Section 6.1
Probability Distribution

A **random variable** is a numerical measure of the outcome of a probability experiment whose value is determined by chance.

The **probability distribution** of a discrete random variable X provides the possible value of the random variable along with their corresponding probabilities. A probability distribution can be in the form of a table, graph, or mathematical formula.

Requirements of Discrete Probability Distributions

$0 \le p(x) \le 1$ for all values of x.

$\sum_{all\ x} p(x) = 1$

Since the probability $p(x)$ is a proportion, it must be between zero impossibility) and one (certainty).

We are guaranteed to get an outcome when we do the experiment.

Note: If a function p does not satisfy both requirements, it cannot be a probability distribution.

Summation Notation $\sum_{all\ x} p(x)$: Add all the $p(x)$ values.

The Mean of a Discrete Random Variable

The mean of a discrete random variable is given by the formula

$$u_X = \sum [x \cdot P(x)]$$

where x is the value of the random variable and P(x) is the probability of observing the random variable x.

Example:

x	P(x)
0	1/8
1	3/8
2	3/8
3	1/8

The expected value is then:

$$\mu = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = 1.5$$

The Variance and Standard Deviation of a Discrete Random Variable

The variance of a discrete random variable is given by the formula

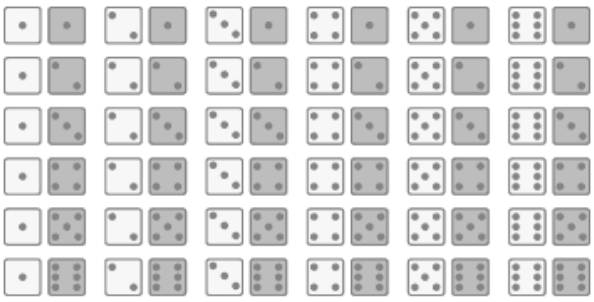
$$\sigma_X^2 = \sum [(x - \mu_x)^2 \cdot P(x)]$$

where x is the value of the random variable and P(x) is the probability of observing the random variable x.

To find the standard deviation of the discrete random variable, take the square root of the variance.

Example:

Let's consider our example again with the two dice.



We know from earlier this section that the expected value is 7. Using that, we can find the variance and standard deviation.

$$\sigma_X^2 = (2-7)^2 \cdot \frac{1}{36} + (3-7)^2 \cdot \frac{2}{36} + \cdots + (11-7)^2 \cdot \frac{2}{36} + (12-7)^2 \cdot \frac{1}{36} \approx 5.83$$

And so the standard deviation is: $\sigma_X = \sqrt{\sigma_X^2} \approx 2.42$