8.2 Distribution of the Sample Proportion MATH 241 THOMPSON

1) Fill in the blanks to complete the following statements.

(a) For the shape of the distribution of the sample proportion to be approximately normal, it is required that $np(1-p) \geq$ _____.

(b) Suppose the proportion of a population that has a certain characteristic is 0.9. The mean of the sampling distribution of $\hat{p}$ from this population is $\mu_{\hat{p}}$ = _____.

(a) For the shape of the distribution of the sample proportion to be approximately normal, it is required that $np(1-p) \geq$ 10 .

(Type an integer or a decimal.)

(b) Suppose the proportion of a population that has a certain characteristic is 0.9. The mean of the sampling distribution of $\hat{p}$ from this population is $\mu_{\hat{p}}$ = .9 .

2) Describe the sampling distribution of $\hat{p}$. Assume the size of the population is 15,000.

$n = 400, p = 0.5$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

✔ D. Approximately normal because $n \leq 0.05N$ and $np(1-p) \geq 10$.

Determine the mean of the sampling distribution of $\hat{p}$.

$\mu_{\hat{p}}$ = .5 (Round to one decimal place as needed.)

Determine the standard deviation of the sampling distribution of $\hat{p}$. $\sqrt{\frac{.5(1-.5)}{400}}$

$\sigma_{\hat{p}}$ = .025 (Round to three decimal places as needed.)

3) Suppose a simple random sample of size n = 75 is obtained from a population whose size is N = 25,000 and whose population proportion with a specified characteristic is p = 0.8.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

✔ D. Approximately normal because $n \leq 0.05N$ and $np(1-p) \geq 10$.

$\mu_{\hat{p}}$ = .8 (Round to one decimal place as needed.)

Determine the standard deviation of the sampling distribution of $\hat{p}$. $\sqrt{\frac{.8(1-.8)}{75}}$

$\sigma_{\hat{p}}$ = .046188 (Round to six decimal places as needed.)

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(b) What is the probability of obtaining x = 66 or more individuals with the characteristic? That is, what is $P(\hat{p} \geq 0.88)$?

$P(\hat{p} \geq 0.88)$ = .0416 (Round to four decimal places as needed.)

Mean: .8 Std. Dev.: .046188

P(X ≥ .88) = 0.04163219

(c) What is the probability of obtaining x = 57 or fewer individuals with the characteristic? That is, what is $P(\hat{p} \leq 0.76)$?

$P(\hat{p} \leq 0.76)$ = .1932 (Round to four decimal places as needed.)

Mean: .8 Std. Dev.: .046188

P(X ≤ .76) = 0.193238

4) Suppose a simple random sample of size n = 1000 is obtained from a population whose size is N = 2,000,000 a whose population proportion with a specified characteristic is p = 0.25. Complete parts (a) through (c) below.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) Describe the sampling distribution of $\hat{p}$.

A. Approximately normal, $\mu_{\hat{p}} = 0.25$ and $\sigma_{\hat{p}} \approx 0.0137$

$\sqrt{\frac{.25(1-.25)}{1000}} = .0137$

(b) What is the probability of obtaining x = 290 or more individuals with the characteristic?

P(x ≥ 290) = .0018 (Round to four decimal places as needed.)

Mean: .25 Std. Dev.: .0137
P(X ≥ .29) = 0.0017518

(c) What is the probability of obtaining x = 220 or fewer individuals with the characteristic?

P(x ≤ 220) = .0143 (Round to four decimal places as needed.)

Mean: .25 Std. Dev.: .0137
P(X ≤ .22) = 0.01427006

5) According to a study, the proportion of people who are satisfied with the way things are going in their lives is 0.88. Suppose that a random sample of 100 people is obtained. Complete parts (a) through (e) below.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) Suppose the random sample of 100 people is asked, "Are you satisfied with the way things are going in your life?" Is the response to this question qualitative or quantitative? Explain.

A. The response is qualitative because the responses can be classified based on the characteristic of being satisfied or not.

(b) Explain why the sample proportion, $\hat{p}$, is a random variable. What is the source of the variability?

A. The sample proportion $\hat{p}$ is a random variable because the value of $\hat{p}$ varies from sample to sample. The variability is due to the fact that different people feel differently regarding their satisfaction.

(c) Describe the sampling distribution of $\hat{p}$, the proportion of people who are satisfied with the way things are going in their life. Be sure to verify the model requirements.

Since the sample size is no more than 5% of the population size and np(1 − p) = 10.56 ≥ 10, the distribution of $\hat{p}$ is approximately normal with $\mu_{\hat{p}}$ = .88 and $\sigma_{\hat{p}}$ = .033.
(Round to three decimal places as needed.)

