9.1 Estimating a Population Proportion MATH 241 THOMPSON

1) A point estimate is the value of a statistic that estimates the value of a parameter.

2) The level of confidence represents the expected proportion of intervals that will contain the parameter if a large number of different samples of size n is obtained. It is denoted $(1-\alpha)\cdot 100\%$.

3) Compute the critical value $z_{\alpha/2}$ that corresponds to a 89% level of confidence.

$z_{\alpha/2} = 1.60$

STAT-CALCULATORS-NORMAL

$1 - 0.89 = \frac{.11}{2} = .055$

Mean: 0 Std. Dev.: 1

P(X ≥ 1.5981931) = .055

4) Compute the critical value $z_{\alpha/2}$ that corresponds to a 83% level of confidence.

$z_{\alpha/2} = 1.37$

$1 - 0.83 = \frac{.17}{2} = .085$

Mean: 0 Std. Dev.: 1

P(X ≥ 1.3722038) = .085

5) Determine the point estimate of the population proportion, the margin of error for the following confidence interval, and the number of individuals in the sample with the specified characteristic, x, for the sample size provided.

Lower bound = 0.348, upper bound = 0.592, n = 1000

The point estimate of the population proportion is .47. $\frac{.348+.592}{2} = .47$
(Round to the nearest thousandth as needed.)

The margin of error is .122. $\frac{.592-.348}{2} = .122$
(Round to the nearest thousandth as needed.)

The number of individuals in the sample with the specified characteristic is 470.

$x = \hat{p}n$ point estimate times n *0.47 X 1000*

6) Determine the point estimate of the population proportion, the margin of error for the following confidence interval, and the number of individuals in the sample with the specified characteristic, x, for the sample size provided.

Lower bound = 0.100, upper bound = 0.450, n = 1200

The point estimate of the population proportion is .275. $\frac{.100+.450}{2} = .275$
(Round to the nearest thousandth as needed.)

$\frac{.450-.100}{2} = .175$

The number of individuals in the sample with the specified characteristic is 330.
(Round to the nearest thousandth as needed.)

point estimate times n *0.275 X 1200*

$x = \hat{p}n$

7) Construct a 90% confidence interval of the population proportion using the given information.

$x = 240$, $n = 300$

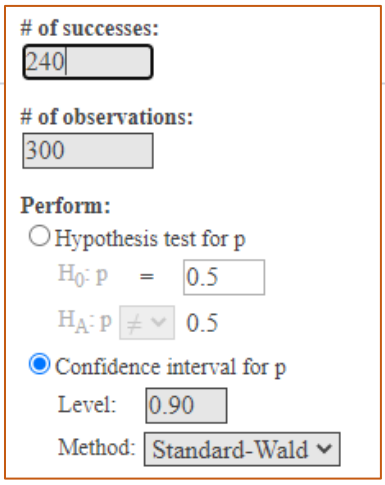
STAT- PROPORTION STATS-ONE SAMPLE –WITH SUMMARY

Click here to view the table of critical values.

The lower bound is .762.

The upper bound is .838.

(Round to three decimal places as needed.)



L. Limit	U. Limit
0.76201373	0.83798627

8) In a survey of 2025 adults in a certain country conducted during a period of economic uncertainty, 67% thought that wages paid to workers in industry were too low. The margin of error was 9 percentage points with 90% confidence. For parts (a) through (d) below, which represent a reasonable interpretation of the survey results? For those that are not reasonable, explain the flaw.

(a) We are 90% confident 67% of adults in the country during the period of economic uncertainty felt wages paid to workers in industry were too low.

Is the interpretation reasonable?

- A. The interpretation is reasonable.
- B. The interpretation is flawed. The interpretation indicates that the level of confidence is varying.
- ✔ C. The interpretation is flawed. The interpretation provides no interval about the population proportion.

(b) We are 81% to 99% confident 67% of adults in the country during the period of economic uncertainty felt wages paid to workers in industry were too low.

Is the interpretation reasonable?

- A. The interpretation is reasonable.
- ✔ B. The interpretation is flawed. The interpretation indicates that the level of confidence is varying.

(c) We are 90% confident that the interval from 0.58 to 0.76 contains the true proportion of adults in the country during the period of economic uncertainty who believed wages paid to workers in industry were too low.

