8.1 Distribution of the Sample Mean MATH 241 THOMPSON

1) Consider a random variable X that is normally distributed. Complete parts (a) through (d) below.

(This is a reading assessment question. Be certain of your answer because you only get one attempt on this question.

(a) If a random variable X is normally distributed, what will be the shape of the distribution of the sample mean?

✓ Normal

(b) If the mean of a random variable X is 50, what will be the mean of the sampling distribution of the sample mean?

$\mu_{\bar{x}} = 50$

(c) As the sample size n increases, what happens to the standard error of the mean?

- A. The standard error of the mean remains the same.
- ✓ B. The standard error of the mean decreases.
- C. The standard error of the mean increases.

(d) If the standard deviation of a random variable X is 10 and a random sample of size n = 21 is obtained, what is the standard deviation of the sampling distribution of the sample mean?

$\sigma_{\bar{x}} = \dfrac{10}{\sqrt{21}}$ (Type an exact answer, using radicals as needed.)

For a new sample size

$$\sigma = \frac{\sigma}{\sqrt{n}}$$

2) A simple random sample of size n = 41 is obtained from a population with μ = 36 and σ = 8. Does the population need to be normally distributed for the sampling distribution of $\bar{x}$ to be approximately normally distributed? Why? What is the sampling distribution of $\bar{x}$?

Does the population need to be normally distributed for the sampling distribution of $\bar{x}$ to be approximately normally distributed? Why?

- A. Yes because the Central Limit Theorem states that only for underlying populations that are normal is the shape of the sampling distribution of $\bar{x}$ normal, regardless of the sample size, n.
- ✓ B. No because the Central Limit Theorem states that regardless of the shape of the underlying population, the sampling distribution of $\bar{x}$ becomes approximately normal as the sample size, n, increases.

What is the sampling distribution of $\bar{x}$? Select the correct choice below and fill in the answer boxes within your choice.
(Type integers or decimals rounded to three decimal places as needed.)

✓ A. The sampling distribution of $\bar{x}$ is normal or approximately normal with $\mu_{\bar{x}} = 36$ and $\sigma_{\bar{x}} = 1.249$.

3) Determine $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ from the given parameters of the population and sample size

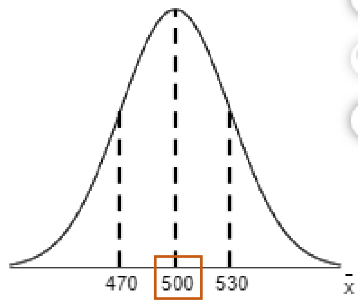
μ = 83, σ = 24, n = 36

$\mu_{\bar{x}} = 83$

$\sigma_{\bar{x}} = 4$ $\qquad \sigma = \frac{\sigma}{\sqrt{n}} \qquad \sigma = \frac{24}{\sqrt{36}} = 4$

4) Complete parts **(a)** through **(d)** for the sampling distribution of the sample mean shown in the accompanying graph.

Click the icon to view the graph.



(a) What is the value of $\mu_{\bar{x}}$?

The value of $\mu_{\bar{x}}$ is 500.

(b) What is the value of $\sigma_{\bar{x}}$?

The value of $\sigma_{\bar{x}}$ is 30.

(c) If the sample size is n = 9, what is likely true about the shape of the population?

A. The shape of the population is skewed right.

B. The shape of the population is approximately normal. ✔

$$\sigma = \frac{\sigma}{\sqrt{n}}$$

(d) If the sample size is n = 9, what is the standard deviation of the population from which the sample was drawn?

The standard deviation of the population from which the sample was drawn is 90.

$$30 = \frac{\sigma}{\sqrt{9}} = 30 \cdot 3 = 90$$

5) Suppose a simple random sample of size n = 36 is obtained from a population with μ = 81 and σ = 6.

(a) Describe the sampling distribution of $\bar{x}$.

(b) What is $P(\bar{x} > 82.15)$?

(c) What is $P(\bar{x} \le 78.7)$?

(d) What is $P(80.5 < \bar{x} < 83.5)$?

(a) Choose the correct description of the shape of the sampling distribution of $\bar{x}$.

A. The distribution is uniform.

B. The distribution is approximately normal. ✔

Find the mean and standard deviation of the sampling distribution of $\bar{x}$.

$\mu_{\bar{x}}$ = 81

$\sigma_{\bar{x}}$ = 1

$$\sigma = \frac{\sigma}{\sqrt{n}} \qquad \sigma = \frac{6}{\sqrt{36}} = 1$$

(b) $P(\bar{x} > 82.15)$ = .1251 (Round to four decimal places as needed.)

(c) $P(\bar{x} \le 78.7)$ = .0107 (Round to four decimal places as needed.)

