- Click here to view graph c.

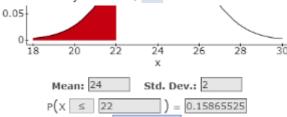


(b) Find and interpret the probability that a randomly selected fertilized egg hatches in less than 22 days.

The probability that a randomly selected fertilized egg hatches in less than 22 days is .1587 . (Round to four decimal places as needed.)

Interpret this probability. Select the correct choice below and fill in the answer box to complete your choice.

- A. In every group of 100 fertilized eggs, eggs will hatch in less than 22 days. (Round to the nearest integer as needed.)

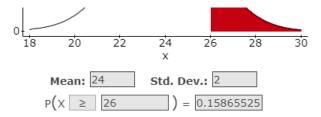


(c) Find and interpret the probability that a randomly selected fertilized egg takes over 26 days to hatch.

The probability that a randomly selected fertilized egg takes over 26 days to hatch is .1587 . (Round to four decimal places as needed.)

Interpret this probability. Select the correct choice below and fill in the answer box to complete your choice.

If 100 fertilized eggs were randomly selected, 16 of them would be expected to take more than 26 days to hatch.



- (d) BETWEEN
- (e) Mean: 24 Std. Dev.: 2 $P(X \le 18) = 0.0013499$

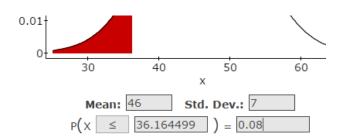
The probability of an egg hatching in less than 17 days is .0013, so it would be unusual, since the probability is less than 0.05.

Assume the random variable X is normally distributed, with mean $\mu = 46$ and standard deviation $\sigma = 7$. Find the 8th percentile.

The 8th percentile is 36.16.

(Round to two decimal places as needed.)

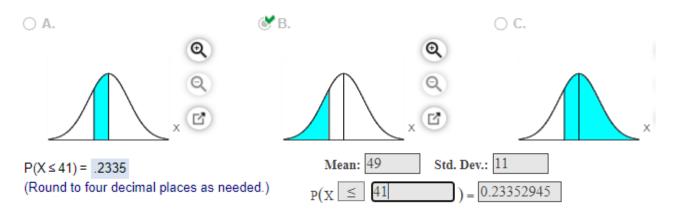
8th percentile means .08



Assume that the random variable X is normally distributed, with mean μ = 49 and standard deviation σ = 11. Co probability. Be sure to draw a normal curve with the area corresponding to the probability shaded.

$$P(X \le 41)$$

Which of the following shaded regions corresponds to $P(X \le 41)$?



15) Scores on a statistics test were normally distributed with a mean of 75 points and a standard deviation of 8 points. What proportion of the scores were below 65 points?

Mean: 75 Std. Dev.: 8

P(X ≤ 65) = 0.10564977

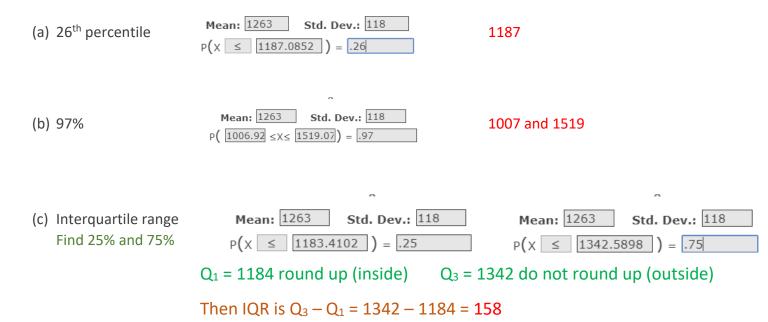
Assume the random variable X is normally distributed, with mean μ = 40 and standard deviation σ = 9. Find the 9th percentile.

The 9th percentile is 27.93. (Round to two decimal places as needed.)

