QUIZ 12 MATH 161 THOMPSON

1) **The size of P of a certain insect population at time t (in days) obeys the function $P(t) = 300e^{0.06t}$**

a) Determine the number of insects at t = 0 days. $300e^{0.06(0)} \approx$ **300**

b) What is the growth rate of the insect population? 6%

c) What is the population after 10 days? $300e^{0.06(10)} \approx$ **547 insects**

d) When will the insect population reach 510? $510 = 300e^{0.06(t)}$ *divide by 300*

$1.7 = e^{.06t} \rightarrow \ln 1.7 = .06t$ **t = 8.8**

e) When will the insect population double? $600 = 300e^{0.06(t)}$ *divide by 300*

$2 = e^{.06t} \rightarrow \ln 27 = .06t$ **t = 11.6**

2) **Find the amount that results from the given investment. $20 invested at 10% compounded continuous after a period of 4 years.**

$\mathbf{20e^{(0.10\cdot 4)} \approx \$29.84}$

3) **Stronium 90 is a radioactive material that decays according to the function $A(t) = A_0e^{-0.0244t}$, where A_0 is the initial amount time t (in years). Assume that a scientist has a sample of 400 grams of strontium 90.**

a) What is the decay rate of strontium 90? **-2.44%**

b) How much strontium 90 is left after 20 years? $400e^{-0.0244(20)} \approx$ **246**

c) When will only 300 grams of strontium 90 be left? $300=400e^{-0.0244t}$

$0.75=e^{-0.0244t}$

$\ln .75= -0.244t$ **t ≈ 11.8**

d) What is the half-life of strontium 90? $200=400e^{-0.0244t}$

$0.5=e^{-0.0244t}$

$\ln .5= -0.244t$ **t ≈ 28.4**

4) Strontium 90 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.0244t}$, where A_0 is the initial amount present and A is the amount present at time t (in years). Assume that a scientist has a sample of 400 grams of strontium 90.

a) What is the decay rate of strontium 90? **-2.44%**

b) How much strontium 90 is left after 10 years?

we just plug in 10 for t* $400e^{-0.0244(00)}$ ≈ **313

c) When will only 100 grams of strontium 90 be left? $100=400e^{-0.0244t}$

300 is what we end with so we divide $\frac{100}{400} = .75$ $0.25=e^{-0.0244t}$

take ln of both sides $\ln .25= -0.244t \rightarrow \frac{ln 0.25}{-0.0244} = \frac{-0.0244t}{-0.0244}t$ **t ≈ 56.8**

d) What is the half-life of strontium 90?

$0.5=e^{-0.0244t}$

take ln of both sides $\ln .5= -0.0244t$ $\frac{ln 0.5}{-0.0244} = \frac{-0.0244t}{-0.0244}t$ **t ≈ 28.4**

5) According to Newton's Law of Cooling, if a body with temperature T_1 is placed in surroundings with temperature T_0, different from that of T_1, the body will either cool or warm to temperature T(t) after t minutes,

$$T(t) = T_0 + (T_1 - T_0)e^{-kt}$$

A cup of coffee with temperature 135°F is placed in a freezer with temperature 0° F. After 5 minutes, the temperature of the coffee is 90°F. Use Newton's Law of Cooling to find the coffee's temperature after 15 minutes.

$T_0=0$ T_1-135

$T(t) = 0 + (135 - 0)e^{-kt}$

$T(t) = 135e^{-kt}$

$90 = 135e^{-5k}$

$\frac{90}{135} = e^{-5k}$

$\frac{\ln(\frac{90}{135})}{-5} = k$

$$T(t) = 135e^{-\frac{\ln(\frac{90}{135})}{-5}(15)} \rightarrow 135e^{3\ln(\frac{90}{135})} = 40°F$$

6) **An employee brings a contagious disease to an office with 100 employees. The number of employees infected by the disease t days after the employees are first exposed to it is given by:** $N = \frac{70}{1+69e^{-0.6t}}$

The number of days until 69 employees have been infected is

$$69 = \frac{70}{1+69e^{-0.6t}}$$

$$69(1 + 69e^{-0.6t}) = 70$$

$$69 + 4761e^{-0.6t} = 70$$

$$-69$$

$$4761e^{-0.6t} = 1$$

$$t = \frac{\ln\left(\frac{1}{4761}\right)}{-0.6} = 14$$

7) **Find the amount that results from the given investment.**

$600 invested at 7% compounded daily after a period of 4 years. *Daily → n = 365*

$$600\left(1 + \frac{0.07}{365}\right)^{(4\cdot365)} \approx \$793.86$$

8) **The sound level, L, in decibels (db), is given by the formula** $L = 10\cdot\log(I \times 10^{12})$ **db, where I is the intensity of the sound in watts per square meter. The sound level is 90db. What value of I gives sound of 90db?**

$$90 = 10\cdot\log(I \times 10^{12})$$

$$9 = \log(I \times 10^{12})$$

$$10^9 = I \times 10^{12}$$

$$I = \frac{10^9}{10^{12}} = .001$$

9) **Find the principal needed now to get the given amount, that is, find the present value to get $300 after 4 years at 9% compounded monthly.** *monthly → n = 12*

$$300 = P\left(1 + \frac{0.09}{12}\right)^{(4\cdot12)} \rightarrow \quad P = 300\left(1 + \frac{0.09}{12}\right)^{(-4\cdot12)}$$

negative exponent means dividing* **P ≈ $209.58

10) How many years will it take for an initial investment of $40,000 to grow to $60,000? Assume rate of interest of 20% compounded continuously.

$$60,000 = 40,000(e)^{(.0.2 \cdot t)}$$

$1.5 = (e)^{(0.2t)}$ 1.5 = $e^{0.2t}$ *then take ln of both sides* →

$\frac{\ln(1.5)}{0.2} = \frac{0.2t}{0.2}$ lne = 1 so its cancelled

t ≈ 2.03

11) What will a $210,000 house cost 5 years from now if the price appreciation for homes over the period averages 3% compounded annually?