Is the interpretation reasonable?

- ✔ A. The interpretation is reasonable.

(d) In 90% of samples of adults in the country during the period of economic uncertainty, the proportion who believed wages paid to workers in industry were too low is between 0.58 and 0.76.

Is the interpretation reasonable?

- A. The interpretation is reasonable.
- ✔ B. The interpretation is flawed. The interpretation suggests that this interval sets the standard for all the other intervals, which is not true.

9) A national survey of 1500 adult citizens of a nation found that 19% dreaded Valentine's Day. The margin of error for the survey was 8.8 percentage points with 90% confidence. Explain what this means.

Which statement below is the best explanation?

A. There is 90% confidence that the proportion of the adult citizens of the nation that dreaded Valentine's Day is between 0.102 and 0.278.

10) A survey of 2323 adults in a certain large country aged 18 and older conducted by a reputable polling organization found that 404 have donated blood in the past two years. Complete parts (a) through (c) below.
Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) Obtain a point estimate for the population proportion of adults in the country aged 18 and older who have donated blood in the past two years.

$\frac{404}{2323} = 0.174$

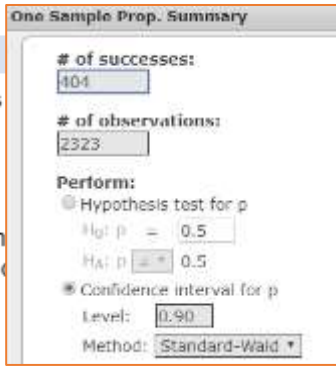
$\hat{p}$ = .174
(Round to three decimal places as needed.)

2323 · .174(1-.174)

(b) Verify that the requirements for constructing a confidence interval about p are satisfied.

The sample can be assumed to be a simple random sample, the value of $n\hat{p}(1-\hat{p})$ is 333.871 which is greater than or equal to 10, and the sample size can be assumed to be less or equal to 5% of the population size.
(Round to three decimal places as needed.)

STAT- PROPORTION STATS-
ONE SAMPLE –WITH SUMMARY



(c) Construct and interpret a 90% confidence interval for the population proportion of adults in the coun who have donated blood in the past two years. Select the correct choice below and fill in any answer b within your choice.
(Type integers or decimals rounded to three decimal places as needed. Use ascending order.)

A. We are 90 % confident the proportion of adults in the country aged 18 and older who have donated blood in the past two years is between 0.161 and 0.187.

L. Limit	U. Limit
0.16097759	0.1868485

11) Construct a confidence interval of the population proportion at the given level of confidence.

x = 540, n = 1100, 96% confidence

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

STAT- PROPORTION STATS-
ONE SAMPLE –WITH SUMMARY

The lower bound of the confidence interval is .460.
(Round to three decimal places as needed.)

The upper bound of the confidence interval is .522.
(Round to three decimal places as needed.)

12) A random sample of 1022 adults in a certain large country was asked "Do you pretty much think television are a necessity or a luxury you could do without?" Of the 1022 adults surveyed, 534 indicated that televisions are a luxury they could do without. Complete parts (a) through (e) below.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) Obtain a point estimate for the population proportion of adults in the country who believe that televisions are a luxury they could do without.

$\hat{p}$ = .523 $\quad \frac{534}{1022} = 0.523$

(Round to three decimal places as needed.)

(b) Verify that the requirements for constructing a confidence interval about p are satisfied.

1022 · .523(1-.523)

The sample is stated to be a simple random sample, the value of $n\hat{p}(1-\hat{p})$ is 254.959, which is greater than or equal to 10, and the sample size can be assumed to be less than or equal to 5% of the population size.

(Round to three decimal places as needed.)

STAT- PROPORTION STATS-ONE SAMPLE –WITH SUMMARY

(c) Construct and interpret a 95% confidence interval for the population proportion of adults in the country who believe that televisions are a luxury they could do without. Select the correct choice below and fill in any answer boxes within your choice.

(Type integers or decimals rounded to three decimal places as needed. Use ascending order.)

A. We are 95 % confident the proportion of adults in the country who believe that televisions are a luxury they could do without is between .492 and .553.

