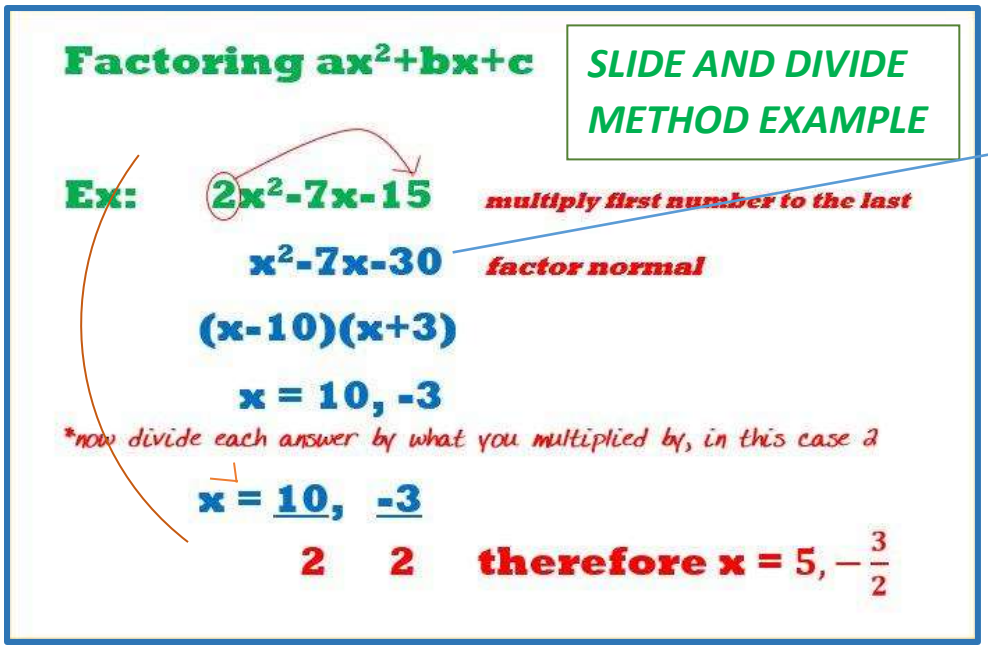
COLLEGE ALGEBRA **MATH 161** **THOMPSON**

Section 1.2 Quadratic Equations



EASY WAY TO GET FACTORS

DOUBLE	HALF
1	30
2	15

can't half 15 then we try 3

3	10
6	5

ALWAYS TRY 2, 3, 5, 7 ON THE LEFT HAND SIDE IF YOU CAN'T HALF THE RIGHT

It helps if you draw a line in the bottom blank space like the orange one when you multiply so that you do not forget to divide.

1) FACTOR $x^2 - 8x + 10$ *Factors of 10 that add to give you 8*

Does not factor so it is prime

2) FACTOR $5x^2 - 6x - 8$ **example above*

slide and divide $x^2 - 6x - 40$ *factors of 40 that subtract to give you 18*

$(x-10)(x+4)$ then divide by 5 and reduce

5 5

Put it back into parentheses $(x-2)(5x+4)$ *bottom # goes in front

3) FACTOR $27 - 3x^2$ GCF then difference between two squares

$3(9 - x^2)$

$3(3-x)(3+x)$

4) FACTOR $80 - 18x + x^2$ *factors of 80 that add to give you 18*

(10-x)(8-x) *signs are both -*

5) $(x+13)^2 - 7(x+13)$ **think of factoring a out of $a^2 - 7a$*

$(x+13)(x+13-7)$ $a(a-7)$

Then simplify $(x+13)(x+6)$

6) Which equation is set up for direct use of the zero-factor property? Solve it.

A. $5x^2 - 8x - 4 = 0$

B. $(8x + 9)^2 = 3$

C. $x^2 + x = 42$

D. $(5x - 2)(x - 8) = 0$

$5x - 2 = 0$ $x - 8 = 0$

$x = \frac{2}{5}, 8$

7) **SOLVE –*means give x = answer*** $x^2 - 15x = 0$

Factor the x out first $x(x - 15) = 0$

Then set each equal to zero $x = 0$ and $x - 15 = 0$

$x = 0$ and $x = 15$

8) **SOLVE –** $x^2 - 225 = 0$

Difference of two squares $(x - 15)(x + 15) = 0$

Then set each equal to zero $x = 15$ and $x = -15$

9) **SOLVE** $z^2 + 2z - 35 = 0$ *Factors of 35 that subtract to get 2, higher # is first sign*

