QUIZ 6 Polynomial Equations **MATH 161** **THOMPSON**

1a) Find a polynomial function f(x) of degree 3 with real coefficients that satisfies the following conditions.

Zero of 0 and zero of 4 having multiplicity 2; f(5) = 20

a. Show the factored format of the polynomial on your answer sheet.

b. Type the expanded form answer on the computer.

$y = ax(x-4)^2$ **x = 5 and y = 20, Find a**

$20 = a(5)(x-4)^2$

$20 = 5a$

$4 = a$

$f(x) = 4x(x-4)^2$ **** you can enter this as the answer**

$4x(x^2 - 8x + 16)$

$4x^3 - 32x^2 + 64x$ if you get it wrong it will give you the answer in expanded form

1b) Find a polynomial function f(x) of degree 3 with real coefficients that satisfies the following conditions.

Zero of 0 and zero of 1 having multiplicity 2; f(2) = 8

$y = ax(x-1)^2$ **x = 2 and y = 8, Find a**

a. Show the factored format of the polynomial on your answer sheet.

b. Type the expanded form answer on the computer.

$8 = a(2)(2-1)^2$

$8 = 2a \quad a = 4$

$f(x) = 4x(x-1)^2$

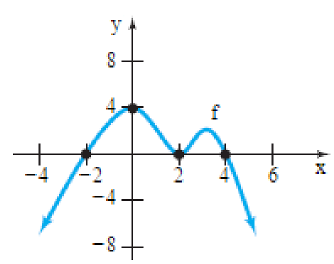
The polynomial function is $f(x) = 4x(x-1)^2$.
(Simplify your answer. Do not leave in factored form. Place expanded form in the blank.)

2) Construct a polynomial function that might have the given graph.

Models $-x^4$

crosses at x=-2 and 4 $(x+2)(x-4)$

touches at x = -2 $(x-2)^2$



*look for $(x-2)^2$

*downward, negative in front

Which of the following is a polynomial function that might have the given graph?

- A. $f(x) = \frac{1}{8}(x+2)(x-2)(x-4)$
- B. $f(x) = -\frac{1}{8}(x+2)(x-2)(x-4)$
- ✔ C. $f(x) = -\frac{1}{8}(x+2)(x-2)^2(x-4)$
- D. $f(x) = -\frac{1}{8}(x+2)^2(x-2)(x-4)^2$
- E. $f(x) = -\frac{1}{8}(x+2)(x-2)(x-4)^2$
- F. $f(x) = \frac{1}{8}(x+2)(x-2)^2(x-4)$

3) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$$g(x) = \frac{9 - x^5}{5}$$

Determine whether g(x) is a polynomial or not. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Write the polynomial in standard form. Then identify the leading term and the constant term. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The polynomial in standard form is $g(x) = -\frac{1}{5}x^5 + \frac{9}{5}$ with the leading term $-\frac{1}{5}x^5$ and the constant $\frac{9}{5}$.

4) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$$g(x) = x^{\frac{7}{2}} - x^2 + 3$$

Determine whether g(x) is a polynomial or not. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. It is not a polynomial because the variable x is raised to the $\frac{7}{2}$ power, which is not a nonnegative integer.
(Type an integer or a fraction.)

B. It is a polynomial of degree ☐.
(Type an integer or a fraction.)

C. It is not a polynomial because the function is the ratio of two distinct polynomials, and the polynomial in the denominator is of positive degree.

Write the polynomial in standard form. Then identify the leading term and the constant term. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The polynomial in standard form is g(x) = ☐ with the leading term ☐ and the constant ☐.
(Use integers or fractions for any numbers in the expressions.)

B. The function is not a polynomial.

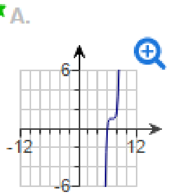
5) Use a transformation of the graph of $y = x^5$ to graph the function.

$$f(x) = (x - 7)^5 + 1$$

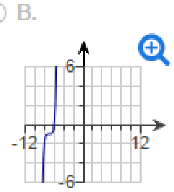
Shifts right 7 units and up 1 unit

Select the graph of $f(x) = (x - 7)^5 + 1$.

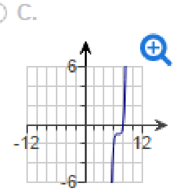
A.



B.



C.



6) Analyze the polynomial function $f(x) = (x+1)^2(x-6)^2$ using parts (a) through (e).

The graph of f behaves like $y = x^4$ for large values of $|x|$.

(b) Find the x- and y-intercepts of the graph of the function.

The x-intercept(s) is/are $-1,6$.
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

The y-intercept is 36.
(Simplify your answer. Type an integer or a fraction.)

