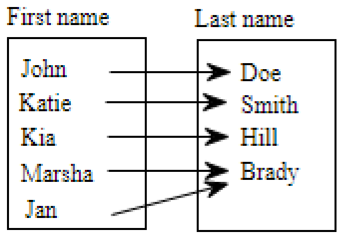
Section 3.1-3.2-3.3 MATH 161 THOMPSON

1) State the domain and range for the following relation. Then determine whether the relation represents a function.

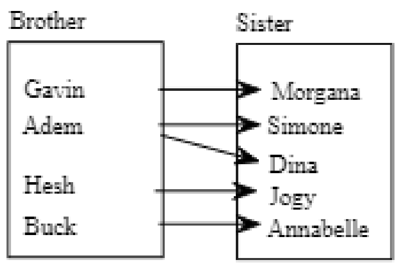


Domain : {John, Katie, Kia, Marsha, Jan}
Range : {Doe, Smith, Hill, Brady}

Choose the correct answer below.

Yes, because each element in the first set corresponds to exactly one element in the second set.

2) State the domain and range for the following relation. Then determine whether the relation represents a function.



Domain : {Adem, Hesh, Buck, Gavin}
Range : {Simone, Dina, Jogy, Annabelle, Morgana}

Does the relation represent a function?

✔ A. The relation in the figure is not a function because the element Adem in the domain corresponds to more than one element in the range.

3) State the domain and range for the following relation. Then determine whether the relation represents a function.

$\{(2,5), (-5,5), (7,9), (2,11)\}$

The domain of the relation is $\{-5,2,7\}$.
(Use a comma to separate answers as needed.)

The range of the relation is $\{5,9,11\}$.
(Use a comma to separate answers as needed.)

Does the relation represent a function? Choose the correct answer below.

○ A. The relation is not a function because there are ordered pairs with 9 as the second element and different first elements.

✔ B. The relation is not a function because there are ordered pairs with 2 as the first element and different second elements.

*can't have duplicating x values in the ordered pairs.

4) Determine whether the equation defines y as a function of x.

$y = 4x^2 - 3x - 5$

Does the equation define y as a function of x?

- [x] Yes

5) Determine whether the equation defines y as a function of x.

$y = \frac{2}{x}$

Does the equation define y as a function of x?

- [x] Yes

6) Determine whether the equation defines y as a function of x.

$y^2 = 8 - x^2$

Does the equation define y as a function of x?

- [ ] Yes
- [x] No

*can't have y^2

7) Determine whether the equation defines y as a function of x.

$x = y^2$

Does the equation define y as a function of x?

- [ ] Yes
- [x] No

8) **$f(x) = 4x^2 + 2x - 4$**

a) Find f(-x) $4(-x)^2 + 2(-x) - 4 = 4x^2 - 2x - 4$

b) Find f(x+1) $4(x+1)^2 + 2(x+1) - 4$ FOIL $(x+1) \rightarrow x^2 + 2x + 1$

$4(x^2 + 2x + 1) + 2(x+1) - 4$ distribute

$4x^2 + 8x + 4 + 2x + 2 - 4$ combine like terms

$4x^2 + 10x + 2$

c) Find f(5x) $4(5x)^2 + 2(5x) - 4 =$

$4(25x^2) + 2(5x) - 4 = 100x^2 + 10x - 4$

d) Find f(x+h) **$4(x+h)^2 + 2(x+h) - 4$** **FOIL (x+h) → $x^2 + xh + xh + h^2$**

$4(x^2 + 2xh + h^2) + 2(x+h) – 7$ **distribute**

$4x^2 + 8xh + 4h^2 + 2x + 2h – 4$ ***no terms combine***

9) Let $g(x) = -x^2 + 4x + 2$. Find and simplify $g(-2)$. **substitute -2 in for all x values**

$-(-2)^2 + 4(-2) + 2$ = $-4 – 8 + 2$ = -10

10) Let $f(x) = -4x + 3$. Find $f\left(\frac{1}{2}\right)$. $-4\left(\frac{1}{2}\right) + 3 = 1$

11) Find the domain of the function: f(x) = -6x + 6

This is a line so the domain is all reals: Interval notation (-∞,∞)

KEY EXAMPLES TO FIND THE DOMAIN: *We only solve the bottom for domain*

Inequality answer –blue Interval answer-red

when the bottom is factored and has two answers make a number line to easily see the intervals to make interval notation answer. (Ex 15 and 17)

12) f(x) = x^2+8x

all reals (-∞,∞)

13) g(x) = $\frac{7x}{x^2-16}$ *(x+4)(x-4)* -4 42

{x|x≠ -4,4} *three intervals* **(-∞,-4)∪(-4,4)∪(4,∞)**

14) h(t) = $\sqrt{6-2t}$

6-2t ≥ 0

{t|t ≤ 3} (-∞,3]

