TRANSLATIONS OF FUNCTIONS

Vertical stretch of 5

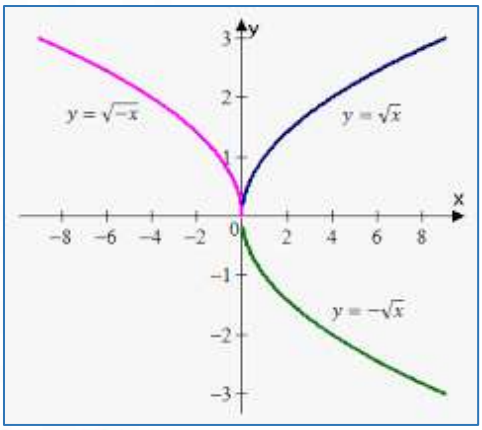
Shrink is a fraction

Shift right 3 units (when you take it out the parenthesis it changes the sign)

$$f(x) = -5(x-3)^2 + 7$$

Negative in front reflects across the x-axis

Shifts up 7 units



REFLECTION:

$f(x) = \sqrt{-x}$ *means it reflects across the* $y-$ *axis*

Horizontal Stretch is fraction inside the function $f(x) = (\frac{1}{5}x)^2$

Horizontal Shrink is whole # inside the function $f(x) = (3x)^2$

1) Suppose that the graph of a function f is known. Then the graph of $y = f(x-2)$ may be obtained by a horizontal shift of the graph of f to the right a distance of 2 units.

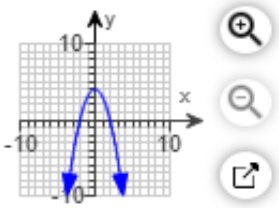
2) Suppose that the graph of a function f is known. Then the graph of $y = f(-x)$ may be obtained by a reflection about the y -axis of the graph of the function $y = f(x)$.

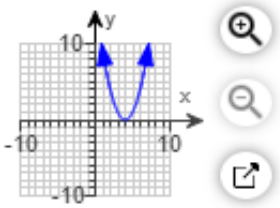
3) Which of the following functions has a graph that is the graph of $y = \sqrt{x}$ shifted up 5 units?

Choose the correct answer below.

- $y = \sqrt{x+5}$
- $y = \sqrt{x-5}$
- ✔ $y = \sqrt{x}+5$
- $y = \sqrt{x}-5$

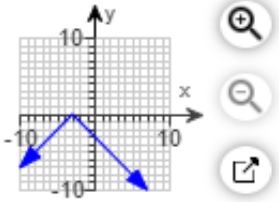
4) Drag the function given above into the appropriate area below to match the graph.

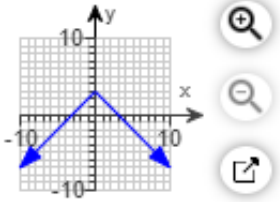




$y = -x^2 + 4$

$y = (x - 4)^2$

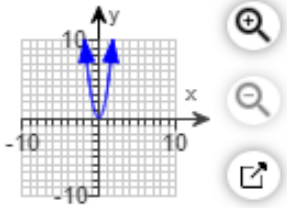


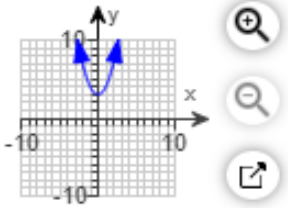


$y = -|x + 3|$

$y = -|x| + 3$

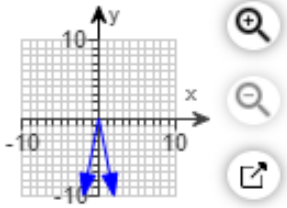
5) Drag the function given above into the appropriate area below to match the graph.

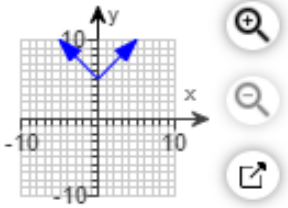




$y = 3x^2$

$y = x^2 + 3$

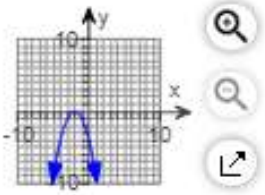




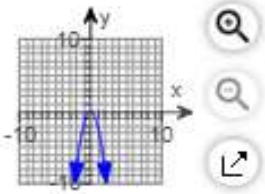
$y = -5|x|$

$y = |x| + 5$

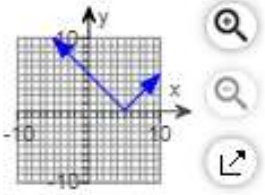
6) Drag the function given above into the appropriate area below to match the graph.



