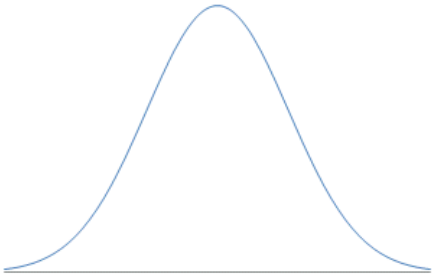
Section 7.1: Properties of the Normal Distribution

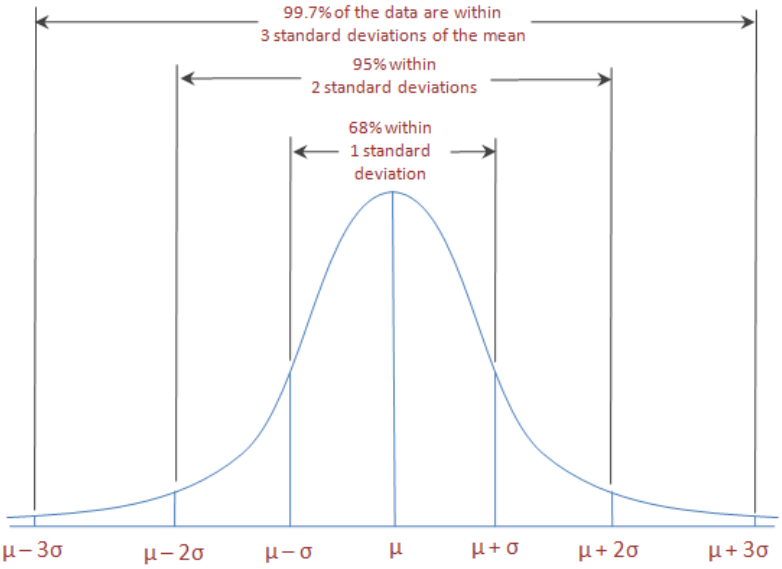
The Normal Curve

Many continuous variables follow a bell-shaped distribution (we introduced this shape back in Section 2.2), like an individuals height, the thickness of tree bark, IQs, or the amount of light emitted by a light bulb. The more formal name of a histogram of this shape is a **normal curve**.



A continuous random variable is **normally distributed** or has a **normal probability distribution** if its relative frequency histogram has the shape of a normal curve.

In Section 3.2, we introduced the Empirical Rule, which said that almost all (99.7%) of the data would be within 3 standard deviations, if the distribution is bell-shaped.



The Standard Normal Distribution

Back in Section 3.4, we introduced the idea of a z-score:

The **z-score** represents the number of standard deviations a data value is from the mean.

$$Z = \frac{x - \mu}{\sigma}$$

We mentioned then that we'd need to remember the z-score later - this is that moment!

The z-score is important, because if the variable X is normally distributed, Z is as well. This brings us to an important fact:

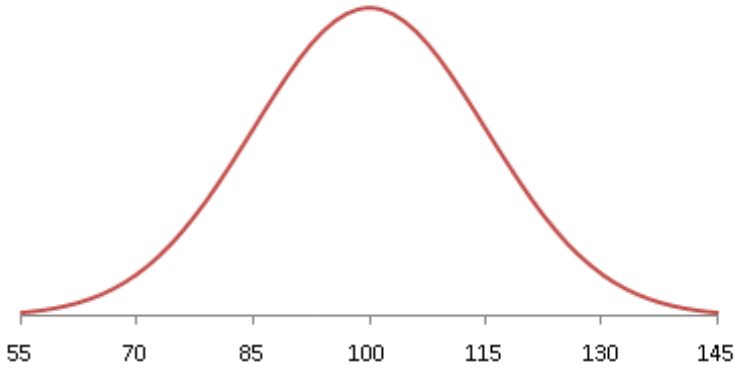
If X is normally distributed with mean μ and standard deviation σ, then