.88·100(1-.88)

$\sqrt{\frac{.88(1-.88)}{100}} = .033$

(d) In the sample obtained in part (a), what is the probability that the proportion who are satisfied with the way things are going in their life exceeds 0.91?

The probability that proportion who are satisfied with the way things are going in their life exceeds 0.91 is .1817.
(Round to four decimal places as needed.)

Mean: .88 Std. Dev.: .033
P(X ≥ .91) = 0.18165107

(e) Using the distribution from part (c), would it be unusual for a survey of 100 people to reveal that 81 or fewer people in the sample are satisfied with their lives?

The probability that 81 or fewer people in the sample are satisfied is .0170, which is unusual because this probability is less than 5 %.
(Round to four decimal places as needed.)

Mean: .88 Std. Dev.: .033
P(X ≤ .81) = 0.01695198

6) According to a survey in a country, 24% of adults do not own a credit card. Suppose a simple random sample of 300 adults is obtained. Complete parts (a) through (d) below.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) Describe the sampling distribution of $\hat{p}$, the sample proportion of adults who do not own a credit card. Choose the phrase that best describes the shape of the sampling distribution of $\hat{p}$ below.

- A. Approximately normal because $n \le 0.05N$ and $np(1-p) < 10$
- B. Approximately normal because $n \le 0.05N$ and $np(1-p) \ge 10$

Determine the mean of the sampling distribution of $\hat{p}$.

$\mu_{\hat{p}} =$ .24 (Round to two decimal places as needed.)

Determine the standard deviation of the sampling distribution of $\hat{p}$.

$\sigma_{\hat{p}} =$ .025 (Round to three decimal places as needed.)

(b) What is the probability that in a random sample of 300 adults, more than 26% do not own a credit card?

The probability is .2119.
(Round to four decimal places as needed.)

Mean: .24 Std. Dev.: .025
P(X ≥ .26) = 0.2118554

Interpret this probability.

If 100 different random samples of 300 adults were obtained, one would expect 21 to result in more than 26% not owning a credit card.

(c) What is the probability that in a random sample of 300 adults, between 21% and 26% do not own a credit card?

The probability is .6731.
(Round to four decimal places as needed.)

Mean: .24 Std. Dev.: .025
P(.21 ≤X≤ .26) = 0.67307493

Interpret this probability.

If 100 different random samples of 300 adults were obtained, one would expect 67 to result in between 21% and 26% not owning a credit card.

(d) Would it be unusual for a random sample of 300 adults to result in 63 or fewer who do not own a credit card? Why? Select the correct choice below and fill in the answer box to complete your choice.
(Round to four decimal places as needed.)

- A. The result is unusual because the probability that $\hat{p}$ is less than or equal to the sample proportion is ____, which is greater than 5%.
- B. The result is unusual because the probability that $\hat{p}$ is less than or equal to the sample proportion is ____, which is less than 5%.
- C. The result is not unusual because the probability that $\hat{p}$ is less than or equal to the sample proportion is .1151, which is greater than 5%.

$\frac{63}{300} = .21$

Mean: .24 Std. Dev.: .025
P(X ≤ .21) = 0.11506967

7) According to a study conducted by an organization, the proportion of Americans who were afraid to fly in 2006 was 0.10. A random sample of 1,200 Americans results in 132 indicating that they are afraid to fly. Explain why this is not necessarily evidence that the proportion of American who are afraid to fly has increased.

$\frac{132}{1200} = .11$

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

Mean: .1 Std. Dev.: .00866

P(X ≥ .11) = 0.1240996

size n is more than 5% of the population.

C. This is not necessarily evidence that the proportion of Americans who are afraid to fly has increased above 0.10 because the probability of obtaining a value equal to or more extreme than the sample proportion is .1241, which is not unusual. (Round to four decimal places as needed.)

Another #7

$\sqrt{\frac{.1(1-.1)}{1200}} = .00866$

$\sqrt{\frac{.1(1-.1)}{1100}} = .009$

According to a study conducted by an organization, the proportion of Americans who were afraid to fly in 2006 was 0.10. A random sample of 1,100 Americans results in 99 indicating that they are afraid to fly. Explain why this is not necessarily evidence that the proportion of Americans who are afraid to fly has decreased.

Mean: .1 Std. Dev.: .009

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

P(X ≤ .09) = 0.13326026

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$\frac{99}{1100} = .09$

C. This is not necessarily evidence that the proportion of Americans who are afraid to fly has decreased below 0.10 because the probability of obtaining a value equal to or more extreme than the sample proportion is .1333, which is not unusual. (Round to four decimal places as needed.)

8) The sample proportion, denoted $\hat{p}$, is given by the formula $\hat{p} = \frac{x}{n}$, where x is the number of individuals with a specified characteristic in a sample of n individuals.

9) *True or False*: The population proportion and sample proportion always have the same value.

Choose the correct answer below.

- False
- True

10) Is the statement below true or false?