(c) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

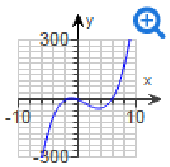
The zero(s) of f is/are $-1,6$.
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

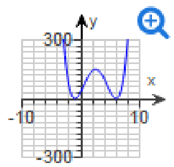
The smaller zero is a zero of multiplicity 2, so the graph of f touches the x-axis at $x = -1$. The larger zero is a zero of multiplicity 2, so the graph of f touches the x-axis at $x = 6$.

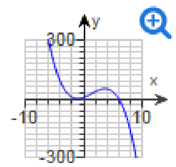
(d) Determine the maximum number of turning points on the graph of the function.

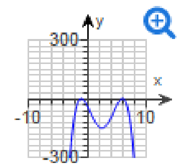
3 (Type a whole number.)

(e) Use the above information to draw a complete graph of the function. Choose the correct graph below.

A.


B. ✓


C.


D.


7) Form a polynomial whose zeros and degree are given.

Zeros: -6, multiplicity 1; 1, multiplicity 2; degree 3

Type a polynomial with integer coefficients and a leading coefficient of 1 in the box below.

$f(x) = (x+6)(x-1)^2$ (Simplify your answer.)

8) If r is a solution to the equation $f(x) = 0$, name three additional statements that can be made about f and r assuming f is a polynomial function.

Choose the correct answer below.

- A. If r is a solution to the equation $f(x) = 0$, then r is a real zero of the polynomial function f, r is a y-intercept of the graph of f, and $x + r$ is a factor of f.
- B. If r is a solution to the equation $f(x) = 0$, then r is a complex zero of the polynomial function f, r is an x-intercept of the graph of f, and $x + r$ is a factor of f.
- ✓ C. If r is a solution to the equation $f(x) = 0$, then r is a real zero of the polynomial function f, r is an x-intercept of the graph of f, and $x - r$ is a factor of f.

9) For the polynomial function below: (a) List each real zero and its multiplicity. (b) Determine whether the graph crosses or touches the x-axis at each x-intercept. (c) Determine the maximum number of turning points on the graph. (d) Determine th end behavior; that is, find the power function that the graph of f resembles for large values of |x|.

$$f(x) = -5\left(x+\frac{3}{5}\right)^2(x+7)^3$$

A. The real zero(s) of f is/are $-\frac{3}{5}, -7$.

(Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

B. There are no real zeros.

The multiplicity of the larger zero is 2.

(Type a whole number.)

The multiplicity of the smaller zero is 3.

(Type a whole number.)

(b) The graph of f touches the x-axis at the larger x-intercept.

The graph of f crosses the x-axis at the smaller x-intercept.

(c) The maximum number of turning points on the graph is 4.

(Type a whole number.)

(d) Type the power function that the graph of f resembles for large values of |x|.

$y = -5x^5$

10) Find the real solutions by factoring.

$2x^3 + 9 = x^2 + 18x$ $\quad 2x^3 - x^2 - 18x + 9 = 0$ move all to the left

factor by grouping $\quad x^2(2x-1) - 9(2x-1) = 0$

$(2x-1)(x^2 - 9) = 0$

Select the correct choice below and fill in any answer boxes within your choice.

$(2x-1)(x-3)(x+3) = 0$

A. The solution set is $\left\{\frac{1}{2}, -3, 3\right\}$.

11)

Find the x-intercepts of the graph of the given equation. Then use the x-intercepts to match the equation with its graph.

$y = (x+2)^2 - 9(x+2) + 18$

$u = x + 2$

$u^2 - 9u + 18 = 0$

$(u-6)(u-3) = 0$

$u = 6, 3$

$x + 2 = 6$ $\quad\quad$ $x + 2 = 3$

$x = 4$ $\quad\quad$ $x = 1$

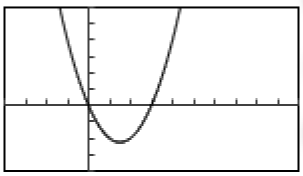
The x-intercepts are 1,4 .

(Simplify your answer. Use a comma to separate answers as needed.)

Choose the correct graph below.

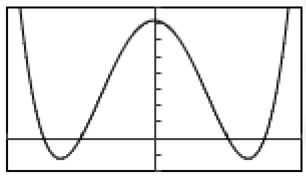
crosses x-axis at 1,4

A.



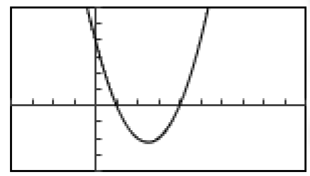
[− 4,10,1] by [− 4,6,1]

B.



[− 4,4,2] by [− 10,40,5]

C.



[− 4,10,1] by [− 4,6,1]

12) Analyze the polynomial function $f(x) = x^2(x-2)$ using parts (a) through (e).