15) j(x) = $\frac{x}{x^2+25}$

doesn't factor so no restrictions **(-∞,∞)**

16) G(x) = $\frac{x-6}{\sqrt{x+7}}$

x+7 > 0 x≠0 nor negative

{x|x>-7} **(-7,∞)**

EX) k(x) = $\frac{x-6}{x^3+x}$ 0

$x(x^2+1)$ {x|x≠0} (-∞,0)∪(0,∞)

can't factor x^2+1

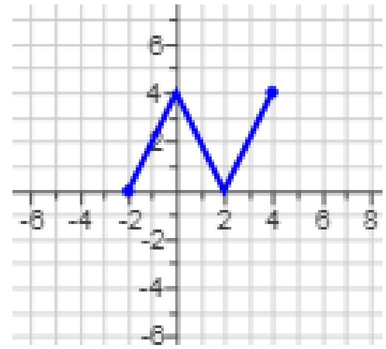
EX) F(x) = $\frac{\sqrt{2x+18}}{x+3}$ **{x|x≥-9,x≠-3}**

x≠-3 *two intervals* **-9** **-3**

2x+18≥ 0 x≥-9 **[-9,-3)∪(-3,∞)**

17) Use the graph of y = f(x) to find each function value.

(a) f(− 2)   (b) f(0)
(c) f(3)   (d) f(4)



(a) f(− 2) = 0   *Find the y when x = -2*

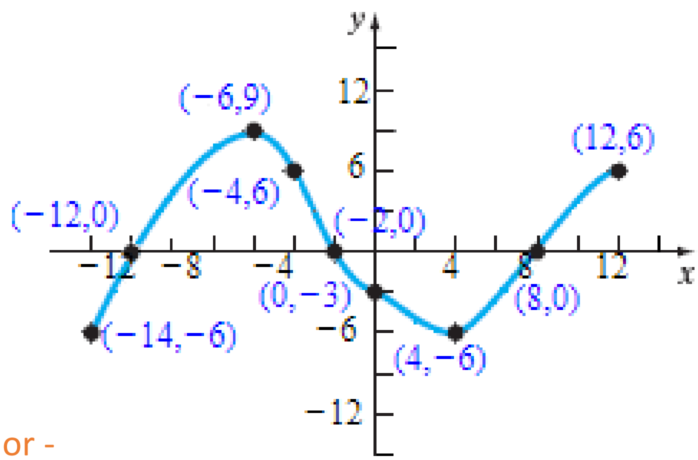
(b) f(0) = 4

(c) f(3) = 2

(d) f(4) = 4

18)

Use the graph of the function f shown to the right to answer parts (a)-(n).



(a) Find f(− 14) and f(− 4).

f(− 14) = -6   Find y when x is -14

f(− 4) = 6   Find y when x is -4

(b) Find f(12) and f(0).

f(12) = 6

f(0) = -3

(c) Is f(4) positive or negative?

At x = 4 is the y value + or -

- Positive
- Negative ✔

(d) Is f(− 4) positive or negative?

- Positive ✔
- Negative

At x = -4 is the y value + or -

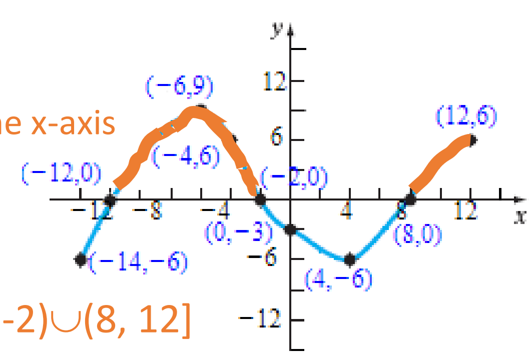
(e) For what value(s) of x is f(x) = 0?

x = -12, -2, 8

(Use a comma to separate answers as needed.)

(f) For what values of x is f(x) > 0?   The intervals above the x-axis

-12<x<-2, 8<x≤12



(-12,-2)∪(8, 12]

(g) What is the domain of f?

The domain of f is { {x|-14≤x≤12} Most left point and most right point

(Type a compound inequality.)

(h) What is the range of f?

The range of f is {y|-6≤y≤9} Lowest and highest point

(Type a compound inequality.)

(i) What are the x-intercept(s)? Where the graph touches the x-axis and same as f(x) = 0 in part e

x = -12, -2, 8,

Where the graph touches the y-axis

(j) What are the y-intercept(s)?

y = -3

(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

(k) How often does the line y = 1 intersect the graph?

If you draw a horizontal line at 1

3 times

(l) How often does the line x = 2 intersect the graph?