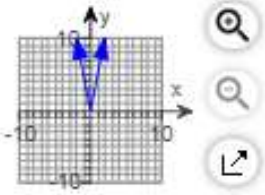
$y = -(x+2)^2$



$y = -2x^2$



$y = |x-5|$



$y = 5|x|$

7) Write the function whose graph is the graph of $y = x^3$, but is shifted to the left 8 units. $y = (x+8)^3$

8) Write the function whose graph is the graph of $y = |x|$, but is shifted up 1 unit. $y = |x|+1$

9) Write the function whose graph is the graph of $y = 2\sqrt{x}$ but is reflected about the y-axis.

The function is $y = 2\sqrt{-x}$.

(Type an exact answer, using radicals as needed.)

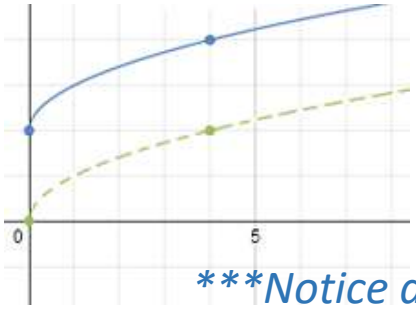
10) Write the function whose graph is the graph of $y = \sqrt{x}$, but is horizontally stretched by a factor of 7.

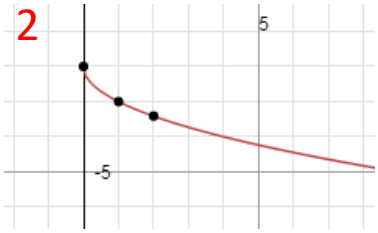
$y = \sqrt{\frac{1}{7}x}$ (Use integers or fractions for any numbers in the expression.)

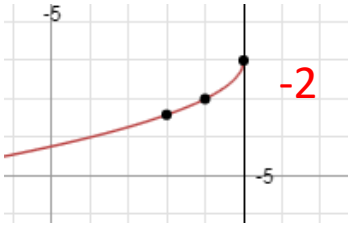
11) Find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$ in the order listed.

(1) Shift up 2 units
(2) Reflect about the x-axis
(3) Reflect about the y-axis

$f(x) = \sqrt{x} + 2$ reflects about x-axis reflects about the y-axis







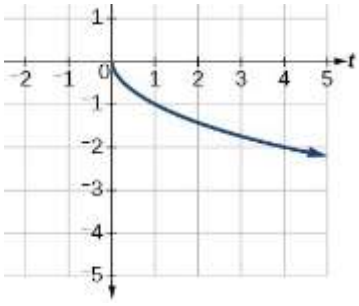
****Notice after the reflection the point shifted down to -2*

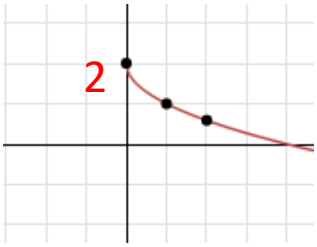
$$y = -\sqrt{-x} - 2$$

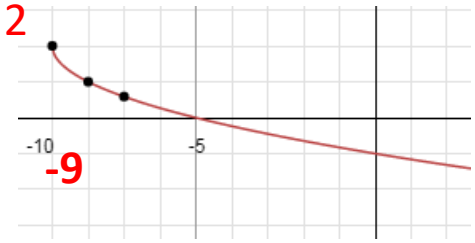
12) Find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$ in the order listed.

(1) Reflect about the x – axis
(2) Shift up 2 units
(3) Shift left 9 units

$f(x) = -\sqrt{x}$ shifts up 2 units then left 9 units







****Notice if the reflection is first does not change the positioning*

$$y = -\sqrt{x + 9} + 2$$

13) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = x^2$ and show all stages. Be sure to identify at least three key points. Find the domain and the range of the function.