$(z-5)(z+7) = 0$ $x = 5$ and $x = -7$

10) **SOLVE** $9x^2 - 80x - 9 = 0$ **example in box top of page*

slide and divide $x^2 - 80x - 81 = 0$

$(x-81)(x+1)$ then divide by 9 and reduce

9 9 $x = 9$ *and* $x = -\frac{1}{9}$

11) **SOLVE** $5x^2 - 45 = 0$ →GCF $5(x^2 - 9) = 0$

Difference of two squares $5(x - 3)(x + 3) = 0$

$x = 3$ and $x = -3$ *drop the 5 because it has no variable*

12) **SOLVE** $x(x - 7) + 10 = 0$ distribute to get into factored form

$x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

$x = 5$ and $x = 2$

13) **SOLVE** $49x^2 + 4 = 28x$ → move 28x to left

$49x^2 - 28x + 4 = 0$ *square root of first and last*

$(7x - 2)(7x - 2) = 0$ *check with FOIL*

$x = \frac{2}{7}$ *do not duplicate answer*

14) $28(p^2 - 1) = 33p$ distribute to get into factored form

$28p^2 - 28 = 33p$ Then move 33p to left

$28p^2 - 33p - 28 = 0$

SAME first and last number: *closest two factors put larger on outside and smaller on the inside negative goes on the right side. Do foil to check*

DOUBLE	HALF
1	28
2	14
4	7

Factor $(7p + 4)(4p - 7) = 0$ $x = -\frac{4}{7}, \frac{7}{4}$

16) $x^2 = 25$ take square root, must have + and - $x = -5, 5$

17) $(2y+3)^2 = 9$ Take the square root of 9, **must have + and -**

$2y+3=$**3** and $2y+3=$ **-3**

$2y = 0$ $2y = -6$

$y = 0$ $y = -3$

18) $(3y+9)^2 = 81$ Take the square root if 81, **must have + and -**

$3y+9=$**9** and $3y+9=$ **-9**

$3y = 0$ $3y = -18$

$y = 0$ $y = -6$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

19) Quadratic formula: x^2+8x-5 = 0

$$\frac{-8\pm\sqrt{64-4\cdot1\cdot(-5)}}{2} = \frac{-8\pm\sqrt{84}}{2} = \frac{-8\pm2\sqrt{21}}{2}$$ (4·21)

$$x = -4 + \sqrt{21}, -4 - \sqrt{21}$$

20) Quadratic formula: x^2-8x-2 = 0

$$\frac{8\pm\sqrt{64-4\cdot1\cdot(-2)}}{2} = \frac{8\pm\sqrt{72}}{2} = \frac{-8\pm6\sqrt{2}}{2}$$ (36·2)

$$x = -4 + 3\sqrt{2}, -4 - 3\sqrt{2}$$

21) Quadratic formula: $9y^2$-y+7 = 0

$$\frac{1\pm\sqrt{1-4\cdot9\cdot7}}{18} = \frac{1\pm\sqrt{-251}}{18}$$

Cannot have a negative under the radical NO SOLUTION

Another example: Quadratic formula: $2y^2$-y+8 = 0

$$\frac{1\pm\sqrt{1-4\cdot2\cdot(8)}}{2} = \frac{8\pm\sqrt{-63}}{2}$$

Cannot have a negative under the radical NO SOLUTION

22) Quadratic formula: $3x^2$ = 1-7x Put in correct form: $3x^2$ + 7x – 1 = 0

$$\frac{-7\pm\sqrt{49-4\cdot3\cdot(-1)}}{2(3)} = \frac{-7\pm\sqrt{61}}{6}$$

23) Quadratic formula: $\frac{2}{9}x^2$-x- $\frac{1}{9}$ = 0 Multiply by 9 first

$2x^2$-9x-1 = 0 $\frac{9\pm\sqrt{81-4\cdot2\cdot(-1)}}{4} = \frac{9\pm\sqrt{89}}{4}$

$$x = \frac{9+\sqrt{89}}{4}, \frac{9-\sqrt{89}}{4}$$

24) Quadratic formula: x^2 - 2x – 1 = 0

$$\frac{2\pm\sqrt{4-4\cdot1\cdot(-1)}}{2} = \frac{2\pm\sqrt{8}}{2} = \frac{2\pm2\sqrt{2}}{2} \quad (36\cdot 2)$$

x =$1+\sqrt{2}, 1-\sqrt{2}$

25) $81x^2$ - 18x – 1 = 0

(9x-1)(9x-1) * since the last number is one we can factor the first term

26) Watch the video that describes Unifying Quadratic Functions.

Click here to watch the video.

If a function f(x) is quadratic, explain how to find the solution to f(x) = 0 graphically.

Choose the correct answer below.

- A. Find any x-intercepts. The x-coordinates are the solutions.
- B. A solution to this equation is 0.
- C. Find any y-intercepts. The y-coordinates are the solutions.

