Section 5.4: Conditional Probability and the General Multiplication Rule

Conditional Probability

Remember in Example 3, in Section 5.3, about rolling two dice? In that example, we said that events E (the first die is a 3) and F (the second die is a 3) were *independent*, because the occurrence of E didn't effect the probability of F. Well, that won't always be the case, which leads us to another type of probability called *conditional probability*.

Conditional Probability

The notation P(F|E) is read "the probability of F given E" and represent the probability that event F occurs, given that event E has already occurred.

Let's look again at Example 1 from that same section.

Example

Consider the experiment where two cards are drawn without replacement. (*Without replacement* means one is drawn and then the second is drawn without putting the first one back.) Define events E and F this way:

E = the first card drawn is a King

F = the second card drawn is a King

Find P(F|E).

[reveal answer]

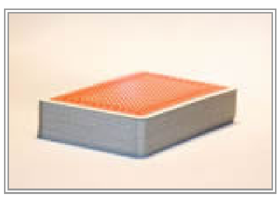
P(F|E) is the probability that the second card is a king given if the first card drawn was a king. In that case, there will be 3 kings left out of 51 cards, so

P(F|E) = 3/51

The General Multiplication Rule

Let's look again at the experiment from Example 1 in Section 5.3.

Example



Source: stock.xchng

Consider the experiment where two cards are drawn without replacement. (*Without replacement* means one is drawn and then the second is drawn without putting the first one back.) Define events E and F this way:

E = the first card drawn is a King

F = the second card drawn is a King

How would we find P(E and F)?

We know from Example 1 that E and F are not independent, so we know we can't use the Multiplication Rule for Independent Events. It's probably not too difficult to see how we might do it, though.

P(E and F) = P(first is King and second is King)
= P(first is King)•P(second is King given first is King)
= (4/52)(3/51)
≈ 0.0045

Or in other words, P(E and F) = P(E) • P(F|E)

This idea is actually a version of the *Multiplication Rule for Independent Events*, and is called the *General Multiplication Rule*.

General Multiplication Rule

The probability that two events E and F both occur is

$$P(E \text{ and } F) = P(E) \bullet P(F|E)$$

Example

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Let's try a new probability experiment. This time, consider a bag of marbles, containing 10 red, 20 blue, and 15 green marbles. Suppose that two marbles are drawn without replacement. (The first marble is not put back in the bag before drawing the second.)

What is the probability that *both* marbles drawn are red?

[reveal answer]

Let's define a couple events:

E = first marble is red

F = second marble is red

We want P(E and F). Using the General Multiplication Rule, we see

$P(E \text{ and } F) = P(E) \bullet P(F|E) = (10/45) \bullet (9/44) \approx 0.0455$

Checking for Independence

If you recall, in Section 5.3, we defined what it meant for two events to be independent:

Two events E and F are **independent** if the occurrence of event E does not affect the probability of event F.

Looking at this in terms of conditional probability, if the occurrence of E doesn't affect the probability of F, then P(F|E) = P(F). This is a good way to test for independence. In fact, we can redefine independence using this concept.

Two events E and F are **independent** if P(F|E) = P(F).

Let's use this new definition in an example to determine if two events are independent.

Example

Let's again use the data from Example 3 and the survey given to 52 students in a Basic Algebra course at ECC, with the following responses to the statement "I enjoy math."

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Total
Male	6	10	3	0	0	19
Female	8	14	7	4	0	33
Total	14	24	10	4	0	52

Suppose a student is selected at random from those surveyed and we define the events E and F as follows:

E = student selected is female

F = student enjoys math

Are events E and F independent?

To answer this, we'll need to see if P(F|E) = P(F).

$P(F) = 38/52 \approx 0.7308$

$P(F|E) = 22/33 \approx 0.6667$

Since $P(F) \neq P(F|E)$, events E and F are dependent.