$f(x) = x^2 - 11$

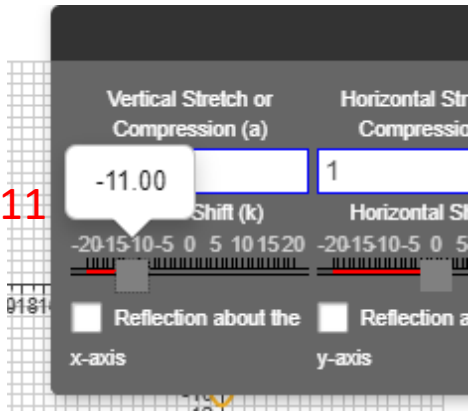
Click parabola

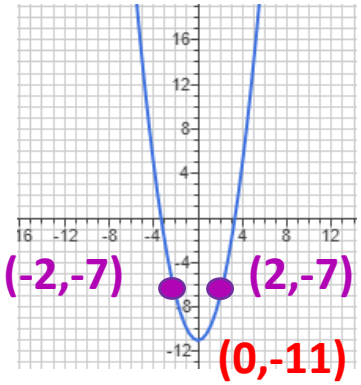
Click anywhere on the graph

Which transformations are needed to graph the function $f(x) = x^2 - 11$? Choose the correct answer below.

A. The graph of $y = x^2$ should be shifted vertically down by 11 units.

Vertical shift of -11





The vertex is now **(0,-11)** Use the graph to determine **one unit to the left** and **one unit to the right**

Points that lie on the graph of $y = x^2$ (Simplify your answers.)	Corresponding points that lie on the graph of $f(x) = x^2 - 11$ (Type ordered pairs. Simplify your answer.)
$(-2, 4)$	$(-2, -7)$
$(0, 0)$	$(0, -11)$
$(2, 4)$	$(2, -7)$

First column

$(-2)^2 = 4$

$(0)^2 = 0$

$(2)^2 = 4$

vertex

The domain of f(x) is $(-\infty, \infty)$. (x) move your pencil from left to right on the graph

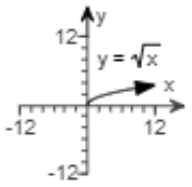
(Type your answer in interval notation.)

The range of f(x) is $[-11, \infty)$. (y)move your pencil from bottom to top on the graph

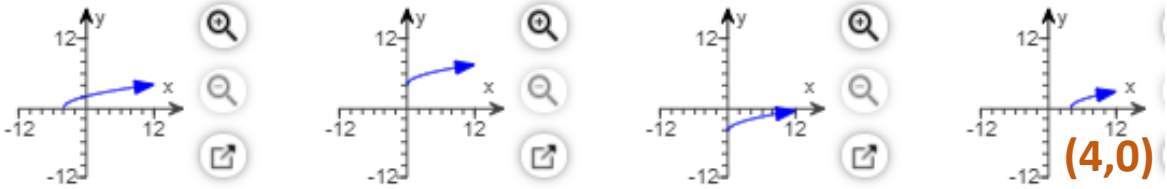
(Type your answer in interval notation.)

14) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function shown to the right. Find the domain and range of the function.

$h(x) = \sqrt{x-4}$



A. B. C. D.



Find the domain of h(x).

$[4,\infty)$ (x)

(Type your answer in interval notation.)

both always to ∞ on $y = \sqrt{x}$

Find the range of h(x).

$[0,\infty)$ (y)

(Type your answer in interval notation.)

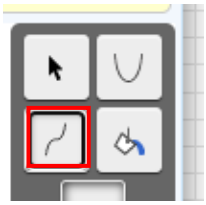
15) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = x^3$ and show all stages. Be sure to identify at least three key points. Find the domain and the range of the function

$f(x) = (x+2)^3 - 1$

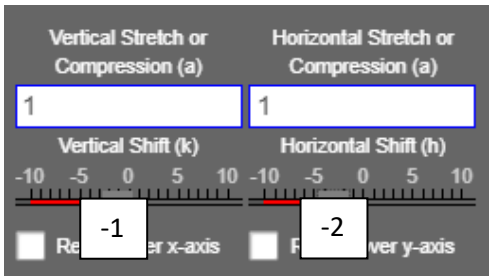
Which transformations are needed to graph the function $f(x) = (x+2)^3 - 1$? Choose the correct answer below.

