Section 5.2: The Addition Rule and Complements

Two events are **disjoint** if they have no outcomes in common. (Also commonly known as **mutually exclusive** events.)

The Addition Rule for Disjoint Events

If E and F are disjoint (mutually exclusive) events, then

P(E or F) = P(E) + P(F)

Example 1

OK - time for an example. Let's use the example from last section about the family with three children, and let's define the following events:

E = the family has exactly two boys
F = the family has exactly one boy

Describe the event "E or F" and find its probability.

[reveal answer]

"E or F" is the event that the family has either one or two boys.

Clearly, it isn't possible for both of these events to occur at the same time, so they are disjoint. The probability of the family having either one or two boys is then:

P(E or F) = P(E) + P(F) = 3/8 + 3/8 = 6/8 = 3/4

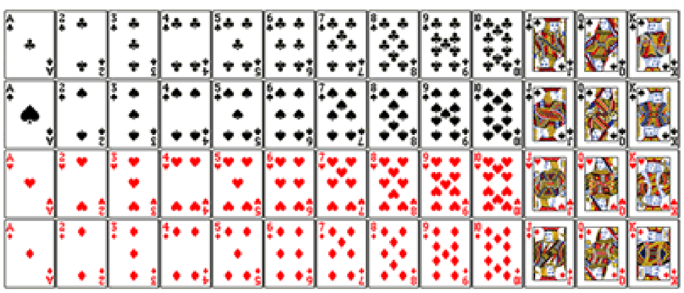
The General Addition Rule

P(E or F) = P(E) + P(F) - P(E and F)

Let's try a couple quick examples.

Example 2

Let's consider a deck of standard playing cards.



Suppose we draw one card at random from the deck and define the following events:

E = the card drawn is an ace
F = the card drawn is a king

Use these definitions to find P(E or F).

[reveal answer]

OK, since E and F have no outcomes in common, we can use the Addition Rule for Disjoint Events:

P(E or F) = P(E) + P(F) = 4/52 + 4/52 = 8/52 = **2/13**

Example 3

Considering the deck of playing cards, where one is drawn at random. Suppose we define the following events:

F = the card drawn is a king
G = the card drawn is a heart

Use these definitions to find P(F or G).

[reveal answer]

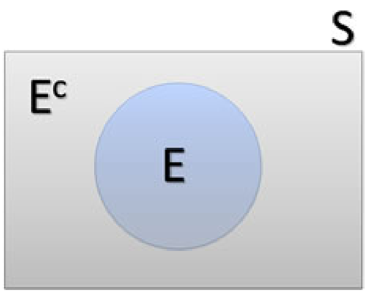
Unlike in the previous example, events F and G, *do* have an outcome in common - the king of hearts - so we'll need to use the General Addition Rule:

P(E or F)
= P(E) + P(F) - P(E and F)
= 4/52 + 13/52 - 1/52
= 16/52 = **4/13**

The **complement of E**, denoted E^C, is all outcomes in the sample space that are not in E.

So essentially, the complement of E is *everything but* the outcomes in E. In fact, some texts actually write it as "not E".

How is the complement helpful? Well, you actually already used the key idea in the example above. Let's look at a Venn diagram.



From Section 5.1, we know that P(S) = 1. Clearly, E and E^C are disjoint, so P(E or E^C) = P(E) + P(E^C) . Combining those two facts, we get:

The Complement Rule

P(E) + P(E^C) = 1

Independence

One of the most important concepts in probability is that of *independent events*.

Two events E and F are **independent** if the occurrence of event E does not affect the probability of event F.

Let's look at a couple examples.

Example 1

Source: stock.xchng

Consider the experiment where two cards are drawn without replacement. (*Without replacement* means one is drawn and then the second is drawn without putting the first one back.) Define events E and F this way:

E = the first card drawn is a King

F = the second card drawn is a King

Are events E and F independent?

[reveal answer]

Example 2

Consider the experiment in which two fair six-sided dice are rolled, and define events E and F as follows:

E = the first die is a 3

F = the second die is a 3

Are events E and F independent?

[reveal answer]

Disjoint vs. Independent

It is very common for students to confuse the concepts of *disjoint* (*mutually exclusive*) events with *independent* events. Recall from the last section:

> Two events are **disjoint** if they have no outcomes in common. (Also commonly known as **mutually exclusive** events.)

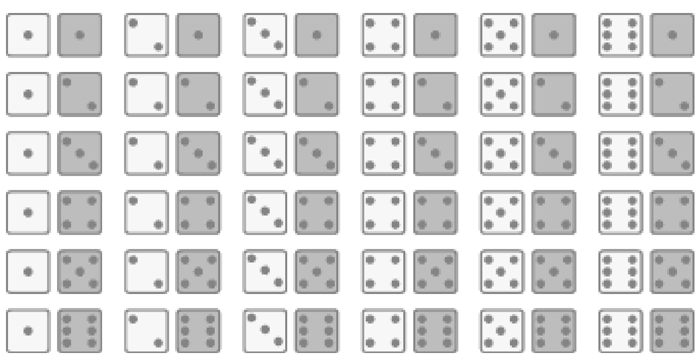
The Multiplication Rule for Independent Events

To introduce the next idea, let's look at the experiment from Example 2, in Section 5.1.

Example 3

The experiment was rolling a fair six-sided die twice. Suppose define the event E:

E = both dice are 2's



The possible outcomes are (1,1), (1,2), (1,3), ... (6,5), and (6,6). Since only one of these is (2,2), we know P(E) = 1/36. Let's look at it another way, though.

$$P(E) = \frac{N(E)}{N(S)} = \frac{1}{36} = \frac{1}{6 \bullet 6} = \frac{1}{6} \bullet \frac{1}{6} = P(E) \bullet P(F)$$

Multiplication Rule for Independent Events

If E and F are independent events, then

$$P(E \text{ and } F) = P(E) \bullet P(F)$$

Example 4

According to data from the American Cancer Society, about 1 in 3 women living in the U.S. will have some form of cancer during their lives.

If three women are randomly selected, what is the probability that they will all contract cancer at some point during their lives?

[reveal answer]

Since the three women are *randomly* selected, we can assume that they are independent of each other. Because of that, we can use the Multiplication Rule for Independent Events:

P(all have breast cancer)
= P(1st does and 2nd does and 3rd does)
= P(1st) • P(2nd) • P(3rd)
= (1/3)(1/3)(1/3)
≈ 0.037

So there is about a 3.7% probability that all 3 of the women will contract cancer at some point.