

1) $f(x) = x^2 + 5$ $g(x) = \sqrt{x - 4}$

a. Find $f \circ g(x) = (\sqrt{x - 4})^2 + 5 = x + 1$ Domain $\{x | x \geq 4\}$

b. Find $g \circ f(x) = \sqrt{x^2 + 5} - 4 = \sqrt{x^2 + 1}$ Domain all reals

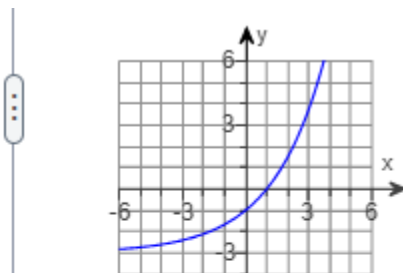
c. Find $f \circ f(x) = (x^2 + 5)^2 + 6 = x^4 + 10x^2 + 31$ Domain all reals

d. Find $g \circ g(x) = \sqrt{\sqrt{x - 4} - 4}$ Domain $\{x | x \geq 20\}$
 **Square 4 then add 4 to get 20

*ALWAYS SQUARE THE # THEN ADD IT TO ITSELF

EX : $\sqrt{\sqrt{x - 8} - 8} \rightarrow 8^2 + 8 = 75$ THEN Domain $\{x | x \geq 72\}$

- 2) The graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.



Is f one-to-one?

- ☐ No
☒ Yes

- 3) A function f has an inverse function. If the graph of f^{-1} lies in quadrant I, in which quadrant does the graph of f lie?

- ☒ Quadrant I

Quadrant I and III stay the same

Quadrants II and IV switch

4) The function $f(x) = \frac{2x+7}{x+2}$ is one-to-one.

(a) Find its inverse and check your answer.

Find the inverse of $f(x) = \frac{2x+7}{x+2}$

Switch x and y then solve for y . $x = \frac{2y+7}{y+2}$ $x(y+2) = 2y+7$

$$xy + 2x = 2y + 7$$

$$xy - 2y = -2x + 7$$

**factor out the y* $y(x-2) = -2x+7$

$$f^{-1} = \frac{-2x+7}{x-2}$$

Domain of f (original) is $\{x|x \neq -2\}$ Range of f is $\{y|y \neq 2\}$

Domain and Range of function and its *inverse* are opposite, THEREFORE...

Domain of f^{-1} is $\{x|x \neq 2\}$ Range of f^{-1} is $\{y|y \neq -2\}$

5) The function $f(x) = x^2 + 2, x \geq 0$ is one-to-one.

(a) Find the inverse of f and check the answer.

(b) Find the domain and the range of f and f^{-1} .

(c) Graph f, f^{-1} , and $y = x$ on the same coordinate axes.

$$X = y^2 + 2$$

$$x-2 = y^2$$

$$\sqrt{x-2} = y' \quad f^{-1}(x) = \sqrt{x-2}$$

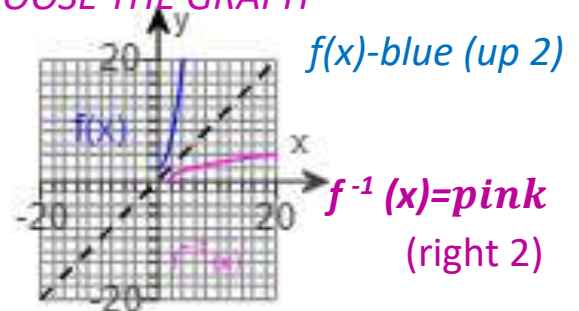
Domain of f is $x \geq 0$

Range of f is $y \geq 2$

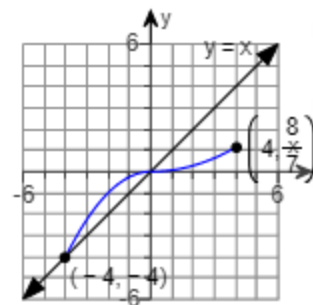
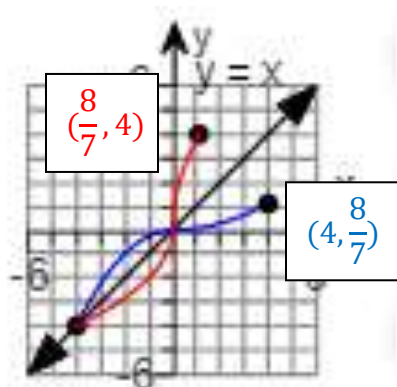
Domain of f^{-1} is $x \geq 2$

Range of f^{-1} is $y \geq 0$ *the domain and range switch

CHOOSE THE GRAPH



- 6) The graph of a one-to-one function is shown to the right. Draw the graph of the inverse function f^{-1} .