(d) Is it possible that a supermajority (more than 60%) of adults in the country believe that television is a luxury they could do without? Is it likely?

It is possible, but not likely that a supermajority of adults in the country believe that television is a luxury they could do without because the 95% confidence interval does not contain 0.6.

(Type an integer or a decimal. Do not round.)

(e) Use the results of part (c) to construct a 95% confidence interval for the population proportion of adults in the country who believe that televisions are a necessity.

The 95% confidence interval is (0.447 , .508).

Use successes as (1022 – 534) = 488 people WITH TVs.

13) In a survey conducted by a reputable marketing agency, 249 of 1000 adults 19 years of age or older confessed to bringing and using their cell phone every trip to the bathroom (confessions included texting a answering phone calls). Complete parts (a) through (f) below.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) What is the sample in this study? What is the population of interest?

Determine the sample in this study. Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. The sample is all adults.
B. The sample is all adults with a cell phone.
C. The sample is the 1000 adults 19 years of age or older.

Determine the population of interest. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- A. The population is all adults.
- B. The population is the ____ adults 19 years of age or older.
  (Type a whole number.)
- C. The population is all adults 19 years of age or older.
- D. The population is all adults with a cell phone.

(b) What is the variable of interest in this study? Is it qualitative or quantitative?

The variable of interest is bringing one's cell phone every trip to the bathroom. This variable is qualitative with two outcomes because individuals are classified based on a characteristic.

(c) Based on the results of this survey, obtain a point estimate for the proportion of adults 19 years of age or older who bring their cell phone every trip to the bathroom.

$\hat{p} =$ .249
(Round to three decimal places as needed.)

(d) Explain why the point estimate found in part (c) is a statistic. Explain why it is a random variable. What is the source of variability in the random variable?

Why is the point estimate found in part (c) a statistic?

- A. The sample size is large.
- B. Its value may change depending on the individuals in the survey.
- C. Its value is based on a sample.

Why is the point estimate found in part (c) a random variable?

- A. The sample size is large.
- B. It is being used to make inferences.
- C. It is information obtained from a survey.
- D. Its value may change depending on the individuals in the survey.
- E. Its value is based on a sample.

What is the source of variability in the random variable?

- A. The sample size
- B. The question asked in the survey
- C. The individuals selected to be in the study
- D. Random errors

(e) Construct and interpret a 95% confidence interval for the population proportion of adults 19 years of age or older who bring their cell phone every trip to the bathroom. Select the correct choice below and in any answer boxes within your choice.
(Type integers or decimals rounded to three decimal places as needed. Use ascending order.)

- A. We are 95 % confident the proportion of adults 19 years of age or older who bring their cell phone every trip to the bathroom is between .222 and .276.

(f) What ensures that the results of this study are representative of all adults 19 years of age or older?

- A. Random sampling

14) A researcher wishes to estimate the proportion of adults who have high-speed Internet access. What size sample should be obtained if she wishes the estimate to be within 0.04 with 95% confidence if
(a) she uses a previous estimate of 0.58?
(b) she does not use any prior estimates?

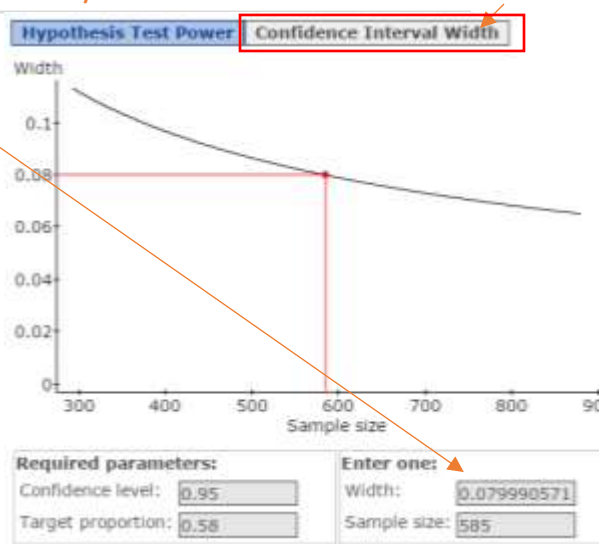
STAT-PROPORTION STATS-ONE SAMPLE
POWER/SAMPLE SIZE –confidence interval

(a) n = 585 (Round up to the nearest integer.)

