Section 8.1: Distribution of the Sample Mean

Objectives

By the end of this lesson, you will be able to...

1. describe the distribution of the sample mean for samples obtained from normal populations
2. describe the distribution of the sample mean for samples obtained from a population that is not normal

Sampling Distributions

Sampling error is the error that results from using a sample to estimate information regarding a population.

The Distribution of the Sample Mean

Let's look again at the definition of a **random variable**, from Section 6.1.

A **random variable** is a numerical measure of the outcome of a probability experiment whose value is determined by chance.

The Law of Large Numbers

As n increases, the difference between $\bar{x}$ and μ approaches zero.

The Central Limit Theorem

The Central Limit Theorem

Regardless of the distribution shape of the population, the sampling distribution of $\bar{x}$ becomes approximately normal as the sample size n increases (conservatively $n \geq 30$).

The Distribution of the Sample Mean

We can even be more specific about the distribution of $\bar{x}$:

The Sampling Distribution of $\bar{x}$

If a simple random sample of size n is drawn from a large population with mean μ and standard deviation σ, the sampling distribution of $\bar{x}$ will have mean and standard deviation:

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where $\sigma_{\bar{x}}$ is the **standard error of the mean.**

Using the Central Limit Theorem

In order to find probabilities about a normal random variable, we need to first know its mean and standard deviation. With the results of the Central Limit Theorem, we now know the distribution of the sample mean, so let's try using that in some examples.

Let's see a couple examples.

Example 1

Let's consider again the distribution of IQs that we looked at in Example 1 in Section 7.1.

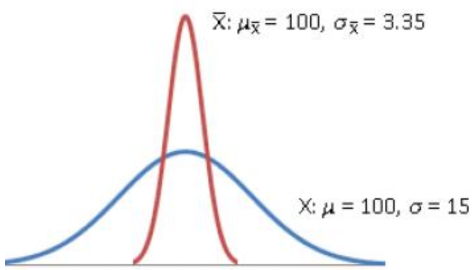
We saw in that example that tests for an individual's intelligence quotient (IQ) are designed to be normally distributed, with a mean of 100 and a standard deviation of 15.

What is the probability that a randomly selected sample of 20 individuals would have a mean IQ of more than 105?

Solution:

To answer this question, we need to find $P(\bar{x} > 105)$, if $n = 20$. Before we can do that, we need to first find the distribution of $\bar{x}$. From the distribution of the sample mean, we know $\mu_{\bar{x}} = \mu = 100$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 15/\sqrt{20} \approx 3.35$.

Here's what the distribution of $\bar{x}$ looks like in relation to the distribution of X.



Example 2

In Example 2 in Section 7.1, we were told that weights of 1-year-old boys are approximately normally distributed, with a mean of 22.8 lbs and a standard deviation of about 2.15. (Source: About.com)

Suppose the sample mean of the 10 1-year-old boys at the Kiddie Care day care center is 22.3 lbs. Is that unusual?

Source: stock.xchng

Solution:

In order to determine if an event is *unusual*, we need to find its probability. If the probability of the event is less than 5%, we can classify it as an unusual event.

In this case, we want to find the probability of observing a sample mean of 22.3 or less. Using the distribution of the sample mean, $\mu_{\bar{x}} = \mu = 22.8$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.15/\sqrt{10} \approx 0.68$. Using StatCrunch...

So we'd observe a sample mean of 22.3 lbs or less from a sample of 10 1-year-old boys about 23% of the time, which is not very unusual at all.