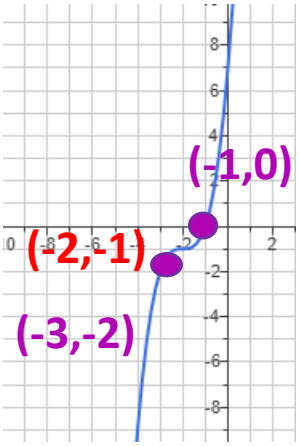
A. The graph of $y = x^3$ should be horizontally shifted to the left by 2 units and shifted vertically down by 1 unit.

Graph is cube root



Click anywhere on the graph





The vertex is now (**-2**,1) from the shifts. Use the graph to determine **one unit to the left** and **one unit to the right**

$(-1)^3 = -1$
$(0)^3 = 0$
$(1)^3 = 1$

Points that lie on the graph of $y = x^3$ (Simplify your answers.)	Corresponding points that lie on the graph of $f(x) = (x+2)^3 - 1$ (Type ordered pairs. Simplify your answer.)
$(-1, -1)$	$(-3, -2)$
$(0, 0)$	$(-2, -1)$
$(1, 1)$	$(-1, 0)$

VERTEX

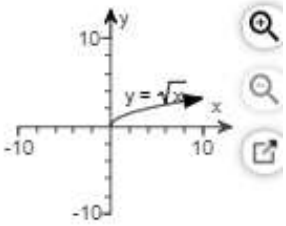
No restrictions on either:

The domain of f(x) is $(-\infty, \infty)$.
(Type your answer in interval notation.)

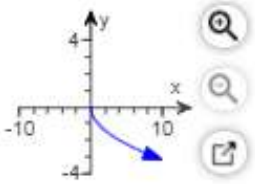
The range of f(x) is $(-\infty, \infty)$.
(Type your answer in interval notation.)

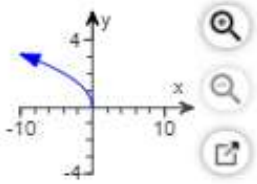
16) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function shown to the right. Find the domain and range of the function.

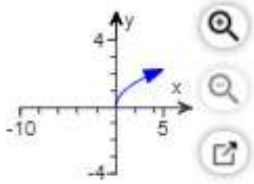
$f(x) = -\sqrt{x}$

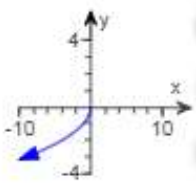


Choose the correct graph below.

A. 

B. 

C. 

D. 

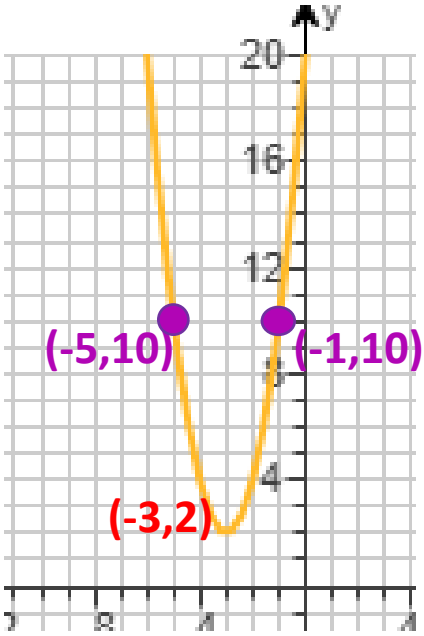
The domain of $y = -\sqrt{x}$ is $[0, \infty)$.
(Type your answer in interval notation.)

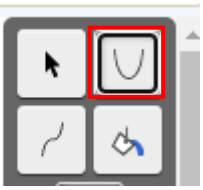
The range of $y = -\sqrt{x}$ is $(-\infty, 0]$.
(Type your answer in interval notation.)

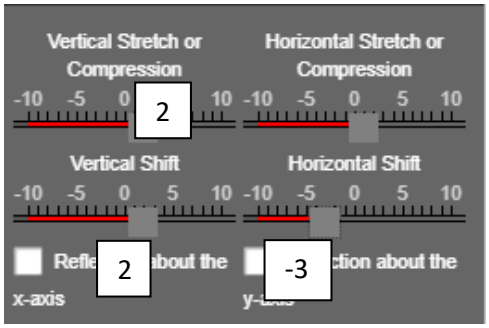
17) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = x^2$ and show all stages. Be sure to identify at least three key points. Find the domain and the range of the function.