If you draw a vertical line at

1 times

(m) For what value(s) of x does f(x) = − 6?

x =-14, 4 Give x where y is -6

(Use a comma to separate answers as needed.)

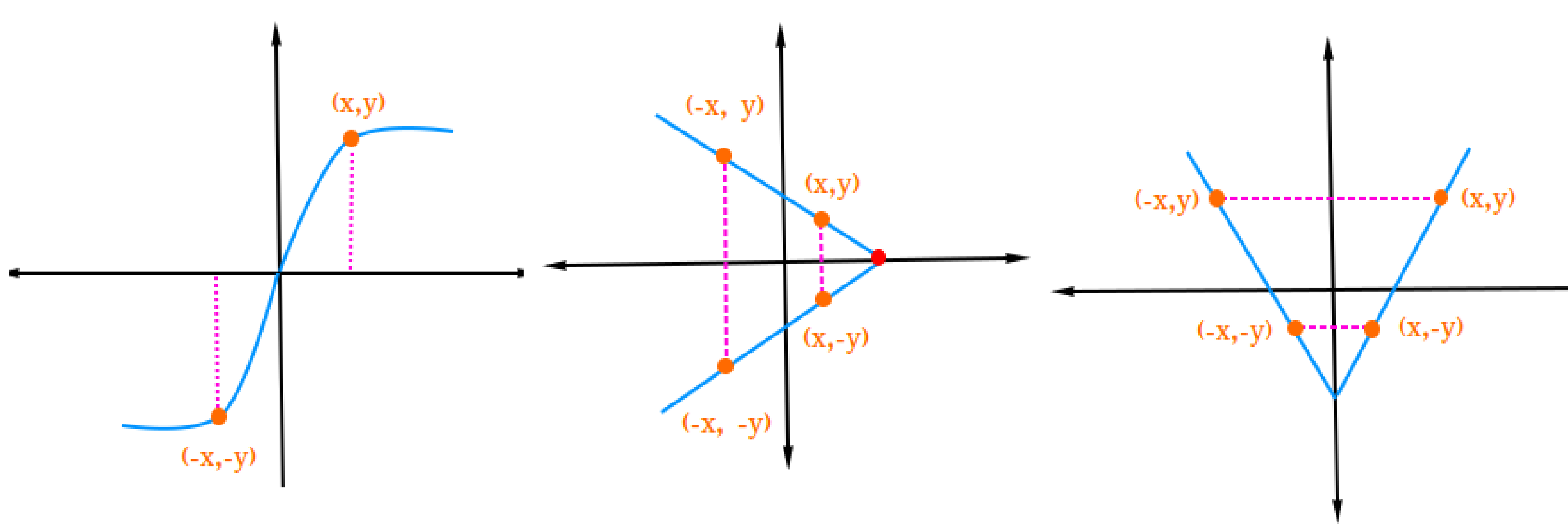
(n) For what value(s) of x does f(x) = 9?

x = -6 Give x where y is 9

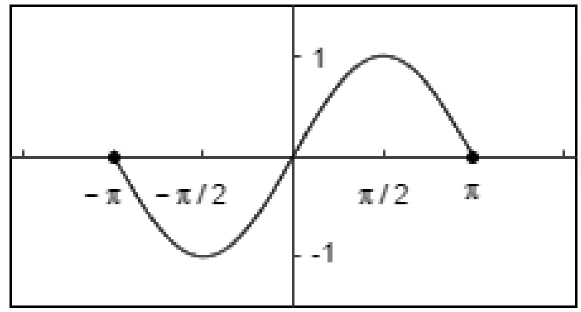
(Use a comma to separate answers as needed.)

KEY EXAMPLES ON SYMMETRY

Symmetry with respect to the origin | Symmetry with respect to x-axis | Symmetry with respect to the y-axis



19) Determine whether the graph below is that of a function by using the vertical-line test. If it is, use the graph to find
(a) its domain and range.
(b) the intercepts, if any.
(c) any symmetry with respect to the x-axis, y-axis, or the origin.



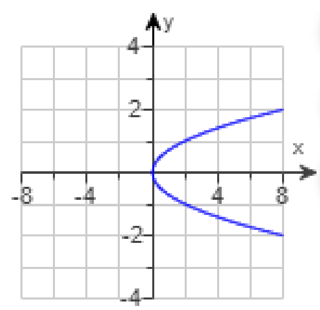
(a) Domain: [-π,π] Take you pencil and move from left to right to see where graph start and ends.
brackets mean solid dots and include the point

Range: [-1,1] Take you pencil and move from bottom to top to see where graph start and ends.

