Section 5.3: Independence and the Multiplication Rule

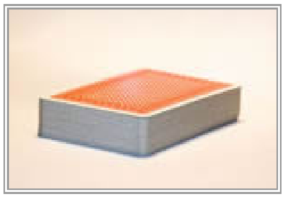
Independence

One of the most important concepts in probability is that of *independent events*.

Two events E and F are **independent** if the occurrence of event E does not affect the probability of event F.

Let's look at a couple examples.

Example 1



Source: stock.xchng

Consider the experiment where two cards are drawn without replacement. (*Without replacement* means one is drawn and then the second is drawn without putting the first one back.) Define events E and F this way:

E = the first card drawn is a King

F = the second card drawn is a King

Are events E and F independent?

[reveal answer]

Definitely not. P(F) depends on what happens with E. If a King is drawn with the first card, P(F) = 3/51, but if a King is not drawn, P(F) = 4/52 = 1/13.

Example 2

Consider the experiment in which two fair six-sided dice are rolled, and define events E and F as follows:

E = the first die is a 3

F = the second die is a 3

Are events E and F independent?

[reveal answer]

Yes. In this case, what happens on the first die does not affect what happens on the second die. P(F) = 1/6, regardless of what happens with the first die.

Disjoint vs. Independent

It is very common for students to confuse the concepts of *disjoint* (*mutually exclusive*) events with *independent* events. Recall from the last section:

Two events are **disjoint** if they have no outcomes in common. (Also commonly known as **mutually exclusive** events.)

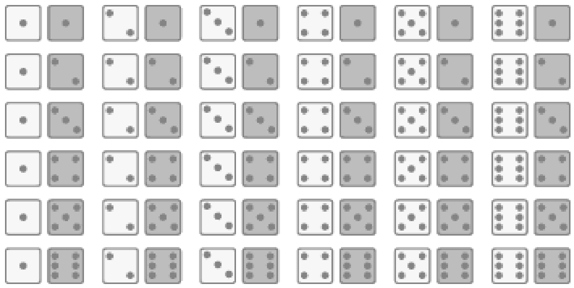
The Multiplication Rule for Independent Events

To introduce the next idea, let's look at the experiment from Example 2, in Section 5.1.

Example 3

The experiment was rolling a fair six-sided die twice. Suppose define the event E:

E = both dice are 2's



The possible outcomes are (1,1), (1,2), (1,3), ... (6,5), and (6,6). Since only one of these is (2,2), we know P(E) = 1/36. Let's look at it another way, though.

$$P(E) = \frac{N(E)}{N(S)} = \frac{1}{36} = \frac{1}{6 \bullet 6} = \frac{1}{6} \bullet \frac{1}{6} = P(E) \bullet P(F)$$

Multiplication Rule for Independent Events

If E and F are independent events, then

$$P(E \text{ and } F) = P(E) \bullet P(F)$$

Example 4

According to data from the American Cancer Society, about 1 in 3 women living in the U.S. will have some form of cancer during their lives.

If three women are randomly selected, what is the probability that they will all contract cancer at some point during their lives?

[reveal answer]

Since the three women are *randomly* selected, we can assume that they are independent of each other. Because of that, we can use the Multiplication Rule for Independent Events:

P(all have breast cancer)
= P(1st does and 2nd does and 3rd does)
= P(1st) • P(2nd) • P(3rd)
= (1/3)(1/3)(1/3)
≈ 0.037

So there is about a 3.7% probability that all 3 of the women will contract cancer at some point.

At-least Probabilities

The phrase "at least" can make a seemingly simple problem much more difficult. For example, suppose we're looking at cancer rates in women. And suppose we have a random sample of 5 women. If we're looking for the probability that *at least one* will have some form of cancer, that's really:

P(1 will have cancer) + P(2 will have cancer) + ... + P(all 5 will have cancer)

Instead, it's much easier to use the *Complement Rule*, from Section 5.2.

The Complement Rule

$$P(E) + P(E^C) = 1$$