$f(x) = 2(x+3)^2 + 2$

D. The graph of $y = x^2$ should be horizontally shifted to the left by 3 units, vertically stretched by a factor of 2, and shifted vertically up by 2 units.







The vertex is now **(-3,2).** Use the graph to determine **TWO unit to the left** and **TWO unit to the right of the vertex**

$(-2)^2 = 4$
$(0)^2 = 0$
$(2)^2 = 4$

Points that lie on the graph of $y = x^2$ (Simplify your answers.)	Corresponding points that lie on the graph of $f(x) = 2(x+3)^2 + 2$ (Type ordered pairs. Simplify your answer.)
$(-2, 4)$	$(-5, 10)$
$(0, 0)$	$(-3, 2)$ VERTEX
$(2, 4)$	$(-1, 10)$

The domain of f(x) is $(-\infty, \infty)$.
(Type your answer in interval notation.)

The range of f(x) is $[2, \infty)$.
(Type your answer in interval notation.)

18) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = \sqrt{x}$ and show all stages. Be sure to show at least three key points. Find the domain and the range of the function.

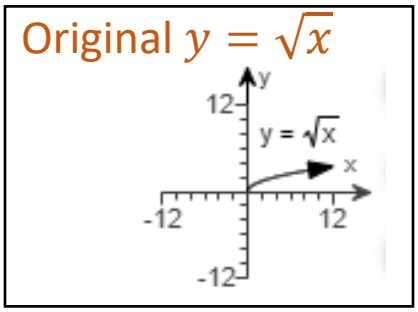
$h(x) = \sqrt{-x} - 6$

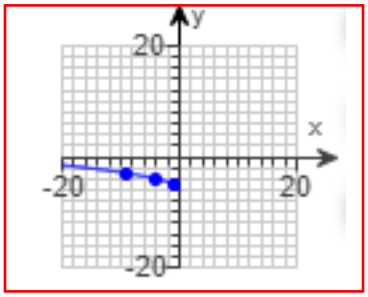
Which transformations are needed to graph the function $h(x) = \sqrt{-x} - 6$? Choose the correct answer below.

C. The graph of $y = \sqrt{x}$ should be vertically shifted down by 6 units, reflected about the y-axis.

Choose the correct graph for $h(x) = \sqrt{-x} - 6$ below.

Original $y = \sqrt{x}$ shifts down 6 and reflects across the y-axis





The domain of h(x) is $(-\infty, 0]$.
(Type your answer in interval notation.)

From left to right, ∞ to 0 including 0

The range of h(x) is $[-6, \infty)$.
(Type your answer in interval notation.)

From bottom to top, includes -6 to ∞

19) Suppose that the x-intercepts of the graph of $y = f(x)$ are 6 and 9.

(a) What are the x-intercepts of the graph of $y = f(x + 7)$?

The x-intercepts of the graph of $y = f(x + 7)$ are $-1, 2$.
(Use a comma to separate answers as needed.)

Left 7 means subtract 7

(b) What are the x-intercepts of the graph of $y = f(x - 4)$?

Right means add 4

The x-intercepts of the graph of $y = f(x - 4)$ are $10, 13$.
(Use a comma to separate answers as needed.)

(c) What are the x-intercepts of the graph of $y = 8f(x)$?

Vertical stretch **doesn't** change x-intercepts because it only stretches

The x-intercepts of the graph of $y = 8f(x)$ are $6, 9$.
(Use a comma to separate answers as needed.)

(d) What are the x-intercepts of the graph of $y = f(-x)$?