The mean of the sampling distribution of $\hat{p}$ is p.

Choose the correct answer below.

- True
- False

According to a study conducted by a statistical organization, the proportion of Americans who are satisfied with the way things are going in their lives is 0.82. Suppose that a random sample of 105 Americans is obtained. Complete parts (a) through (c).

(a) Describe the sampling distribution of $\hat{p}$. Choose the correct answer below.

- A. The distribution of $\hat{p}$ cannot be approximated by the normal distribution. Instead, a binomial distribution with n = 105 and p = 0.82 should be used.
- B. The distribution of $\hat{p}$ can be approximated by a normal distribution with $\mu_{\hat{p}} = 0.82$ and $\sigma_{\hat{p}} = 0.074986$.
- C. The distribution of $\hat{p}$ is uniform.
- ✔ D. The distribution of $\hat{p}$ can be approximated by a normal distribution with $\mu_{\hat{p}} = 0.82$ and $\sigma_{\hat{p}} = 0.037493$.

(b) Using the distribution from part (a), what is the probability that at least 82 Americans in the sample are satisfied with their lives?

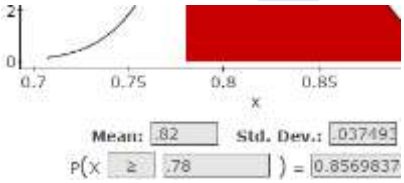
The probability that at least 82 Americans in the sample are satisfied is .8570.
(Round to four decimal places as needed.)

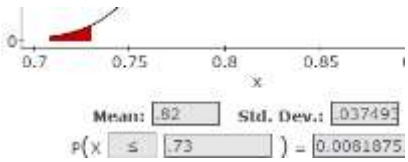
(c) Using the distribution from part (a), what is the probability that 77 or fewer Americans in the sample are satisfied with their lives?

The probability that 77 or fewer Americans in the sample are satisfied is .0082.
(Round to four decimal places as needed.)

$$x \geq \frac{82}{105} = .78$$

$$x \leq \frac{77}{105} = .73$$





According to a survey in a country, 37% of adults do not have any credit cards. Suppose a simple random sample of 600 adults is obtained.

(a) Describe the sampling distribution of $\hat{p}$, the sample proportion of adults who do not have a credit card. Choose the phrase that best describes the shape of the sampling distribution of $\hat{p}$ below.

- A. Not normal because $n \leq 0.05N$ and $np(1-p) \geq 10$.
- B. Not normal because $n \leq 0.05N$ and $np(1-p) < 10$.
- ✔ C. Approximately normal because $n \leq 0.05N$ and $np(1-p) \geq 10$.
- D. Approximately normal because $n \leq 0.05N$ and $np(1-p) < 10$.

Determine the mean of the sampling distribution of $\hat{p}$.

$\mu_{\hat{p}} =$ .37 (Round to two decimal places as needed.)

Determine the standard deviation of the sampling distribution of $\hat{p}$.

$\sigma_{\hat{p}} =$ 0.02 (Round to three decimal places as needed.)

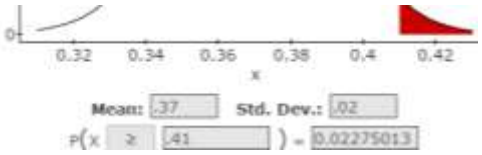
(b) In a random sample of 600 adults, what is the probability that less than 36% have no credit cards?

The probability is .3085. (Round to four decimal places as needed.)

(c) Would it be unusual if a random sample of 600 adults results in 246 or more having no credit cards?

- A. The result is not unusual because the probability that $\hat{p}$ is greater than or equal to this sample proportion is greater than 5%.
- ✔ B. The result is unusual because the probability that $\hat{p}$ is greater than or equal to this sample proportion is less than 5%.

$$x \geq \frac{246}{600} = .41$$



Unusual because .022 is 2% which is less than 5%

According to a study conducted by an organization, the proportion of Americans who were afraid to fly in 2006 was 0.10. A random sample of 1,100 Americans results in 121 indicating that they are afraid to fly. Explain why this is not necessarily evidence that the proportion of Americans who are afraid to fly has increased.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. This is not necessarily evidence that the proportion of Americans who are afraid to fly has increased above 0.10 because the sample size n is more than 5% of the population.
- B. This is not necessarily evidence that the proportion of Americans who are afraid to fly has increased above 0.10 because the value of $np(1-p)$ is less than 10.
- C. This is not necessarily evidence that the proportion of Americans who are afraid to fly has increased above 0.10 because the probability of obtaining a value equal to or more extreme than the sample proportion is 0.1345, which is not unusual.

$x \geq \frac{121}{1100} = .11$

Mean: .10 Std. Dev.: .009045

P(X ≥ .11) = 0.13445348

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$