

Section 1.4 PART 2

Equations of Quadratic Type

USE U SUBSTITUTION FOR X

1) $x^4 - 10x^2 + 9 = 0$ factor and take half of 1st exponent
 $u = x^2$ $u^4 - 10u^2 + 9 = 0$
 $(u^2 - 9)(u^2 - 1) = 0$
 $(u+3)(u-3)(u-1)(u+1) = 0$ PUT X BACK IN FOR U
 $x = -3, -1, 1, 3$

2) $8x^4 - 2x^2 - 1 = 0$ SLIDE AND DIVIDE and take half of 1st exponent
 $u = x^2$ $8u^2 - 2u - 1 = 0$
 $u^2 - 2u - 8 = 0$
 $(u-4)(u+2) = 0$ divide by 8
 $u = \frac{1}{2}, -\frac{1}{4}$ can't take square root of a negative number
 $x^2 = \frac{1}{2}$ ~~$x^2 = -\frac{1}{4}$~~ PUT X BACK IN FOR U
 $x = \sqrt{\frac{1}{2}} \rightarrow \sqrt{\frac{1}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

3) $x^6 - 26x^3 - 27 = 0$ $u = x^3$ $u^6 - 26u - 27 = 0$ take half of 1st exponent
 $(u^3 - 27)(u^3 + 1) = 0$ set both equal to zero
 $u = -1, 27$ PUT X BACK IN FOR U
 $x^3 = -1$ $x^3 = 27$
 $x = -1, 3$

4) $(x + 2)^2 + 9(x + 2) + 14 = 0$ u is always the middle term

$u = x+2$ $u^2 + 9u + 14 = 0$

$(u+7)(u+2) = 0$

$u = -7, -2$ PUT X BACK IN FOR U

$x+2 = -7$ and $x+2 = -2$

$x = -9, -4$

5) $(2x + 4)^2 + 4(2x + 4) + 4 = 0$ u is always the middle term

$u = 2x+4$ $u^2 + 4u + 4 = 0$

*if nothing is in front of the middle term just use u $(u+2)(u+2) = 0$

$u = -2$

$2x+4 = -2$

$x = -3$

6) $2(s + 7)^2 - 13(s + 7) = 7$ $u = s+7$ $2u^2 - 13u - 7 = 0$

$u^2 - 13u - 14 = 0$

$(u-14)(u+1) = 0$

$u = \frac{14}{2}$ may convert to 7, $\frac{-1}{2}$

$s+7 = 7$ and $s+7 = \frac{-1}{2}$

$x = 0, -7\frac{1}{2}$

7) $x - 12x\sqrt{x} = 0$ move one term to the right

$(x = 12x\sqrt{x})^2$ square both sides

$x^2 = 144x^3$ move back to left to factor

$x^2 - 144x^3 = 0$

$x^2(1 - 144x) = 0$ set each part equal to zero

$x^2 = 0$ and $1 - 144x = 0$

$x = 0, \frac{1}{144}$

$$8) \quad x + \sqrt{x} = 72$$

$$u = \sqrt{x}$$

$$u^2 + u - 72 = 0$$

$$(u-8)(u+9)=0 \quad u = -9, 8$$

$$\sqrt{x} = -9$$

no solution

$$\sqrt{x} = 8$$

$$x = 64$$

$$9) \quad 4x^{1/2} - 9x^{1/4} + 3 = 0$$

$$u = x^{1/4} \quad 4u^2 - 9u + 3 = 0 \quad \text{Use quadratic equation because it doesn't factor}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{9 \pm \sqrt{81 - 4 \cdot 4 \cdot 3}}{8}$$

$$u = \frac{9 \pm \sqrt{33}}{8}$$

The opposite of $\frac{1}{4}$ root is raised to the 4th power, put answer in exactly like this

$$x = \left(\frac{9 + \sqrt{33}}{8}\right)^4, \left(\frac{9 - \sqrt{33}}{8}\right)^4$$

$$10) \quad (\sqrt[4]{7x^2 - 6} = x)^4 \quad \text{raise both sides to the 4th power}$$

$$7x^2 - 6 = x^4 \quad \text{move everything to the right side}$$

$$x^4 - 7x^2 - 6 = 0 \quad \text{FACTOR, half of 1st exponent}$$

$$(x^2 - 6)(x^2 - 1) = 0 \quad \text{set both sides equal to zero}$$

$$x^2 - 6 = 0 \quad \text{and} \quad x^2 - 1 = 0 \quad \text{take square root of both}$$

$$x = \sqrt{6}, 1$$

* if $x = \sqrt{8}$ must reduce radical

$$\sqrt{2 \cdot 4} \rightarrow x = 2\sqrt{2}$$

$$11) \quad x^2 + 8x + 3\sqrt{x^2 + 8x} = 18$$

$$u = \sqrt{x^2 + 8x}$$

$$u^2 + 3u - 18 = 0$$

$$(u-3)(u+6)=0 \quad u = -6, 3$$

$$\sqrt{x^2 + 8x} = -6$$

can't have negative

$$\sqrt{x^2 + 8x} = 3$$

$$x^2 + 8x = 9 \quad \text{move 9 to left to factor}$$

$$x^2 + 8x - 9 = 0 \quad \text{FACTOR}$$

$$(x+9)(x-1)=0 \quad \text{set both sides equal to zero}$$

$$x = -9, 1$$

12) $x^{-2} - 6x^{-1} + 8 = 0$ $u = x^{-1}$ $u^2 - 6u + 8 = 0$
 $(u-2)(u-4) = 0$ $u = 2, 4$
 $x^{-1} = 2$ $x^{-1} = 4$ negative exponent makes answer a fraction
 $x = \frac{1}{2}, \frac{1}{4}$

13) $5x^{2/3} - 34x^{1/3} - 7 = 0$ $u = x^{1/3}$ $5u^2 - 34u - 7 = 0$ USE SLIDE AND DIVIDE
 $u^2 - 34u - 35 = 0$
 $(u-35)(u+1) = 0$ divide by 5
 $u = 7, -1/5$
 $x^{1/3} = 7$ $x^{1/3} = -1/5$
 opposite of 1/3 exponent is cube so cube both answers
 $x = 343, -\frac{1}{125}$

14) $\left(\frac{v}{v+1}\right)^2 + \frac{3v}{v+1} = 18$ $u = \frac{v}{v+1}$ the 3 is the coefficient in front of u
 $u^2 + 3u - 18 = 0$
 $(u+6)(u-3) = 0$ $u = -6, 3$
 $\frac{v}{v+1} = -6$ $\frac{v}{v+1} = 3$
 $-6v - 6 = v$ $3v + 3 = v$
 $-6 = -7v$ $3 = -2v$
 $x = -\frac{6}{7}$ and $-\frac{3}{2}$