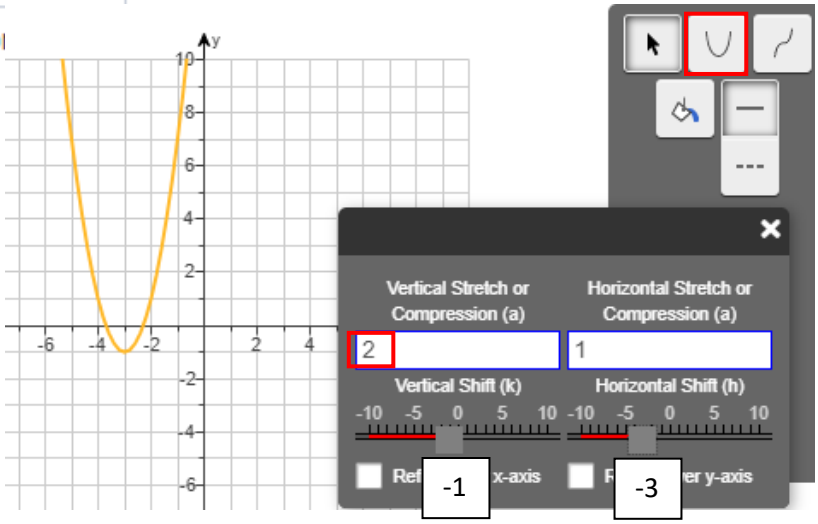
Reflects across the x-axis
Changes sign of intercepts

The x-intercepts of the graph of $y = f(-x)$ are $-6, -9$.
(Use a comma to separate answers as needed.)

20) Starting with the graph of a basic function, graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Find the domain and range of the function.

$$g(x) = 2(x+3)^2 - 1$$

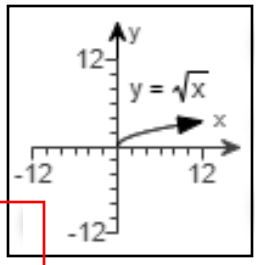
Use the graphing tool to graph the equatio



The domain of g(x) $(-\infty,\infty)$.
(Type your answer in interval notation.)

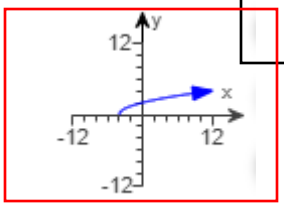
The range of g(x) $[-1,\infty)$.
(Type your answer in interval notation.)

21) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function shown to the right. Find the domain and range of the function.



$$h(x) = \sqrt{x+4}$$

shifts left 4 units



Find the domain of h(x).

$[-4,\infty)$ left to right, starts at -4 (bracket) to ∞(always parenthesis)
(Type your answer in interval notation.)

Find the range of h(x).

$[0,\infty)$ bottom to top, starts at 0 (bracket) to ∞(always parenthesis)
(Type your answer in interval notation.)

22) Find the domain and range of the function.

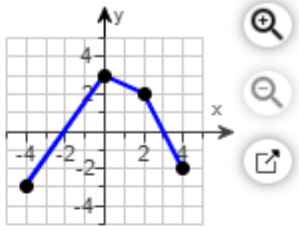
$f(x) = \sqrt{x-6} + 8$ Domain is inside the absolute value [6,∞) *change sign

Range is outside the absolute value [8,∞)

23) The graph of a function f is illustrated. Use the graph of f as the first step toward graphing each of the following functions.

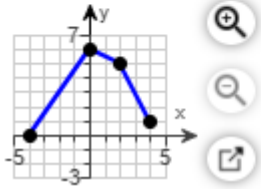
(a) $Q(x) = f(x+3)$

(b) $P(x) = -f(x)$

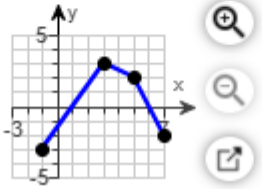


(a) Choose the graph of $Q(x) = f(x+3)$ below. left 3

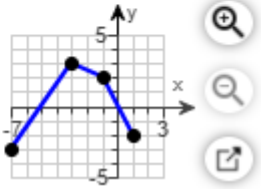
○ A.



○ B.

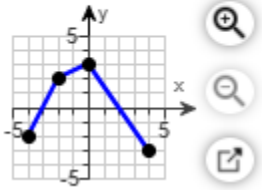


✔ C.

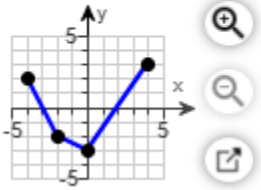


(b) Choose the graph of $P(x) = -f(x)$ below. reflects across the x-axis

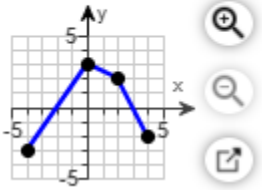
○ A.



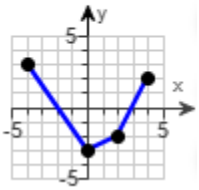
○ B.