(a) Determine the end behavior of the graph of the function.

The graph of f behaves like $y = x^3$ for large values of $|x|$.

(b) Find the x- and y-intercepts of the graph of the function.

The x-intercept(s) is/are 0,2 .

(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

The y-intercept is 0 .

(Simplify your answer. Type an integer or a fraction.)

(c) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

The zero(s) of f is/are 0,2 .

(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

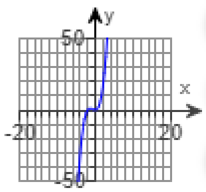
The lesser zero of the function is of multiplicity 2 , so the graph of f touches the x-axis at x = 0 . The greater zero of the function is of multiplicity 1 , so the graph of f crosses the x-axis at x = 2 .

(d) Determine the maximum number of turning points on the graph of the function.

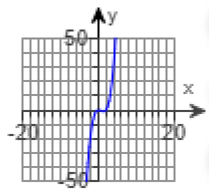
2 (Type a whole number.)

(e) Use the above information to draw a complete graph of the function. Choose the correct graph below.

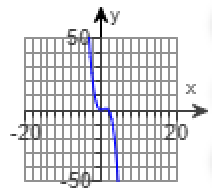
A.



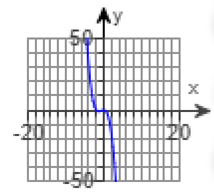
B. ✔



C.



D.



13) For the polynomial function $f(x) = 5\left(x^2+1\right)^2(x-3)^3$ answer the following questions. $f(x) = 5x^7$

(a) List each real zero and its multiplicity.
(b) Determine whether the graph crosses or touches the x-axis at each x-intercept.
(c) Determine the maximum number of turning points on the graph.
(d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of

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(a) Find any real zeros of f. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

✔ A. The real zero of f is 3 with multiplicity of 3.
(Simplify your answer. Type integers or fractions. Type each answer only once.)

B. The smallest zero of f is ___ with multiplicity of ___ and the largest zero of f is ___ with multiplicity of ___.
(Simplify your answers. Type integers or fractions. Type each answer only once.)

C. There are no real zeros.

(b) Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

✔ A. The graph crosses the x-axis at 3.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. The graph touches the x-axis at ___.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

C. The graph touches the x-axis at ___ and crosses at ___.
(Type integers or simplified fractions. Use a comma to separate answers as needed.)

D. The graph neither crosses nor touches the x-axis.

(c) The maximum number of turning points on the graph is ___.
(Type a whole number.)

(d) The power function that the graph of f resembles for large values of $|x|$ is y = 5x.

14) Find the real solutions by factoring.

$2x^3 = 5x^2$

$2x^3 - 5x^2 = 0$

$x^2(2x-5) = 0$

What is the solution set? Select the correct choice below and fill in any answer boxes within your choice.