(d) $P(80.5 < \bar{x} < 83.5)$ = .6853 (Round to four decimal places as nee

STATS CALCULATORS NORMAL

Mean: 81 Std. Dev.: 1

P(X ≥ 82.15) = 0.12507194

BETWEEN

Mean: 81 Std. Dev.: 1

P(80.5 ≤X≤ 83.5) = 0.6852528

6) Suppose a simple random sample of size n = 45 is obtained from a population with $\mu = 68$ and $\sigma = 19$.

(a) What must be true regarding the distribution of the population in order to use the normal model to compute probabilities regarding the sample mean? Assuming the normal model can be used, describe the sampling distribution $\bar{x}$.
(b) Assuming the normal model can be used, determine $P(\bar{x} < 71.7)$.
(c) Assuming the normal model can be used, determine $P(\bar{x} \geq 69.3)$.

For a new sample size

$$\sigma = \frac{\sigma}{\sqrt{n}}$$

B. Since the sample size is large enough, the population distribution does not need to be normal.

C. Approximately normal, with $\mu_{\bar{x}} = 68$ and $\sigma_{\bar{x}} = \frac{19}{\sqrt{45}} = 2.832$

STATS CALCULATORS NORMAL

(b) $P(\bar{x} < 71.7) =$ .9043 (Round to four decimal places as needed.)

Mean: 68 Std. Dev.: 2.832
P(X ≤ 71.7) = 0.90430823
Compute

(c) $P(\bar{x} \geq 69.3) =$ .3231 (Round to four decimal places as needed.)

Mean: 68 Std. Dev.: 2.832
P(X ≥ 69.3) = 0.32310288
Compute

7) Suppose the lengths of the pregnancies of a certain animal are approximately normally distributed with mean $\mu = 293$ days and standard deviation $\sigma = 21$ days. Complete parts (a) through (f) below.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

STATS CALCULATORS NORMAL

(a) What is the probability that a randomly selected pregnancy lasts less than 285 days?

The probability that a randomly selected pregnancy lasts less than 285 days is approximately .3516. (Round to four decimal places as needed.)

Mean: 293 Std. Dev.: 21
P(X ≤ 285) = 0.35161929

Interpret this probability. Select the correct choice below and fill in tl
(Round to the nearest integer as needed.)

C. If 100 pregnant individuals were selected independently from this population, we would expect 35 pregnancies to last less than 285 days.

(b) Suppose a random sample of 19 pregnancies is obtained. Describe the sampling distribution of the sample mean length of pregnancies.

The sampling distribution of $\bar{x}$ is normal with $\mu_{\bar{x}} = 293$ and $\sigma_{\bar{x}} = 4.8177$.
(Round to four decimal places as needed.)

$$\sigma = \frac{21}{\sqrt{19}} = 4.8177$$

(c) What is the probability that a random sample of 19 pregnancies has a mean gestation period of 285 days or less?

The probability that the mean of a random sample of 19 pregnancies is less than 285 days is approximately .0484.
(Round to four decimal places as needed.)

Mean: 293 Std. Dev.: 4.8177
P(X ≤ 285) = 0.04840259

Interpret this probability. Select the correct choice below and fill in the ans
(Round to the nearest integer as needed.)

A. If 100 independent random samples of size n = 19 pregnancies were obtained from this population, we would expect 5 sample(s) to have a sample mean of 285 days or less.

(d) What is the probability that a random sample of 44 pregnancies has a mean gestation period of 285 days or less?

The probability that the mean of a random sample of 44 pregnancies is less than 285 days is approximately .0058 .
(Round to four decimal places as needed.)

new $\sigma = \frac{21}{\sqrt{44}} = 3.1659$

Mean: 293 Std. Dev.: 3.1659

P(X ≤ 285) = 0.00575326

Compute

Interpret this probability. Select the correct choice below and fill in the answer box within your choice.
(Round to the nearest integer as needed.)

multiply you answer by 100 and round to whole number

A. If 100 independent random samples of size n = 44 pregnancies were obtained from this population, we would expect 1 sample(s) to have a sample mean of 285 days or less.

0.0057 rounds to 1

(e) What might you conclude if a random sample of 44 pregnancies resulted in a mean gestation period of 285 days or less?

P < 0.05 is an unusual event

This result would be unusual, so the sample likely came from a population whose mean gestation period is less than 293 days.

(f) What is the probability a random sample of size 15 will have a mean gestation period within 9 days of the mean?

The probability that a random sample of size 15 will have a mean gestation period within 9 days of the mean is .9030 .
(Round to four decimal places as needed.)

$\sigma = \frac{21}{\sqrt{15}} = 5.42277$

Mean: 293 Std. Dev.: 5.42277

BETWEEN --- add and subtract 9 from the mean

P(284 ≤X≤ 302) = 0.90301882

293 – 9 293 + 9

8) The reading speed of second grade students in a large city is approximately normal, with a mean of 88 words per minute (wpm) and a standard deviation of 10 wpm. Complete parts (a through (f).

STATS CALCULATORS NORMAL

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) What is the probability a randomly selected student in the city will read more than 93 words per minute?