○ C.



✔ D.



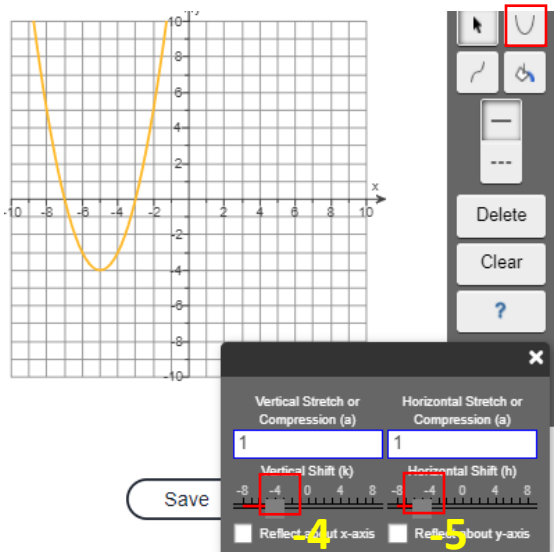
24) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f in the form $f(x) = a(x-h)^2 + k$.]

$f(x) = (x+5)^2 - 4$

Which transformations are needed to graph the function $f(x) = (x+5)^2 - 4$? Choose the correct answer below.

○ A. The graph of $y = x^2$ should be horizontally shifted to the left by 4 units and shifted vertically up by 5 units.

✔ B. The graph of $y = x^2$ should be horizontally shifted to the left by 5 units and shifted vertically down by 4 units.



25) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f in the form $f(x) = a(x - h)^2 + k$.]

$f(x) = x^2 + 2x - 3$

Find x value of vertex using $x = \frac{-b}{2a} = \frac{-2}{2} = -1$

Which transformations are needed to graph the function $f(x) = x^2 + 2x - 3$? Choose the correct answer below.

Find y by plugging -1 in for x

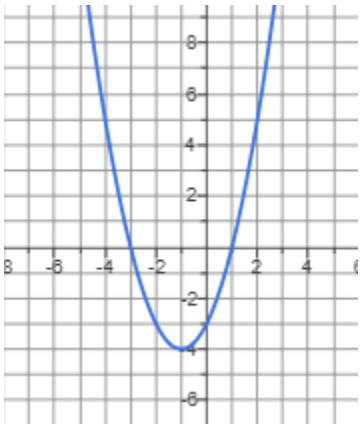
$y = (-1)^2 + 2(-1) - 3 = -4$

Vertex is (-1,-4)

Moves left 1 and down 4 using vertex

A. The graph of $y = x^2$ should be horizontally shifted to the right by 1 unit and shifted vertically up by 4 units.

B. The graph of $y = x^2$ should be horizontally shifted to the left by 1 unit and shifted vertically down by 4 units.



Vertical shift is -4

Horizontal shift is -1

26) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f in the form $f(x) = a(x - h)^2 + k$.]

$f(x) = 2x^2 - 12x + 17$

Find x value of vertex using $x = \frac{-b}{2a} = \frac{12}{4} = 3$

Which transformations are needed to graph the function $f(x) = 2x^2 - 12x + 17$? Choose the correct answer below.

Find y by plugging -1 in for x

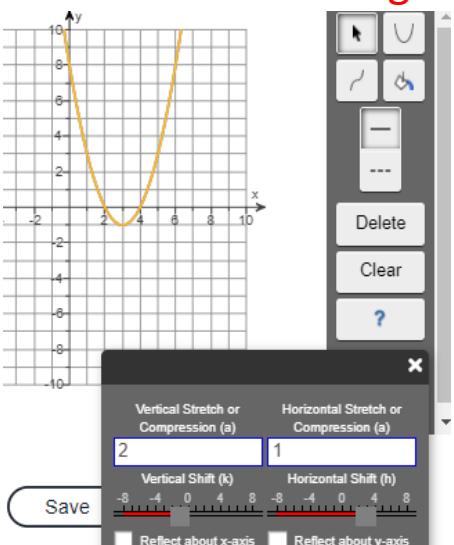
$y = 2(3)^2 - 12(3) + 17 = -1$

Vertex is (3,-1)

Moves right 3 and down 1 using vertex

A. The graph of $y = x^2$ should be stretched vertically by a factor of 2, horizontally shifted to the left by 3 units, and shifted vertically up by 1 unit.