Mean: 40 Std. Dev.: 9 $P(X \le 27.933205) = .09$

 17) The number of chocolate chips in an 18-ounce bag of chocolate chip cookies is approximately normally distributed with mean 1252 and standard deviation 129 chips. (a) What is the probability that a randomly selected bag contains between 1000 and 1500 chocolate chips? (b) What is the probability that a randomly selected bag contains fewer than 1125 chocolate chips? (c) What proportion of bags contains more than 1175 chocolate chips? (d) What is the percentile rank of a bag that contains 1050 chocolate chips? 								
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	(a) The probability that a randomly selected bag contains between 1000 and 1500 chocolate chips is .9473 . (Round to four decimal places as needed.)							
	(b) The probability that a randomly selected bag contains fewer than 1125 chocolate chips is .1624. (Round to four decimal places as needed.)							
	(c) The proportion of bags that contains more than 1175 chocolate chips is .7247. (Round to four decimal places as needed.)							
	(d) A bag that contains 1050 chocolate chips is in the 6 th percentile. Mean: 1252 Std. Dev.: 129 (Round to the nearest integer as needed.) $P(X \le 1050) = 0.05868701$							
18)	 The lengths of a particular animal's pregnancies are approximately normally distributed, with mean μ = 255 days and standard deviation σ = 12 days. (a) What proportion of pregnancies lasts more than 270 days? (b) What proportion of pregnancies lasts between 237 and 264 days? (c) What is the probability that a randomly selected pregnancy lasts no more than 243 days? (d) A "very preterm" baby is one whose gestation period is less than 228 days. Are very preterm babies unusual? 							
	() The second of the second o							
	(a) The proportion of pregnancies that last more than 270 days is .1056. (Round to four decimal places as needed.)							
(b) The proportion of pregnancies that last between 237 and 264 days is .7066. (Round to four decimal places as needed.)								
(c) The probability that a randomly selected pregnancy lasts no more than 243 days is .1587. (Round to four decimal places as needed.)								
	(d) A "very preterm" baby is one whose gestation period is less than 228 days. Are very preterm babies unusual?							
	The probability of this event is .0122, so it would be unusual because the probability is less than 0.05. (Round to four decimal places as needed.)							
19) The mean gas mileage for a hybrid car is 57 miles per gallon. Suppose that the gasoline mileage is approxim normally distributed with a standard deviation of 3.5 miles per gallon. (a) What proportion of hybrids gets over miles per gallon? (b) What proportion of hybrids gets 53 miles per gallon or less? (c) What proportion of hybrid gets between 59 and 61 miles per gallon? (d) What is the probability that a randomly selected hybrid gets less than 46 miles per gallon?								
	Click the icon to view a table of areas under the normal curve.							
	(a) The proportion of hybrids that gets over 60 miles per gallon is .1957 . (Round to four decimal places as needed.)							
	(b) The proportion of hybrids that gets 53 miles per gallon or less is .1265 . (Round to four decimal places as needed.)							

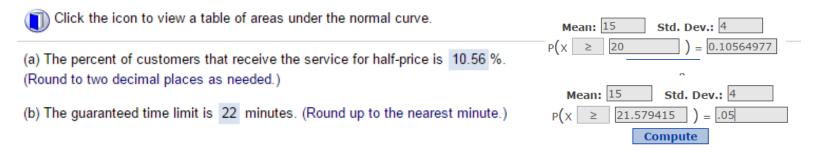
- (c) The proportion of hybrids that gets between 59 and 61 miles per gallon is .1573 . (Round to four decimal places as needed.)
- (d) The probability that a randomly selected hybrid gets less than 46 miles per gallon is .0008. (Round to four decimal places as needed.)
- 20) The number of chocolate chips in a bag of chocolate chip cookies is approximately normally distributed with mean 1263 and a standard deviation of 118.
 - (a) Determine the 26th percentile for the number of chocolate chips in a bag.
 - (b) Determine the number of chocolate chips in a bag that make up the middle 97% of bags.
 - (c) What is the interquartile range of the number of chocolate chips in a bag of chocolate chip cookies? Click here to view the standard normal distribution table (page 1). Click here to view the standard normal distribution table (page 2).



The time required for an automotive center to complete an oil change service on an automobile approximately follows a normal distribution, with a mean of 15 minutes and a standard deviation of 4 minutes.

- (a) The automotive center guarantees customers that the service will take no longer than 20 minutes. If it does take longer, the customer will receive the service for half-price. What percent of customers receive the service for half-price?

 (b) If the automotive center does not want to give the discount to more than 5% of its customers, how long should it make the
- (b) If the automotive center does not want to give the discount to more than 5% of its customers, how long should it make the guaranteed time limit?



There are two college entrance exams that are often taken by students, Exam A and Exam B. The composite score on Exam A is approximately normally distributed with mean 21 and standard deviation 4.6. The composite score on Exam B is approximately normally distributed with mean 1025 and standard deviation 203. Suppose you scored 23 on Exam A and 1225 on Exam B. Which exam did you score better on? Justify your reasoning using the normal model.

Choose the correct answer below.

$\cap \wedge$	The score on Evam A is	hotter because the	difference between t	he score and the mean	is lower than it is for Exam E	R
	. The score on ⊏xam A is	s better, because the	amerence between t	ne score and the mean	is lower than it is for exam t	Δ

- B. The score on Exam A is better, because the percentile for the Exam A score is higher.

