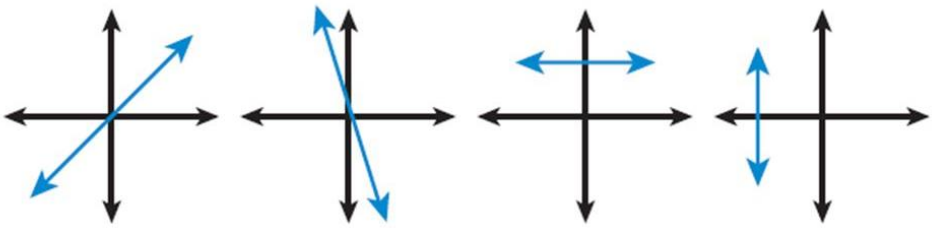
Equations of Lines

Horizontal line has slope of zero and equation is x=3
Vertical line has undefined slope and equation is y = -3



Positive Negative Zero Undefined

horizontal **vertical**

Find the x- and y-intercepts.

$-3x + 5y = 9$

x-intercept: Plug in **0** for y.

$-3x - 5(0) = 9$

$-3x = 9$

$x = -3$; **(-3, 0)**

y-intercept: Plug in **0** for x.

$-3(0) + 5y = 9$

$5y = 9$

$y = \frac{9}{5}$; $(0, \frac{9}{5})$

Various Forms of an Equation of a Line.

Slope-Intercept Form ⇒ $y = mx + b$; m = slope of the line; $b = y$-intercept

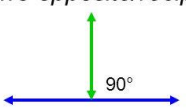
Standard Form ⇒ $Ax + By = C$; A, B, and C are integers; $A > 0$, A must be postive

Point-Slope Form ⇒ $y - y_1 = m(x - x_1)$; m = slope of the line; (x_1, y_1) is any point

Parallel vs. Perpendicular

- Parallel – Lines have the *same* slope
  - //
  - ex: $\frac{-2}{1}$ and $\frac{-2}{1}$

- Perpendicular – Lines have *opposite/reciprocal* slopes
  - ⊥
  - ex: $\frac{-2}{1}$ and $\frac{1}{2}$



SLOPE FORMULA

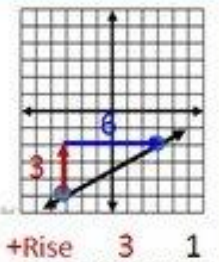
$x_1\ y_1 \quad x_2\ y_2$

(3, -2) (-3, -5)

$\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{-5 + 2}{-3 - 3} = \frac{-3}{-6}$

Slope = $\frac{1}{2}$



$\frac{+\text{Rise}}{\text{Run}\rightarrow} \quad \frac{3}{6} = \frac{1}{2}$

EX: 1

Write an Equation in Slope-Intercept Form

A line passing through (2,2) and (3,4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-2}{3-2} = 2$$

$y=mx+b$

$4=2(3)+b$

$4=6+b$

$-2=b$

$y=mx+b$

$\boxed{y=2x-2}$

EX: 2

Find Line Thru (1,10), Parallel to $2x-y=2$

Solve for y first to find the slope →

L_1 → given line

$y=mx+b$ $\quad 2x-y=2$

$-y=2-2x$

$y=2x-2 \leftarrow y=-2+2x$

Parallel lines so, SLOPE = 2

L_2 → parallel line

$y=2x+b$

$10=2\cdot1+b$

$10-2=2+b-2$

$8=b$

$\boxed{y=2x+8}$

EX: 3

Find the equation of a line Through (1,10) and Perpendicular to $2x-y=2$

$\boxed{2x-y=2}$

$y=2x-2$

Perpendicular lines so, flip 2 and change the sign

$y=\frac{-1}{2}x+b$ $\quad (1,10)$

$10=\frac{-1}{2}(1)+b$

$b=\frac{21}{2}$

$\boxed{y=\frac{-1}{2}x+\frac{21}{2}}$

EX: 4

Find the equation of the line that has x-intercept of 4 and y-intercept of -3

Point-Slope Form $y-y_1 = m(x-x_1)$

Two points are (4,0) and (0,-3)

$m = \frac{3}{4}$ $\quad$ $y-0 = \frac{3}{4}(x-4)$

$y = \frac{3}{4}x-3$

EX: 5

Find the equation of the line that has undefined slope and passes through the point (5,-4)

undefined slope means vertical line and equation is x=

$x = 5$

EX: 6

Find the equation of a horizontal line that passes through the point (1,-3) and put in general form.

horizontal line is a slope of zero and equation y=

$y = -3$

EX: 7

Find the equation of the line perpendicular to the line x = 5 and contains the point (-4,6)

The line x= is vertical and has undefined slope;
perpendicular to that is slope of zero and line y=

$y = 6$

EX: 8

Find the equation of the line that passes through the points (–2, 4) and (1, 2). Point-Slope Form $y-y_1 = m(x-x_1)$

Given two points, I can always find the slope:

$$m = \frac{(4)-(2)}{(-2)-(1)} = \frac{2}{-3} = -\frac{2}{3}$$

Then I can use either point as my (x_1, y_1), along with this slope Ive just calculated, and plug in to the point-slope form. Using $(-2, 4)$ as the (x_1, y_1), I get:

$$y - y_1 = m(x - x_1)$$
$$y - (4) = (-{}^{2}/_{3})(x - (-2))$$

Point-Slope Form → $y - 4 = (-{}^{2}/_{3})(x + 2)$

$$y - 4 = (-{}^{2}/_{3})x - {}^{4}/_{3}$$
$$y = (-{}^{2}/_{3})x - {}^{4}/_{3} + 4$$
$$y = (-{}^{2}/_{3})x - {}^{4}/_{3} + {}^{12}/_{3}$$

Slope-intercept Form → $y = (-{}^{2}/_{3})x + {}^{8}/_{3}$

General Form → $\frac{2}{3}x + y = \frac{8}{3}$