B. The graph of $y = x^2$ should be stretched vertically by a factor of 2, horizontally shifted to the right by 3 units, and shifted vertically down by 1 unit.



27) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f i the form $f(x) = a(x - h)^2 + k$.]

$f(x) = -x^2 - 2x$

Find x value of vertex using $x = \frac{-b}{2a} = \frac{2}{-2} = -1$

Which transformations are needed to graph the function $f(x) = -x^2 - 2x$? Choose the correct answer below.

Find y by plugging -1 in for x

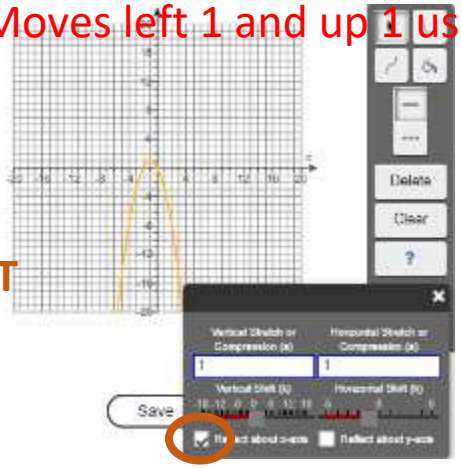
$y = -(-1)^2 - 2(-1) = 1$

A. The graph of $y = x^2$ should be horizontally shifted to the right by 1 unit, reflected about the y-axis, and shifted vertically down 1 unit.

B. The graph of $y = x^2$ should be horizontally shifted to the left by 1 unit, reflected about the x-axis, and shifted vertically up 1 unit.

Vertex is (-1,1)

Moves left 1 and up 1 using vertex



Make sure to check reflection FIRST

28) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflection). [Hint: If necessary, write f ir the form $f(x) = a(x - h)^2 + k$.]

$f(x) = \frac{1}{2}x^2 + x + 2$

Find x value of vertex using $x = \frac{-b}{2a} = \frac{-1}{1} = -1$

Find y by plugging -1 in for x

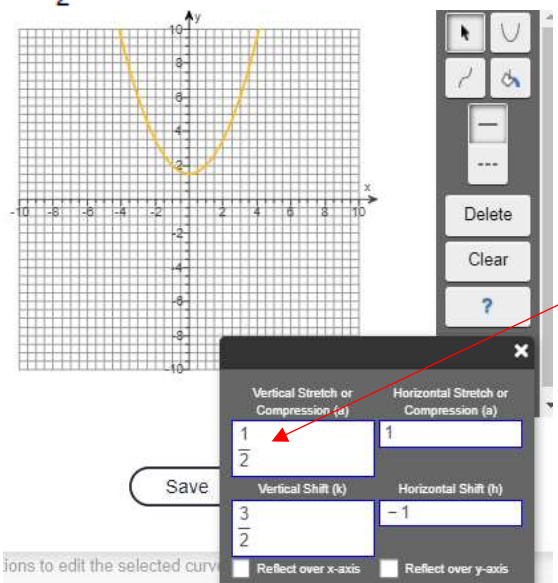
$y = ½ (-1)^2 + (-1) + 2 = \frac{3}{2}$

B. The graph of $y = x^2$ should be compressed vertically by a factor of $\frac{1}{2}$, horizontally shifted to the left by 1 unit, and shifted vertically up by $\frac{3}{2}$ units.

Vertex is $(-1, \frac{3}{2})$

horizontal shift -1, vertical shift $\frac{3}{2}$

Vertical shrink of $\frac{1}{2}$



29) Graph the following function by starting with a function from the library of functions and then combining shifting, compressing, stretching, and/or reflecting techniques.

$f(x) = 2(x + 7)^2 - 4$

the given function using a function from the library of functions.

- [ ] A. Stretch the graph horizontally by a factor of 2.
- [ ] B. Reflect the graph about the y-axis.
- [ ] C. Shift the graph 7 units to the right.
- [x] D. Stretch the graph vertically by a factor of 2.
- [x] E. Shift the graph 7 units to the left.
- [x] F. Shift the graph 4 units down.
- [ ] G. Reflect the graph about the x-axis.
- [ ] H. Shift the graph 4 units up.