(b) Intercepts: (-π,0),(0,0),(π,0) *list as ordered pairs*

(c) symmetrical with respect to the origin (graph examples above)

20) Determine whether the graph is that of a function by using the vertical-line test. If it is, use the graph to find
(a) its domain and range.
(b) the intercepts, if any.
(c) any symmetry with respect to the x-axis, y-axis, or the origin.

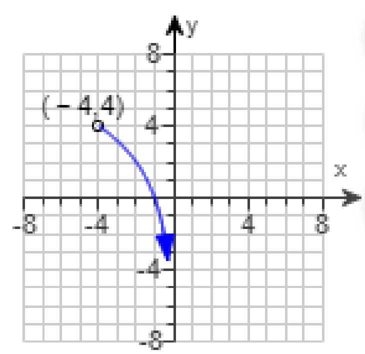


vertical line test crosses it more than once

The graph is not a function for ALL answers

21) Determine whether the graph on the right is that of a function by using the vertical-line test. If it is, use the graph to find the following.

(a) the domain and range (assume that the curve approaches but never intersects the y-axis)
(b) the intercepts, if any
(c) any symmetry with respect to the x-axis, y-axis, or the origin



Yes, the graph is a function because every vertical line intersects in at most one point

(a) Domain: (-4,0) Take you pencil and move from left to right to see where graph start and ends. The graph curves and gets close to 0 parenthesis because it has open circle

Range: (-∞4) Take you pencil and move from bottom to top to see where graph start and ends.

parenthesis because it has open circle and infinity – arrow points down

(b) Intercepts: (-1,0)

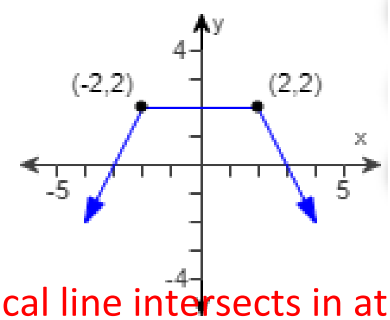
(c) Has no symmetry

22) Determine whether the graph is that of a function by using the vertical-line test. If it is, use the graph to find

(a) its domain and range.

(b) the intercepts, if any.

(c) any symmetry with respect to the x-axis, y-axis, or the origin.



Yes, the graph is a function because every vertical line intersects in at most one point

(a) Domain: (-∞,∞) arrows both left and right with straight lines

Range: (-∞,2] arrow down and stops at 2 with closed circle

parenthesis because it has open circle and infinity – arrow points down

(b) Intercepts: (3,0),(0,2),(-3,0) *make sure to include both x and y intercepts*

(c) Symmetrical with respect to the y-axis (graph examples above)

23) $f(x) = 3x^2 - x - 2$

a) Is the point (-1,2) on the graph of f? $3(-1)^2 - (-1) - 2 = 2$

Yes, because substituting x=-1 into the equation results in 2

b) If x=2, what is f(x)? $3(2)^2 - (2) - 2 = 8$

list the point(s) on the graph where x=2 (2,8)

c) If f(x) = -2, what is x? $-\cancel{2} = 3x^2 - x - \cancel{2}$

$0 = 3x^2 - x$ then factor out an x

$0 = x(3x-1)$ set each part = 0

$x = 0$ $3x-1 = 0$ $x = 0, \frac{1}{3}$

list the point(s) on the graph where f(x)=-2 $(0,-2), (\frac{1}{3},-2)$

d) What is the domain of f: the graph is a parabola $(-\infty,\infty)$

e) What are the x=intercepts? factor $3x^2 - x - 2 = 0$ using slide and divide

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$ divide by 3 $x = 1, -\frac{2}{3}$

f) What are the y=intercepts? substitute 0 in for all x values

$y = 3(0)^2 - (0) - 2$ $y = -2$

24) $f(x) = \frac{x+11}{x-3}$

a) Is the point (6,8) on the graph of f? $\frac{6+11}{6-3} = \frac{17}{3}$ NO, substituting x=6 doesn't =8

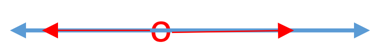
b) If x=2, what is f(x)? $\frac{2+11}{2-3} = \frac{13}{-1}$ f(x) = -13 List the point $(2,-13)$

c) If f(x) = 2, what is x? $\frac{x+11}{x-3} = 2$ cross multiply

$2x-6 = x+11 \rightarrow x=17$ List the point (17,2)