✔ A. $\left\{0, \frac{5}{2}\right\}$

a) Solve the following inequality. *solve first then factor*

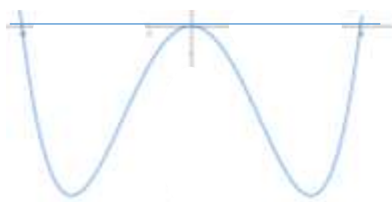
$x^4 > 36x^2$

$x^4 - 36x^2 > 0$

$x^2(x^2-36)>0$

$x^2(x+6)(x-6)>0$

*easiest to look at the graph of x^4 and touches at 0 crosses at -6 and 6



We are looking for above the x-axis since it is > 0

$(-\infty,-6)\cup(6, \infty)$

15) Zeros -3, -1, 4 and f(-2) = 24

$y = a(x+3)(x+1)(x-4)$ find a

$24 = a(-2+3)(-2+1)(-2-4)$

$24 = 6a$

$4 = a$

$y = 4(x+3)(x+1)(x-4)$ multiply out

$(4x+12)(x^2 - 3x - 4)$

$4x^3 - 12x^2 - 16x + 12x^2 - 36x - 48$

$4x^3 - 52x - 48$

16) Zeros are 0 and 2 of multiplicity 2, f(3) = 18

Put into factored form: $y = x(x-2)^2$

plug in 3 for x and 18 for y and solve for a

$18 = a(3)(3 - 2)^2$

$18 = 3a$ $a = 6$

then write equation with a = 6

$y = 6x(x-2)^2$

Expand the equation: $y = 6x(x^2 - 4x + 4)$

$y = 6x^3 - 24x^2 + 24x$

17) Zeros are -3, -1, 4 f(-2) = -18

Put into factored form: $y = (x+3)(x+1)(x-4)$

plug in -2 for x and -18 for y and solve for a

$-18 = a(-2+3)(-2+1)(-2-4)$

$-18 = 6a$ $a = -3$

then write equation with a = 6

$y = -3(x+3)(x+1)(x-4)$

Expand the equation: $y = (-3x - 9)(x^2 - 3x - 4)$

$y = -3x^3 + 9x^2 + 12x - 9x^2 + 27x + 26$

$y = -3x^3 + 39x + 26$

18) $x - 12x\sqrt{x} = 0$ move one term to the right

$(x = 12x\sqrt{x})^2$ square both sides

$x^2 = 144x^3$ move back to left to factor

$x^2 - 144x^3 = 0$

$x^2 (1-144x) = 0$ set each part equal to zero

$x^2 = 0$ and $1 - 144x = 0$

$x = 0, \frac{1}{144}$

19) $x + \sqrt{x} = 72$ $u = \sqrt{x}$ $u^2 + u - 72 = 0$

$(u-8)(u+9) = 0$ $u = -9, 8$

$\sqrt{x} = -9$ $\sqrt{x} = 8$

no solution $x = 64$

20) $4x^{1/2} - 9x^{1/4} + 3 = 0$

$u = x^{1/4}$ $4u^2 - 9u + 3 = 0$ *Use quadratic equation because it doesn't factor*

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\frac{9 \pm \sqrt{81 - 4 \cdot 4 \cdot 3}}{8}$ $u = \frac{9 \pm \sqrt{33}}{8}$

The opposite of ¼ root is raised to the 4th power, put answer in exactly like this

$x = (\frac{9+\sqrt{33}^4}{8}), (\frac{9-\sqrt{33}^4}{8})$

21) $\left(\sqrt[4]{7x^2 - 6} = x\right)^4$ raise both sides to the 4^{th} power

$7x^2-6 = x^4$ move everything to the right side0

$x^4 - 7x^2 - 6 = 0$ FACTOR, half of 1^{st} exponent

$(x^2-6)(x^2-1)=0$ set both sides equal to zero

$x^2-6=0$ and $x^2-1=0$ take square root of both

$x = \sqrt{6}$, 1

** if $x = \sqrt{8}$ must reduce radical*

$\sqrt{2 \cdot 4} \rightarrow x = 2\sqrt{2}$

22) $x^2 + 8x + 3\sqrt{x^2 + 8x} = 18$ $u=\sqrt{x^2 + 8x}$ $u^2+3u-18=0$

$(u-3)(u+6)=0$ $u = -6,3$

$\sqrt{x^2 + 8x} = -6$ $\sqrt{x^2 + 8x} = 3$

can't have negative

$x^2+8x=9$ move 9 to left to factor

$x^2+8x-9=0$ FACTOR

$(x+9)(x-1)=0$ set both sides equal to zero

$x = -9,1$

23) $x^{-2} - 6x^{-1} + 8 = 0$ $u=x^{-1}$ $u^2-6u+8=0$

$(u-2)(u-4)=0$ $u = 2,4$

$x^{-1} = 2$ $x^{-1} = 4$ negative exponent makes answer a fraction

$x = ½, ¼$

24) $5x^{2/3} - 34x^{1/3} - 7 = 0$ $u=x^{1/3}$ $5u^2-34u-7=0$ USE SLIDE AND DIVIDE

$u^2-34u-35=0$

$(u-35)(u+1)=0$ divide by 5

$u = 7, -1/5$

$x^{1/3} = 7$ $x^{1/3} = -1/5$

opposite of 1/3 exponent is cube so cube both answers

$x = 343, -\frac{1}{125}$

25) $\left(\frac{v}{v+1}\right)^2 + \frac{3v}{v+1} = 18$ u= $\frac{v}{v+1}$ the 3 is the coefficient in front of u

$u^2+3u-18=0$

$(u+6)(u-3)=0$ u = -6,3

$\frac{v}{v+1} = -6$ $\frac{v}{v+1} = 3$

-6v-6=v 3v+3=v

-6 = -7v 3 = -2v

x = $-\frac{6}{7}$ and $-\frac{3}{2}$

26) Find real solutions by factoring $x^3 - 2x^2 + 49x - 98 = 0$

Factor GCF from each highlighted part $x^2(x-2)+49(x-2)=0$

$(x^2+49)(x-2)=0$

$x^2+49= 0$ and x-2 = 0

square can never = 0 ~~x^2~~ = -49 *no solution* **x = 2**

27) Find real solutions by factoring $6x^3 + 36x = 5x^2 + 30$

move everything to the left

Factor GCF from each highlighted part $6x^3 - 5x^2 - 36x - 30=0$

Factor out the (6x-5) $x^2(6x - 5)- 6(6x - 5)=0$

$(x^2-6)(6x-5)=0$

$x^2-6= 0$ and $6x-5 = 0$ $x=-\sqrt{6}, \sqrt{6}, \frac{5}{6}$