FXTRA PROBLEMS:

Steel rods are manufactured with a mean length of 20 centimeter (cm). Because of variability in the manufacturing process, the lengths of the rods are approximately normally distributed with a standard deviation of 0.09 cm.



Click the icon to view a table of areas under the normal curve.

(a) What proportion of rods has a length less than 19.9 cm?

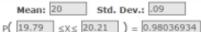
0.1333 (Round to four decimal places as needed.)

(b) Any rods that are shorter than 19.79 cm or longer than 20.21 cm are discarded. What proportion of rods will be discarded?

STAT -CALCULATORS - NORMAL - (PUT BETWEEN 19.79 AND 20.21)

0.0196 (Round to four decimal places as needed.)

$$1 - 0.98036934 = .0196$$



(c) Using the results of part (b), if 5000 rods are manufactured in a day, how many should the plant manager expect to discard?

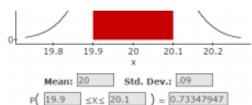
98

(Use the answer from part b to find this answer. Round to the nearest integer as needed.)

(d) If an order comes in for 10,000 steel rods, how many rods should the plant manager expect to manufacture if the order states that all rods must be between 19.9 cm and 20.1 cm?

13,634 (Round up to the nearest integer.)

Explanation: x is what to order to end up with 10,000



Put in calculator:

$$X = \frac{10000}{0.733479} = 13633.6555$$

The number of chocolate chips in a bag of chocolate chip cookies is approximately normally distributed with mean 1263 and a standard deviation of 118. (a) Determine the 30th percentile for the number of chocolate chips in a bag. (b) Determine the number of chocolate chips in a bag that make up the middle 97% of bags.

Click the icon to view a table of areas under the normal curve.

(a) The 30th percentile for the number of chocolate chips in a bag of chocolate chip cookies is 1201 chocolate chips.

(b) The number of chocolate chips in a bag that make up the middle 97% of bags is 1007 to 1519 chocolate chips.

STAT – CALC – NORMAL - BETWEEN

(Round to the nearest whole number as needed.)

Mean: 1263 Std. Dev.: 118

P($1006.92 \le X \le 1519.07$) = .97

The mean incubation time for a type of fertilized egg kept at 100.2°F is 20 days. Suppose that the incubation times are approximately normally distributed with a standard deviation of 2 days.

- (a) What is the probability that a randomly selected fertilized egg hatches in less than 16 days?
- (b) What is the probability that a randomly selected fertilized egg takes over 24 days to hatch?
- (c) What is the probability that a randomly selected fertilized egg hatches between 18 and 20 days?
- (d) Would it be unusual for an egg to hatch in less than 15 days? Why?
- Click the icon to view a table of areas under the normal curve.
- (a) The probability that a randomly selected fertilized egg hatches in less than 16 days is .0228. (Round to four decimal places as needed.)
- (b) The probability that a randomly selected fertilized egg takes over 24 days to hatch is .0228.
 (Round to four decimal places as needed.)
- (c) The probability that a randomly selected fertilized egg hatches between 18 and 20 days is .3413 . (Round to four decimal places as needed.)
- (d) Would it be unusual for an egg to hatch in less than 15 days? Why?

The probability of this event is .0062, so it would be unusual because the probability is less than 0.05. (Round to four decimal places as needed.)

A study found that the mean amount of time cars spent in drive-throughs of a certain fast-food restaurant was 137.9 seconds. Assuming drive-through times are normally distributed with a standard deviation of 30 seconds, complete parts (a) through (d) below.

Click here to view the standard normal distribution table (page 1). Click here to view the standard normal distribution table (page 2).

STATS - CALCULATORS - NORMAL

...

(a) What is the probability that a randomly selected car will get through the restaurant's drive-through in less than 85 seconds?

The probability that a randomly selected car will get through the restaurant's drive-through in less than 85 seconds is (Round to four decimal places as needed.)

(b) What is the probability that a randomly selected car will spend more than 187 seconds in the restaurant's drive-through?

The probability that a randomly selected car will spend more than 187 seconds in the restaurant's drive-through is (Round to four decimal places as needed.)

(c) What proportion of cars spend between 2 and 3 minutes in the restaurant's drive-through?

use 120 and 180

The proportion of cars that spend between 2 and 3 minutes in the restaurant's drive-through is 0.6444. for seconds (Round to four decimal places as needed.)

(d) Would it be unusual for a car to spend more than 3 minutes in the restaurant's drive-through? Why?

The probability that a car spends more than 3 minutes in the restaurant's drive-through is 0.0803, so it would not be unusual, since the probability is greater than 0.05. (Round to four decimal places as needed.)