(b) n = 601 (Round up to the nearest integer.)

Width=0.04x2=0.08

(b) No estimates – target = 0.5



15) A researcher wishes to estimate the percentage of adults who support abolishing the penny. What size sample should be obtained if he wishes the estimate to be within 5 percentage points with 99% confidence if
(a) he uses a previous estimate of 32%?
(b) he does not use any prior estimates?

STAT-PROPORTION STATS-ONE SAMPLE
POWER/SAMPLE SIZE –confidence interval

(a) n = 578 (Round up to the nearest integer.)

(b) n = 664 (Round up to the nearest integer.)

5% is 0.05

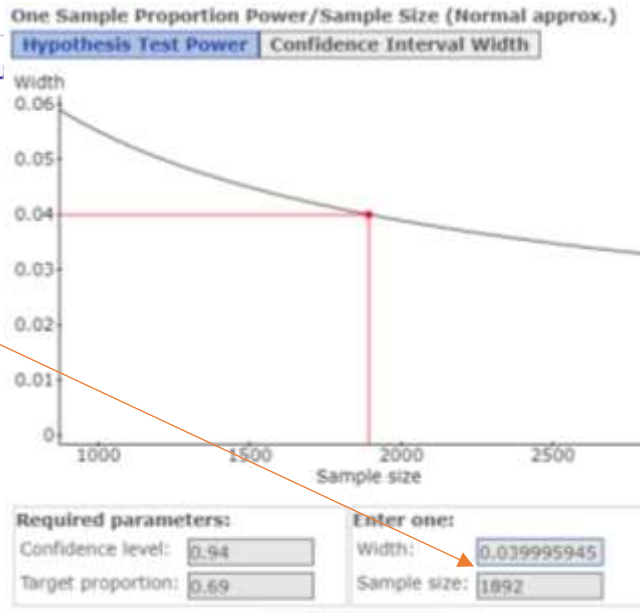
Width=0.05x2=0.1

No estimates – target = 0.5

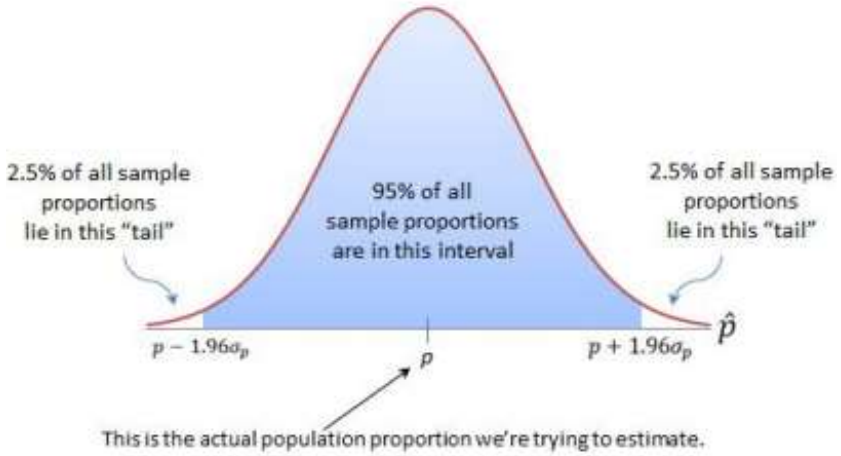
16) In a poll, 69% of the people polled answered yes to the question "Are you in favor of the death penalty for a person convicted of murder?" The margin of error in the poll was 2%, and the estimate was made with 94% confidence. At least how many people were surveyed?

The minimum number of surveyed people was 1892 . (Round u

Width=0.02x2=0.04



17)



Fill in the blanks to complete the sentences below.

(a) As the number of samples increases, the proportion of 95% confidence intervals that include the population proportion approaches _____.

(b) If a 95% confidence interval results in a sample proportion that does not include the population proportion, then the sample proportion is more than _____ standard errors from the population proportion.

(This is a reading assessment question. Be certain of your answer because you only get one attempt on this question.)

(a) As the number of samples increases, the proportion of 95% confidence intervals that include the population proportion approaches 0.95.

(b) If a 95% confidence interval results in a sample proportion that does not include the population proportion, then the sample proportion is more than 1.96 standard errors from the population proportion.