d) Give the domain: set denominator =0 $\{x \mid x \neq 3\}$

make a number line



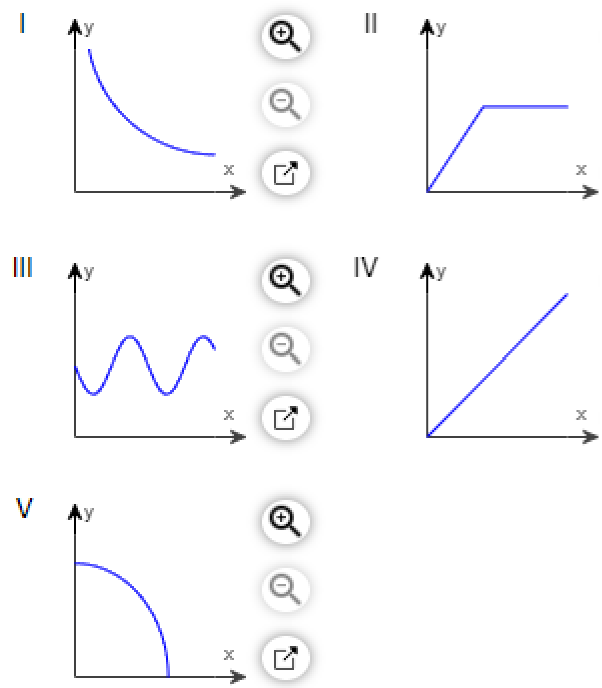
3 $(-\infty,3)\cup(3,\infty)$

e) What are the x=intercepts? Set the numerator = 0 $x + 11 = 0$ -11

f) What are the y=intercepts? Set x values = 0 $\frac{0+11}{0-3} = 0$ $-\frac{11}{3}$

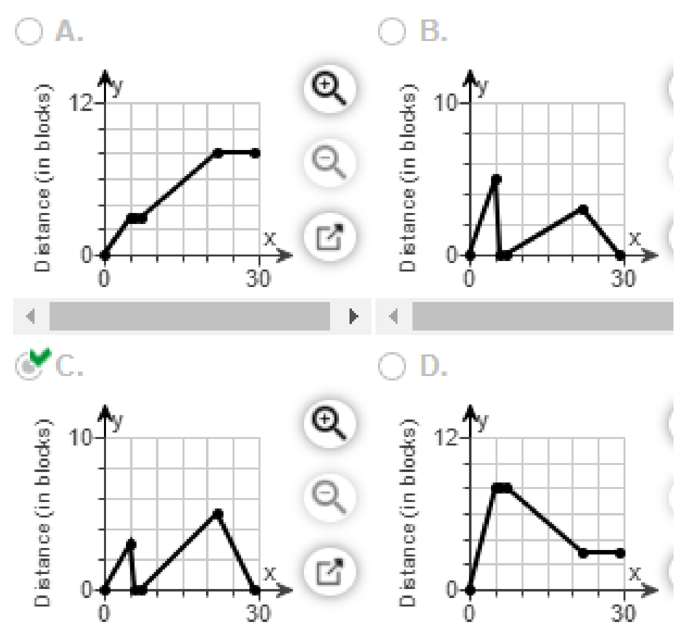
25) Match the following functions with the graph.
(a) The cost of painting a wall as a function of its square footage.
(b) The height of an egg dropped from a 220-foot building as a function of time.
(c) The height of a human as a function of time.
(d) The demand for hamburger as a function of price.
(e) The height of a child on a swing as a function of time.

(a) IV
(b) V
(c) II
(d) I
(e) III



26) A person decides to take a walk. He leaves home, walks 3 blocks in 5 minutes at a constant speed, and realizes that he forgot to lock the door. So he runs home in 1 minute. While at his doorstep, it takes him 1 minute to find his keys and lock the door. He walks 5 blocks in 15 minutes and then decides to jog home. It takes him 7 minutes to get home. Draw a graph of his distance from home (in blocks) as a function of time.

Choose the correct graph below.

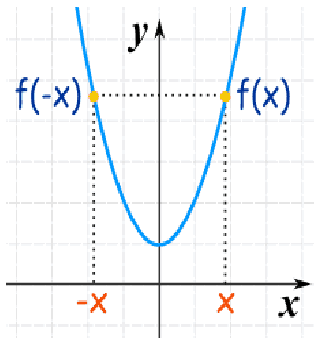


Even Functions

A function is "even" when:

$f(x) = f(-x)$ for all x

In other words there is symmetry about the y-axis (like a reflection):



This is the curve $f(x) = x^2+1$

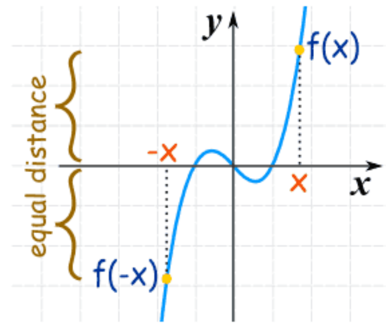
Odd Functions

A function is "odd" when:

$-f(x) = f(-x)$ for all x

Note the minus in front of f: $-\mathbf{f(x)}$.