26) Quadratic formula: $2x(x+1) = 3$ Put in correct form: $2x^2 + 2x - 3 = 0$

$$\frac{-2\pm\sqrt{4-4\cdot2\cdot(-3)}}{2(2)} = \frac{-2\pm\sqrt{28}}{4} = \frac{-2\pm2\sqrt{7}}{4} \quad (4\cdot 7)$$

x =$\frac{-1+\sqrt{7}}{2}, \frac{-1-\sqrt{7}}{2}$

27) A rectangular garden has length of 9 feet greater than the width. Find the length and width if the area is 400ft^2?

Let l = w+9 w (w+9) = 400 w^2 +9x – 400 = 0

(w+25) (w-16) then w = 16

width is 16ft and length is 25ft

28) A ball is thrown vertically upward from a building 128 ft tall with initial velocity of 112 ft/sec. The distance s (in feet) of the ball from the ground after t seconds is $s = 128 + 112t - 16t^2$

a) After how many seconds does the ball hit the ground?

$-16t^2 + 112t + 128 = 0$

$-16(t^2 - 7t - 8) = 0$

$(t-8)(t-1) = 0$ $t = 8$ seconds

b) After how many seconds does the ball pass the top of the building on its way down?

$-16t^2 + 112t + 128 = 128$

$112t = 16t^2$

$16t(t-7) = 0$ $t = 7$ seconds

MORE EXAMPLES USING PATTERNS:

a) $25x^2 - 40x + 16 = 0$

$(5x - 4)(5x - 4) = 0$ *square root of first and last*

FOIL to check

$5x-4 = 0$ $5x-4 = 0$

$x = \frac{4}{5}$

b) $45x^2 - 56x - 45 = 0$

$(9x + 5)(5x - 9) = 0$

same and first #, take closest two factors with larger on outside negative goes on the right

$9x+5 = 0$ $5x-9 = 0$

$x = -\frac{5}{9}, \frac{9}{5}$

c) $72x - 17 = \frac{72}{x}$ Multiply all by x $72x^2 - 17x - 72 = 0$

$(9x + 8)(8x - 9) = 0$

$x = -\frac{5}{9}, \frac{9}{5}$

d) $\frac{7(x-3)}{x-4}+\frac{2}{x}=\frac{-2}{x(x-4)}$ Multiply all by x(x-4) to get

7x(x-3) + 2(x-4) = -2 Then solve for $7x^2$-21x+2x-8=-2

$7x^2$-19x-6=0

(7x+2)(x-3) $x = -\frac{2}{7}, 3$

e) Quadratic formula: $3x^2 = 4x$ *do not need quadratic*

$3x^2$ - 4x = 0

x(3x – 4) = 0 $x = 0, \frac{4}{3}$

f) Quadratic formula: $5-\frac{1}{x}-\frac{3}{x^2}$ Multiply all by x^2 $5x^2 – x – 3 = 0$

$\frac{1\pm\sqrt{1-4\cdot5\cdot(-3)}}{2(5)} = \frac{1\pm\sqrt{61}}{10}$

g) Quadratic formula: $\frac{3x}{x-2}+\frac{1}{x}=4$ Multiply by x(x-2) first

$3x^2 + x - 2 = 4x^2 – 8x$ → $0 = x^2 – 9x + 2$

$\frac{9\pm\sqrt{81-4\cdot1\cdot(2)}}{2}$

$x = \frac{9+\sqrt{73}}{2}, \frac{9-\sqrt{73}}{2}$

h) $x^2 – 14 = 0$ take square root $x^2 = 14$ $x = -\sqrt{14}, \sqrt{14}$

i) $9x^2$- 6x + 1 = 0 *slide and divide* x^2- 6x + 9 = 0

(x - 3)(x - 3) = 0 *divide by 9*

$x = \frac{1}{3}$

j) $10x^2$- 19x - 15 = 0 *slide and divide* x^2- 19x - 150 = 0

(x - 25)(x + 6) = 0 *divide by 10*

$x = -\frac{5}{2}, \frac{3}{5}$

k) 5 + 23z = 10 x^2 → 0 = 10z^2 - 23x - 5 = 0 move everything to the left

slide and divide z^2 - 23x – 50 - 0

(z + 2)(z - 25) = 0 *divide by 10*

$x = -\frac{1}{5}, \frac{5}{2}$

l) $x^2 - x = 1$ Put in correct form: $x^2 - x - 1 = 0$

$\frac{1 \pm \sqrt{1-4 \cdot 1 \cdot (-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$

m) $\left(\frac{x}{x-4} + \frac{4}{x+1} = \frac{19x-1}{x^2-3x-4}\right)$ $\boxed{(x-4)(x+1)}$

(x-4)(x+1) *multiply each numerator by what is in the green but not in the denominator*

x(x+1)+4(x-4) = 19x-1

Then solve for x^2+x+4x-16=19x-1

x^2-14x-15=0

(x+1)(x-15) = 0

x = -1 but (-1+1)=0 which makes it undefined; therefore **x =** 15