18) Fill in the blank to complete the sentence below.

If the normality requirement is not satisfied (that is, $np(1 - p)$ is not at least 10), then a 95% confidence interval about the population proportion will include the population proportion in _____ 95% of the intervals.

(This is a reading assessment question. Be certain of your answer because you only get one attempt on this question.)

If the normality requirement is not satisfied (that is, $np(1 - p)$ is not at least 10), then a 95% confidence interval about the population proportion will include the population proportion in less than 95% of the intervals.

More Examples

In a survey of 2025 adults in a certain country conducted during a period of economic uncertainty, 67% thought that wages paid to workers in industry were too low. The margin of error was 7 percentage points with 90% confidence. For parts (a) through (d) below, which represent a reasonable interpretation of the survey results? For those that are not reasonable, explain the flaw.

(a) We are 90% confident 67% of adults in the country during the period of economic uncertainty felt wages paid to workers in industry were too low.

67 – 7 →

67 + 7 →

Lower bound = 60 %

Upper bound = 74 %

Thus, lower bound is $\hat{p} - E$ and the upper bound is $\hat{p} + E$ where $\hat{p}$ is the point estimate and E is the margin of error.

Identify the values for $\hat{p}$ and E.

$\hat{p}$ = 67 % and E = 7 %.

The 90% confidence interval for the proportion is (0.60, 0.74). Now use this information to evaluate the interpretation, "We are 90% confident 67% of adults in the country during the period of economic uncertainty felt wages paid to workers in industry were too low."

The interpretation is flawed. The interpretation provides no interval about the population proportion.

(b) We are 83% to 97% confident 67% of adults in the country during the period of economic uncertainty felt wages paid to workers in industry were too low.

Use the 90% confidence interval for the proportion, (0.60, 0.74), and the information in the previous steps to evaluate the interpretation

The interpretation is flawed. The interpretation indicates that the level of confidence is varying

(c) We are 90% confident that the interval from 0.60 to 0.74 contains the true proportion of adults in the country during the period of economic uncertainty who believed wages paid to workers in industry were too low.

Use the 90% confidence interval for the proportion, (0.60, 0.74), and the information in the previous steps to evaluate the interpretation

The interpretation is reasonable

(d) In 90% of samples of adults in the country during the period of economic uncertainty, the proportion who believed wages paid to workers in industry were too low is between 0.60 and 0.74.

Use the 90% confidence interval for the proportion, (0.60, 0.74), and the information in the previous steps to evaluate the interpretation

The interpretation is flawed. The interpretation suggests that the interval sets the standard for all the other intervals, which is not true.

A national survey of 1000 adult citizens of a nation found that 21% dreaded Valentine's Day. The margin of error for the survey was 6.9 percentage points with 85% confidence. Explain what this means.

Which statement below is the best explanation?

p = .21 and E = 0.069

- A. There is 85% confidence that 21% of the adult citizens of the nation dreaded Valentine's Day.
- B. There is 85% confidence that the proportion of the adult citizens of the nation that dreaded Valentine's Day is between 0.141 and 0.279. Lower (.21-.067) =.141 Upper (.21+0.069)=.279
- C. In 85% of samples of adult citizens of the nation, the proportion that dreaded Valentine's Day is between 0.141 and 0.279.
- D. There is 78.1% to 91.9% confidence that 21% of the adult citizens of the nation dreaded Valentine's Day.

A poll of 1027 adults in a certain country found that 23% identified themselves as the followers of some religion. The margin of error was 3 percentage points with 95% confidence.

Which of the following represents a reasonable interpretation of the survey results?

- A. There is between 92% and 98% confidence that 23% of adults in a certain country identify themselves as the followers of some religion.
- B. In 95% of samples of adults in a certain country, the proportion who identify themselves as the followers of some religion is between 20% and 26%.
- C. There is 95% confidence that the proportion of adults in a certain country who identify themselves as the followers of some religion is between 20% and 26%.
- D. There is 95% confidence that 23% of adults in a certain country identify themselves as the followers of some religion.

In a trial of 125 patients who received 10-mg doses of a drug daily, 45 reported headache as a side effect. Use this information to complete parts (a) through (d) below.