And we get origin symmetry:



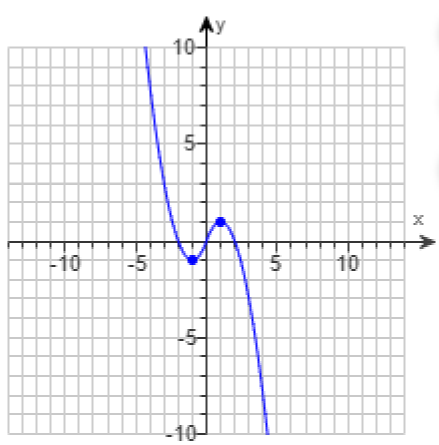
This is the curve $f(x) = x^3-x$

27) An even function f is one for which $f(-x) = f(x)$ for every x in the domain of f; an odd function f is one for which $f(-x) = -f(x)$ for every x in the domain of f.

28) Even functions have graphs that are symmetric with respect to the y-axis.

29) An odd function is symmetric with respect to the origin.

30) Use the graph of the function f given below to answer the questions.



Is there a local maximum at x = 1?

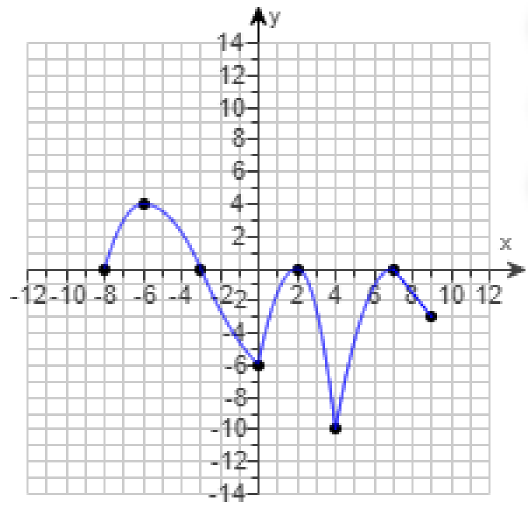
- ✔ Yes
- ○ No *Is the graph the lowest at x=1*

If there is a local maximum at x = 1, what is it? Select the correct choice below and fill in any answer boxes within your choice.

✔ A. The local maximum is y = 1.
(Type an integer.) *What is the y value at x=1*

○ B. There is no local maximum at x = 1.

31) Find the absolute maximum of f on $[-8,9]$.



Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

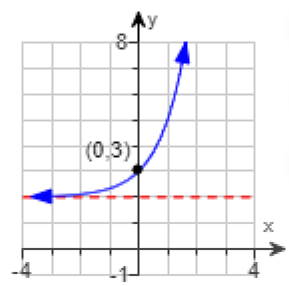
the highest point at x=-6 is y=4

A. The absolute maximum of f is $f(-6) = 4$.
(Type integers or fractions.)

B. There is no absolute maximum.

32) Using the given graph of the function f, find the following.

(a) the intercepts, if any
(b) its domain and range
(c) the intervals on which it is increasing, decreasing, or constant
(d) whether it is even, odd, or neither



(a) What are the intercepts?

(0,3)

(b) The domain is $(-\infty,\infty)$.

(c) On which interval(s) is the graph increasing?

A. The graph is increasing on $(-\infty,\infty)$.

On which interval(s) is the graph decreasing?

B. There is no interval on which the graph is decreasing.

On which interval(s) is the graph constant?

B. There is no interval on which the graph is constant.

(d) The function is neither even nor odd.

Even Functions

A function is "even" when:

$f(x) = f(-x)$ for all x

Odd Functions

A function is "odd" when:

$-f(x) = f(-x)$ for all x

33) Determine algebraically whether the given function is even, odd, or neither.

$f(x) = -4x^4$

- Neither *Hint* If exponent is even then even function always*
- Even
- Odd

34) Determine algebraically whether the given function is even, odd, or neither.

$g(x) = 8x^3 + 2$

- Odd
- Even
- ✔ Neither

Hint If exponent is odd then odd function unless there is a constant therefore the 2 makes it neither*

35) Determine algebraically whether the given function is even, odd, or neither.

$f(x) = \sqrt[9]{3x}$

- Even
- Neither
- ✔ Odd

*Hint *If exponent is odd then odd function unless there is a constant*

36) Determine algebraically whether the given function is even, odd, or neither.

$f(x) = 2x^2 + |-9x|$

- ✔ Even
- Neither
- Odd

Need to plug (-x) in for x

$2(-x)^2 + |-9(-x)|$

$2x^2 + 9x$ gives you the same function output so even

37) Determine algebraically whether the given function is even, odd, or neither.