$A = 210000\left(1 + \left(\frac{0.03}{1}\right)\right)^{(5)}$ **A ≈ $243,447.56**

12) Find the principal needed now to get the given amount; that is, find the present value

To get $200 after 4 years at 4% compounded quarterly

$200 = P\left(1 + \left(\frac{0.04}{4}\right)\right)^{(4 \cdot 4)}$ → $P = 200\left(1 + \left(\frac{0.04}{4}\right)\right)^{(-4 \cdot 4)}$ *** to get use negative exponent**

**negative exponent means dividing to find P*

P ≈ $170.56

13) Find the principal needed now to get the given amount; that is, find the present value.

To get $60 after $2\frac{1}{4}$ years at 5% compounded continuously

$60 = Pe^{(0.05 \cdot 2.25)}$ → $60e^{(-0.05 \cdot 2.25)}$ *** to get use negative exponent**

negative exponent means dividing* **P ≈ $53.62

14) Find the amount that results from the given investment.

$60 invested at 8% compounded continuously after a period of 4 years

$60e^{(0.8 \cdot 4)} \approx \$82.63$

15) If Tanisha has \$ 100 to invest at 9% per annum compounded monthly, how long will it be before she has \$ 250? If the compounding is continuous, how long will it be?

(a) monthly → *n = 12*

$$250 = 100\left(1+\frac{0.09}{12}\right)^{(12t)}$$ *divide by 100 2.5 = $(1.0075)^{4t}$ *then take ln of both sides*

$$\rightarrow \frac{\ln 2.5}{(4\ln 1.0075)} = \frac{(4t)\ln 1.0075}{4\ln 1.0075}$$

divide right side to get t:

$$\left(\frac{\ln(2)}{(4\ln 1.02)}\right) \rightarrow t \approx 10.22$$

(b) compounded continuously, how long will it be?

$$250 = 100(e)^{(.09t)}$$ *divide by 100 2.5 = $e^{0.09t}$ *then take ln of both sides* →

$$\frac{\ln(2.5)}{.09} = \frac{.09t}{.09}$$

t ≈ 10.18

16) Jerome will be buying a used car for \$10,000 in 2 years. How much money should he ask his parents for now so that, if he invests it at 9% compounded continuously, he will have enough to buy the car?

$$10,000 = P(e)^{(.09\cdot 2)}$$ **negative exponent means dividing*

$$10,000(e)^{(-.09\cdot 2)}$$ **P≈ \$8352.70**

17) Find the amount that results from the given investment.

\$500 invested at 9% compounded daily after a period of 3 years

Have to put all parenthesis $500\left(1+\left(\frac{0.09}{365}\right)\right)^{(3\cdot 365)} \approx \$654.96$

**exponent also has to be in parenthesis after the ^ key*

18) How many years will it take for an initial investment of \$10,000 to grow to \$25,000? Assume a rate of interest of 9% compounded continuously.

$$25,000 = 10,000(e)^{(.0.09\cdot t)}$$

$$2.5 = (e)^{(0.09t)}$$ 2.5 = $e^{0.09t}$ *then take ln of both sides* →

$$\frac{\ln(2.5)}{0.09} = \frac{0.09t}{0.09}$$

t ≈ 10.18

19) The half-life of carbon-14 is 5600 years. If a piece of charcoal made from the wood of a tree shows only 61% of the carbon-14 expected in living matter, when did the tree die?

Find rate (k) first: $\ln 0.5 = e^{5600k}$ $\quad .61 = e^{\left(\frac{\ln 0.5}{5600}\right)t}$

$k = \frac{\ln 0.5}{5600}$ $\quad \ln .61 = \frac{\ln 0.5}{5600}t$ *multiply by reciprocal*

$\frac{(5600 \cdot \ln 0.61)}{\ln 0.5} = t \approx$ **3993**

15) The population of a colony of mosquitoes obeys the law of uninhibited growth. If there are 1000 mosquitoes initially and there are 1400 after 1 day, what is the size of the colony after 4 days? How long is it until there are 50,000 mosquitoes?

**k replaces r in the formula and find k first*

Find **rate (k)** first: $1400 = 1000e^{k(1)}$ $\quad$ 4 days → $1000e^{(4 \cdot \ln 1.4)}$

divide by 1000 $\quad 1.4 = e^{k}$ $\quad \approx$ **3842 mosquitoes**

$\ln 1.4 = k$

$50000 = 1000e^{\ln 1.7t}$

divide by 1000 $\quad 50 = e^{\ln 1.7t}$

take ln of both sides $\quad \ln 50 = \ln 1.4t \quad t = \frac{\ln 50}{\ln 1.4}$ **t ≈ 11.6 days**