Mean: 88 Std. Dev.: 10

The probability is .3085 .
(Round to four decimal places as needed.)

P(X ≥ 93) = 0.30853754

C. If 100 different students were chosen from this population, we would expect 31 to read more than 93 words per minute.

(b) What is the probability that a random sample of 10 second grade students from the city results in a mean reading rate of more than 93 words per minute?

Mean: 88 Std. Dev.: 3.1623

The probability is .0569 .

$\frac{10}{\sqrt{10}} = 3.1623$

P(X ≥ 93) = 0.05692443

multiply you answer by 100 and round to whole number 0.0596 is 6

B. If 100 different samples of n = 10 students were chosen from this population, we would expect 6 sample(s) to have a sample mean reading rate of more than 93 words per minute.

(c) What is the probability that a random sample of 20 second grade students from the city results in a mean reading rate of more than 93 words per minute?

$$\frac{10}{\sqrt{20}} = 2.23607$$

Mean: 88 **Std. Dev.:** 2.23607

P(X ≥ 93) = 0.01267373

The probability is .0127 .

(Round to four decimal places as needed.)

Interpret this probability. Select the correct choice below and fill in the answer box within your choice.

multiply you answer by 100 and round to whole number 0.0127 is 1

A. If 100 different samples of n = 20 students were chosen from this population, we would expect 1 sample(s) to have a sample mean reading rate of more than 93 words per minute.

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result.

D. Increasing the sample size decreases the probability because $\sigma_{\bar{x}}$ decreases as n increases. check your sample size and probability, it may be decrease and decrease

(e) A teacher instituted a new reading program at school. After 10 weeks in the program, it was found that the mean reading speed of a random sample of 19 second grade students was 90.9 wpm. What might you conclude based on this result? Select the correct choice below and fill in the answer boxes within your choice.

(Type integers or decimals rounded to four decimal places as needed.)

$$\frac{10}{\sqrt{19}} = 2.2942$$

Mean: 88 **Std. Dev.:** 2.2942

P(X ≥ 90.9) = 0.10310475

B. A mean reading rate of 90.9 wpm is not unusual since the probability of obtaining a result of 90.9 wpm or more is .1031 . This means that we would expect a mean reading rate of 90.9 or higher from a population whose mean reading rate is 88 in 10 of every 100 random samples of size n = 19 students. The new program is not abundantly more effective than the old program.

(f) There is a 5% chance that the mean reading speed of a random sample of 25 second grade students will exceed what value?

$$\frac{10}{\sqrt{25}} = 2$$

There is a 5% chance that the mean reading speed of a random sample of 25 second grade students will exceed 91.29 wpm. (Round to two decimal places as needed.)

Mean: 88 **Std. Dev.:** 2

P(X ≥ 91.289707) = 0.05

leave blank put 0.05 for 5%

9) Suppose a geyser has a mean time between eruptions of 76 minutes. If the interval of time between the eruptions is normally distributed with standard deviation 20 minutes, answer the following questions.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

Mean: 76 Std. Dev.: 20
P(X ≥ 86) = 0.30853754

(a) What is the probability that a randomly selected time interval between eruptions is longer than 86 minutes?

The probability that a randomly selected time interval is longer than 86 minutes is approximately .3085. (Round to four decimal places as needed.)

(b) What is the probability that a random sample of 10 time intervals between eruptions has a mean longer than 86 minutes?

$$\frac{20}{\sqrt{10}} = 6.324$$

Mean: 76 Std. Dev.: 6.324
P(X ≥ 86) = 0.05690728

The probability that the mean of a random sample of 10 time intervals is more than 86 minutes is approximately .0569. (Round to four decimal places as needed.)

(c) What is the probability that a random sample of 16 time intervals between eruptions has a mean longer than 86 minutes?

The probability that the mean of a random sample of 16 time intervals is more than 86 minutes is approximately .0228. (Round to four decimal places as needed.)

$$\frac{20}{\sqrt{16}} = 5$$

Mean: 76 Std. Dev.: 5
P(X ≥ 86) = 0.02275013

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result. Choose the correct answer below.

D. The probability decreases because the variability in the sample mean decreases as the sample size increases.

(e) What might you conclude if a random sample of 16 time intervals between eruptions has a mean longer than 86 minutes? Choose the best answer below.

A. The population mean may be greater than 76.

D. The population mean is 75 minutes, and this is just a rare sampling.

(f) same as #8 part (f)

10) The sampling distribution of $\bar{x}$ has mean $\mu_{\bar{x}}$ = μ and standard deviation $\sigma_{\bar{x}}$ = $\frac{\sigma}{\sqrt{n}}$.

11) The standard deviation of the sampling distribution of $\bar{x}$, denoted $\sigma_{\bar{x}}$, is called the standard error of the mean.

The distribution of the sample mean, $\bar{x}$, will be normally distributed if the sample is obtained from a population that is normally distributed, regardless of the sample size.

Choose the correct answer below.

- ○ False
- ✔ True

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