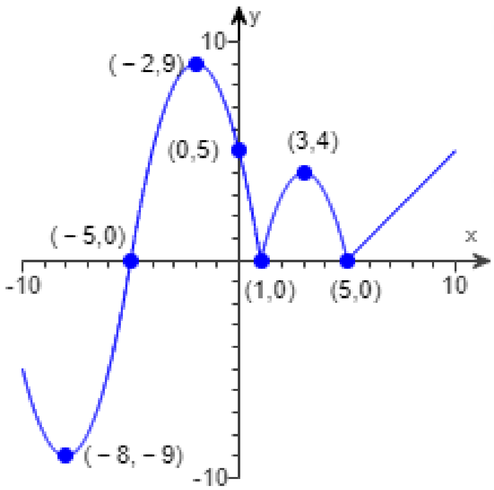
$f(x) = \frac{2}{x^4}$

Is the given function even, odd, or neither?

- ✔ A. Even
- B. Odd
- C. Neither

Hint If exponent is even then even function always*

38) Use the graph of the function f given below to answer the question.

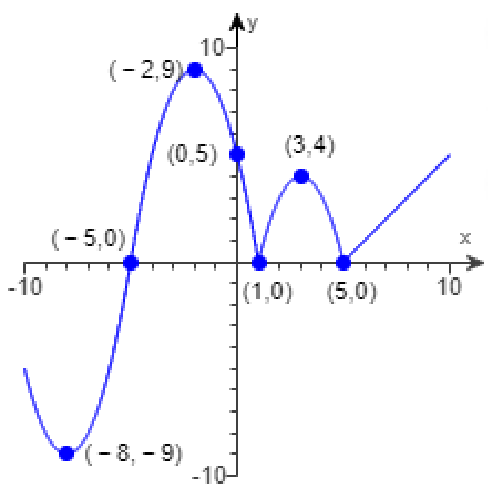


Is f strictly decreasing on the interval (− 2,0)?

- ✔ Yes
- No

The graph is only decreasing when looking at x = -2 to x = 1

39) Use the graph of the function f given below to answer the question.

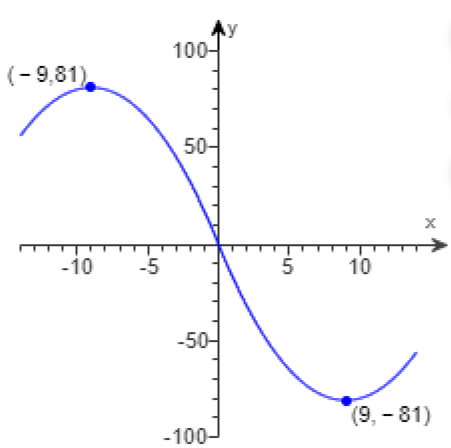


Is f strictly decreasing on the interval (0,3)?

- ✔ No
- Yes

The graph is decreasing from x= 0 to 1 then increases x = 1 to 3

40) List the intervals on which f is decreasing.



(− 9,9)

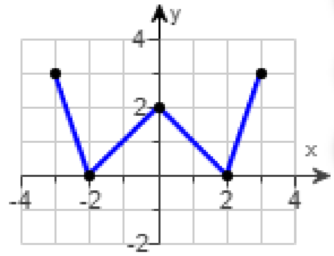
(Type your answer in interval notation. Use a comma to separate answers as needed.)

The graph is decreasing x = -9 to 9

Increasing and decreasing intervals always have parenthesis because they are constant at the end points of the interval

41) Using the given graph of the function f, find the following.

(a) the intercepts, if any
(b) its domain and range
(c) the intervals on which it is increasing, decreasing, or constant
(d) whether it is even, odd, or neither



(a) intercepts: (-2,0),(0,2),(2,0) make sure to list x and y intercepts

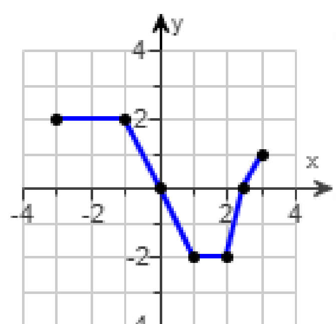
(b) Domain: [-3,3] brackets because of solid points
Range: [0,3] brackets because of solid points

(c) Increasing: (-2,0),(2,3) brackets because of solid points
Decreasing: (-3,-2),(0,2) brackets because of solid points
The graph is not constant

(d) Even because it is symmetrical with respect to the y-axis

42) Using the given graph of the function f, find the following.