(a) Obtain a point estimate for the population proportion of patients who received 10-mg doses of a drug daily and reported headache as a side effect.

$\hat{p}$ = .36 (Round to two decimal places as needed.)

(b) Verify that the requirements for constructing a confidence interval about p are satisfied.

Are the requirements for constructing a confidence satisfied? p < .05 – yes

- A. Yes, the requirements for constructing a confidence interval are satisfied.
- B. No, the requirement that the sample size is no more than 5% of the population is not satisfied.
- C. No, the requirement that $n\hat{p}(1-\hat{p})$ is greater than 10 is not satisfied.
- D. No, the requirement that each trial be independent is not satisfied.

(c) Construct a 99% confidence interval for the population proportion of patients who receive the drug and report headache as a side effect.

The 99% confidence interval is (.249 , .471).
(Round to three decimal places as needed.)

L. Limit	U. Limit
0.2494132	0.4705868

(d) Interpret the confidence interval. Which statement below best interprets the interval?

- A. We are 99% confident that the interval contains the true value of p.

An interactive poll found that 371 of 2,245 adults aged 18 or older have at least one tattoo.
(a) Obtain a point estimate for the proportion of adults who have at least one tattoo.
(b) Construct a 90% confidence interval for the proportion of adults with at least one tattoo.
(c) Construct a 98% confidence interval for the proportion of adults with at least one tattoo.
(d) What is the effect of increasing the level of confidence on the width of the interval?

(a) $\hat{p}$ = .165 (Round to three decimal places as needed.)

(b) Construct the 90% confidence interval. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- A. Lower bound: .152
  Upper bound: .178
  (Round to three decimal places as needed.)
- B. The requirements for constructing a confidence interval are not satisfied.

(c) Construct the 98% confidence interval. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- A. Lower bound: .147
  Upper bound: .183
  (Round to three decimal places as needed.)
- B. The requirements for constructing a confidence interval are not satisfied.

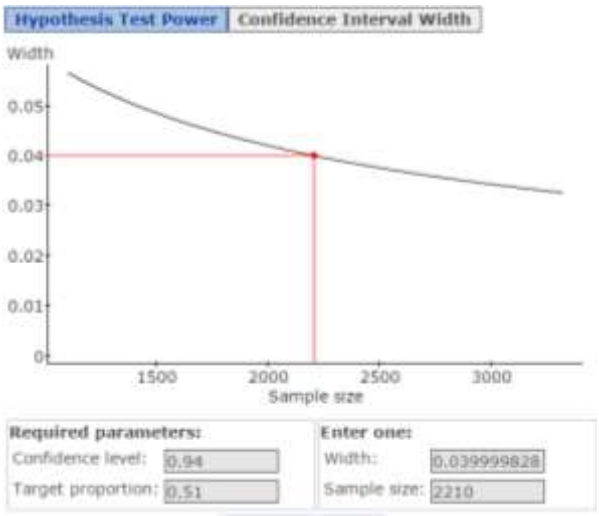
(d) Choose the correct answer below.

- A. Increasing the level of confidence has no effect on the interval.
- B. Increasing the level of confidence widens the interval.
- C. Increasing the level of confidence narrows the interval.
- D. It is not possible to tell the effect of increasing the level of confidence on the width of the interval since the requirements for constructing a confidence interval in parts (b) and (c) were not met.

In a poll, 51% of the people polled answered yes to the question "Are you in favor of the death penalty for a person convicted of murder?" The margin of error in the poll was 2%, and the estimate was made with 94% confidence. At least how many people were surveyed?

The minimum number of surveyed people was 2210. (Round up to the nearest integer.)

Width - .02x2 = .04



Why does the margin of error increase as the level of confidence increases?

Choose the correct answer below.

- A. The margin of error increases as the level of confidence increases because the smaller the expected proportion of intervals that will contain the parameter, the larger the margin of error.
- B. The margin of error increases as the level of confidence increases because the larger the expected proportion of intervals that will contain the parameter, the larger the margin of error.

Why does the margin of error decrease as the sample size n increases?

Choose the correct answer below.

- A. The margin of error decreases as the sample size n increases because the difference between the statistic and the parameter decreases. This is a consequence of the Law of Large Numbers.