Switch x and y on the point

- 7) Find functions f and g so that $f \circ g = H$.

$$H(x) = |7x + 9|$$

$g(x)$ goes into $f(x)$

- ☐ A. $f(x) = |-x|$, $g(x) = \frac{x-9}{7}$
- ☒ B. $f(x) = |x|$, $g(x) = 7x + 9$
- ☐ C. $f(x) = 7x + 9$, $g(x) = |x|$
- ☐ D. $f(x) = \frac{x-9}{7}$, $g(x) = |-x|$

- 8) The domain of a one-to-one function f is $[4, \infty)$, and its range is $[-2, \infty)$. State the domain and the range of f^{-1} .

Switch domain and range

What is the domain of f^{-1} ?

The domain of f^{-1} is $[-2, \infty)$.

(Type your answer in interval notation.)

What is the range of f^{-1} ?

The range of f^{-1} is $[4, \infty)$.

(Type your answer in interval notation.)

9) Given $f(x) = 5x$ and $g(x) = 8x^2 + 6$, find the following expressions.

- (a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

...

(a) $(f \circ g)(4) = 670$ (Simplify your answer.) $g(4)=134 \rightarrow f(134) = 670$

(b) $(g \circ f)(2) = 806$ (Simplify your answer.) $f(2)=10 \rightarrow g(10) = 806$

(c) $(f \circ f)(1) = 25$ (Simplify your answer.) $f(1)= 5 \rightarrow f(5) = 25$

(d) $(g \circ g)(0) = 294$ (Simplify your answer.) $g(0)=6 \rightarrow g(6) = 294$

10) For $f(x) = x^2$ and $g(x) = x^2 + 1$, find the following composite functions and state the domain of each.

(a) $f \circ g$	(b) $g \circ f$	(c) $f \circ f$	(d) $g \circ g$
$(x^2+1)^2$	$(x^2)^2+1$	$(x^2)^2$	$(x^2+1)^2+1$
x^4+2x^2+1	x^4+1	x^4	x^4+2x^2+2

🟢 B. The domain of $g \circ f$ is all real numbers. For all parts because they are parabolas

11) The function $f(x) = 2x - 1$ is one-to-one.

(a) Find the inverse of f and check the answer.

(b) Find the domain and the range of f and f^{-1} .

(c) Graph f , f^{-1} , and $y = x$ on the same coordinate axes.

$$x = 2y - 1$$

$$\frac{x+1}{2} = y' \quad f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$$

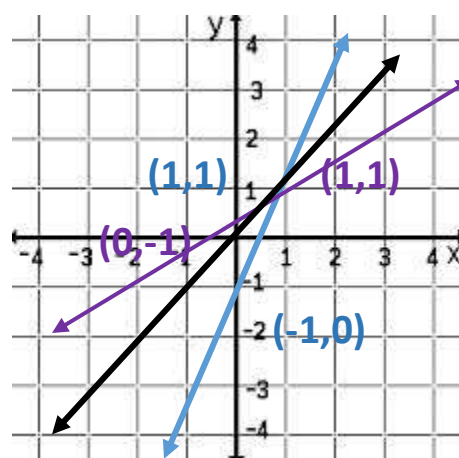
Domain and Range are all reals

Can't plot fractions on the inverse function

find points on the original line and switch the coordinates for the inverse line.

Don't forget to plot the $y = x$ line plot $(0,0)$ and $(1,1)$ to get the line

$$f(x) = 2x - 1$$



12) Find the inverse of $f(x) = \frac{8x}{x+3}$

Switch x and y then solve for y . $x = \frac{8y}{y+3}$ $x(y+3) = 8y$

$$xy + 3x = 8y$$

$$xy - 8y = -3x$$

**factor out the y* $y(x - 8) = -3x$

$$f^{-1} = \frac{-3x}{x-8}$$

Domain of f is $\{x | x \neq -3\}$ Range of f is $\{y | y \neq 8\}$

Domain and Range of function and its inverse are opposite, THEREFORE...

Domain of f^{-1} is $\{x | x \neq 8\}$ Range of f^{-1} is $\{y | y \neq -3\}$

13) A function f has an inverse function. If the graph of f^{-1} lies in quadrant IV, in which quadrant does the graph of f lie?



Quadrant II

Quadrant I and III stay the same

Quadrants II and IV switch