(a) the intercepts, if any
(b) its domain and range
(c) the intervals on which it is increasing, decreasing, or constant
(d) whether it is even, odd, or neither



(a) intercepts: (0,0),($\frac{5}{2}$, 0) make sure to list x and y intercepts

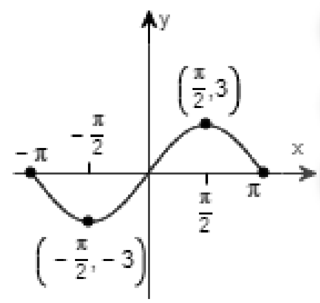
(b) Domain: [-3,3] brackets because of solid points
Range: [-2,2] brackets because of solid points

(c) Increasing: (2,3) brackets because of solid points
Decreasing: (-1,1) brackets because of solid points
Constant: (-3,-1),(1,2)

(d) Neither

43) Using the given graph of the function f, find the following.

(a) The numbers, if any, at which f has a local maximum. What are these local maxima?
(b) The numbers, if any, at which f has a local minimum. What are these local minima?



(a) local max at x = $\frac{\pi}{2}$ and the max is 3 (y value at that point)

(a) local min at x = $-\frac{\pi}{2}$ and the min is -3 (y value at that point)

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \qquad g(x) \neq 0$$

EX 1: f(x) = $x^2 - 7$ g(x) = 6x - 1

a) Find f(x)+g(x) **$x^2 - 7 + 6x - 1 = x^2 + 6x - 8$**

b) Find f(x)-f(x) **$x^2 - 7 - (6x-1) \rightarrow x^2 - 7 - 6x + 1 = x^2 - 6x - 6$**

c) Find f(x)·g(x) **$(x^2 - 7)(6x-1)$ FOIL = $6x^3 - x^2 - 42x + 7$**

d) Find $\frac{f(x)}{g(x)}$ $\frac{x^2 - 7}{6x - 1}$

EX 2: $f(x) = 2 + \frac{3}{x}$ $\quad\quad$ $g(x) = \frac{3}{x}$

a) Find f(x)+g(x) $\quad 2 + \frac{3}{x} + \frac{3}{x} = 2 + \frac{6}{x}$

b) Find f(x)-g(x) $\quad 2 + \frac{3}{x} - \frac{3}{x} = 2$

c) Find f(x)·g(x) $\quad (2 + \frac{3}{x})\frac{3}{x} = \frac{6}{x} + \frac{9}{x^2}$

$$\frac{6x}{x^2} + \frac{9}{x^2} = \frac{6x+9}{x^2}$$

d) Find $\frac{f(x)}{g(x)}$ $\quad \frac{2+\frac{3}{x}}{\frac{3}{x}} = \left(2 + \frac{3}{x}\right)\frac{x}{3} = \frac{2x}{3} + 1$

EX 3: $f(x) = 6x + 4$

h) Find $\frac{f(x+h)-f(x)}{h}$ $\quad \frac{6(x+h)+4-(6x+4)}{h}$

Simplify the top: $6x + 6h + 4 - 6x - 4$

$$\frac{6h}{h} = 6$$

EX 4: $f(x) = x^2 - 3x + 2$

h) Find $\frac{f(x+h)-f(x)}{h}$ $\qquad \frac{(x+h)^2 - 3(x+h)+2 - (x^2 - 3x+2)}{h}$

Simplify the top: $x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2$

$$\frac{2xh+h^2-3h}{h} = 2x + h - 3$$

EX 5: $f(x) = \frac{3}{x-5}$ $\qquad g(x) = \frac{x}{x+3}$

a) Find f(x)+g(x)

$$\frac{3}{x-5} + \frac{x}{x+3}$$

Common denominator

$$\frac{3(x+3)}{(x-5)(x+3)} + \frac{x(x-5)}{(x-5)(x+3)}$$

$$\frac{3x+9}{(x-5)(x+3)} + \frac{x^2-5x}{(x-5)(x+3)} = \frac{x^2-2x+9}{(x-5)(x+3)}$$

b) Find f(x)-g(x)

$$\frac{3}{x-5} - \frac{x}{x+3}$$

Common denominator

$$\frac{3(x+3)}{(x-5)(x+3)} - \frac{x(x-5)}{(x-5)(x+3)}$$

$$\frac{3x+9}{(x-5)(x+3)} - \frac{x^2-5x}{(x-5)(x+3)} = \frac{3x+9-x^2+5x}{(x-5)(x+3)} = \frac{-x^2+8x+9}{(x-5)(x+3)}$$

c) Find f(x)·g(x)

$$\frac{3}{x-5} \cdot \frac{x}{x+3} = \frac{3x}{(x-5)(x+3)}$$

d) Find $\frac{f(x)}{g(x)}$

$$\frac{\frac{3}{x-5}}{\frac{x}{x+3}} = \frac{3}{x-5} \cdot \frac{x+3}{x} = \frac{3(x+3)}{x(x-5)} = \frac{3x+9}{x^2-5x}$$

e) Find -f(x)

$$-\frac{3}{x-5}$$