$$Z = \frac{x - \mu}{\sigma}$$

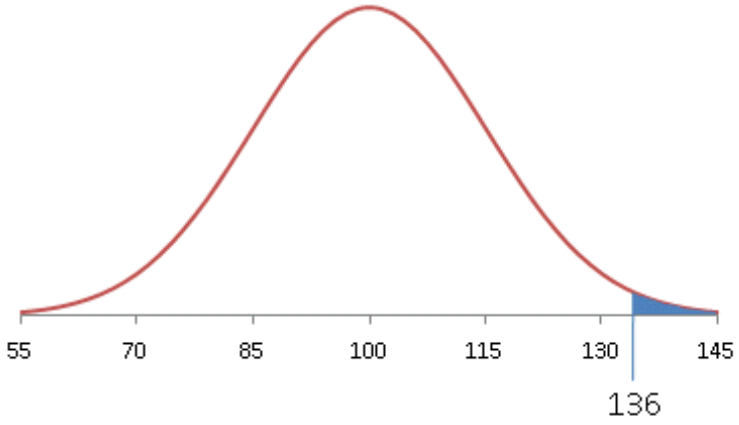
is normally distributed with a mean of 0 and a standard deviation of 1. We say that Z has the **standard normal distribution.**

Example 1

Most tests that gauge one's intelligence quotient (IQ) are designed to have a mean of 100 and a standard deviation of 15. It's also known that IQs are normally distributed. So what would the distribution look like for IQs?



There is no universal agreement on what IQ constitutes a "genius", though in 1916, psychologist Lewis M. Thurman set a guideline of 140 (scaled to 136 in today's tests) for "potential genius".



Suppose the area to the right of 136 is about 0.0082. What are two interpretations of that area?

[reveal answer]

1. About 0.82% of all individuals can be classified as a "potential genius" according to Dr. Thurman.
2. If an individual is selected at random, there is a probability of about 0.0082 that the individual is a "potential genius".

Example 2

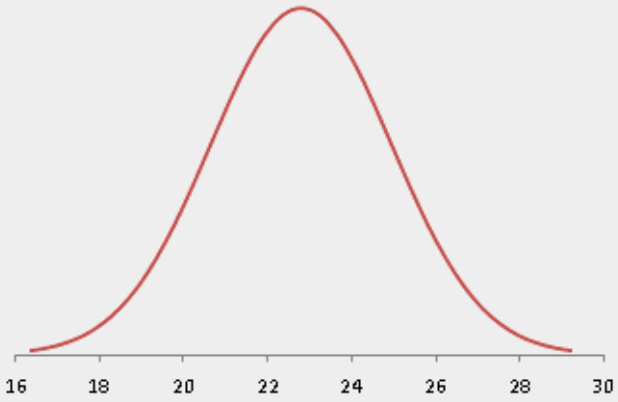
Source: stock.xchng

Weights of 1-year-old boys are approximately normally distributed, with a mean of 22.8 lbs and a standard deviation of about 2.15. (Source: About.com)

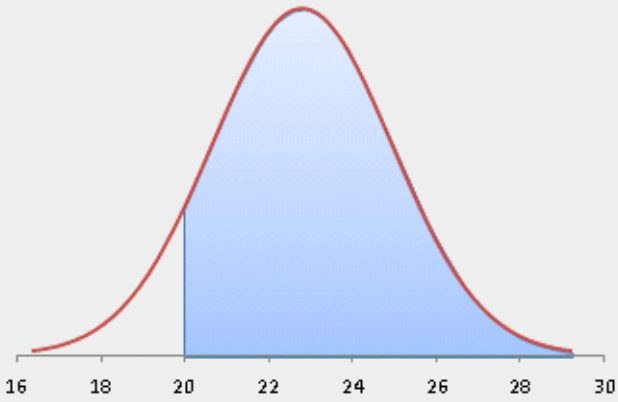
a. Draw a quick sketch of the normal curve for the weights of 1-year-old boys.
b. Shade the area representing the boys who are at least 20 pounds.
c. The area is approximately 0.9036. Give two interpretations of this result.

[reveal answer]

a.



b.



c. Two interpretations would be (1) approximately 90% of all 1-year-old boys weigh at least 20 pounds; and (2) the probability that a randomly selected 1-year-old boy weighs at least 20 pounds is about 0.9036.