

Lesson 4-1 Linear Functions and Properties MATH 161 THOMPSON

1) The slope of a vertical line is **undefined**; the slope of a horizontal line is **0**

2) For the line $2x + 3y = 6$, the x-intercepts is 3 and the y-intercept is 2

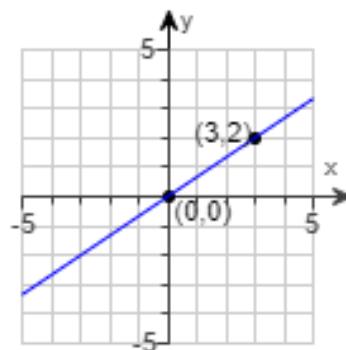
To find x-intercepts, set $y = 0$ To find y-intercepts, set $x=0$

3) (a) Find the slope of the line and (b) interpret the slope.

Slope is $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Graph shifts up 2 and right 3 $m = \frac{2}{3}$

(b) For every 3-unit change in x, the change in y is 2 units



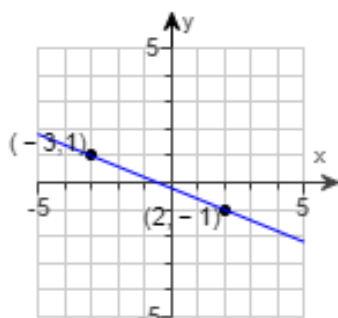
4) (a) Find the slope of the line and (b) interpret the slope.

Slope is $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

slope will be a negative when shifting down

Graph shifts down 2 and right 5 $m = -\frac{2}{5}$

(b) For every 5-unit change in x, the change in y is -2 units



5) Plot the pair of points and determine the slope of the line containing them. Graph the line.

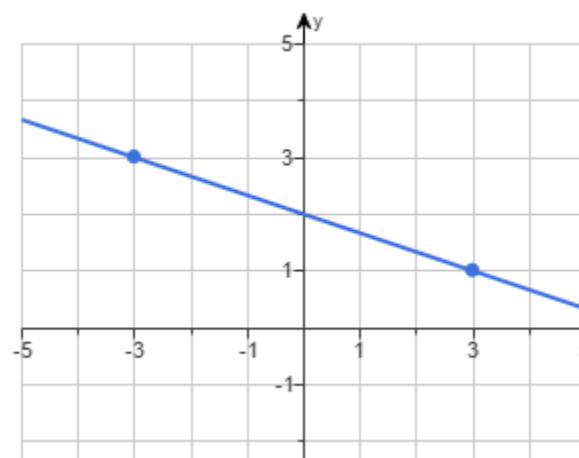
$(-3,3); (3,1)$

What is the slope of the line containing the points $(-3,3)$ and $(3,1)$? Select the correct choice below and fill in any answer boxes within your choice.

☒ A. The slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$
(Type an integer or a simplified fraction.)

☐ B. The slope is undefined.

$$m = \frac{1-3}{3+3} = \frac{-2}{6} = -\frac{1}{3}$$



To graph plot both points

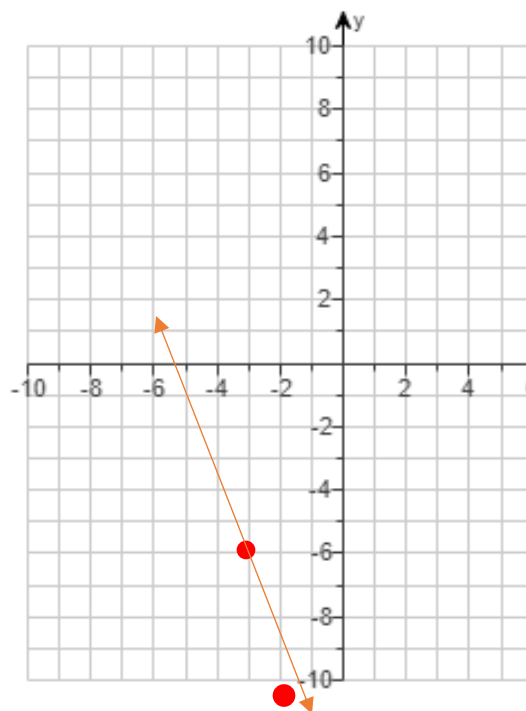
Graph the line containing the point P and having slope m.

6) $P = (-3, -6); m = -3$

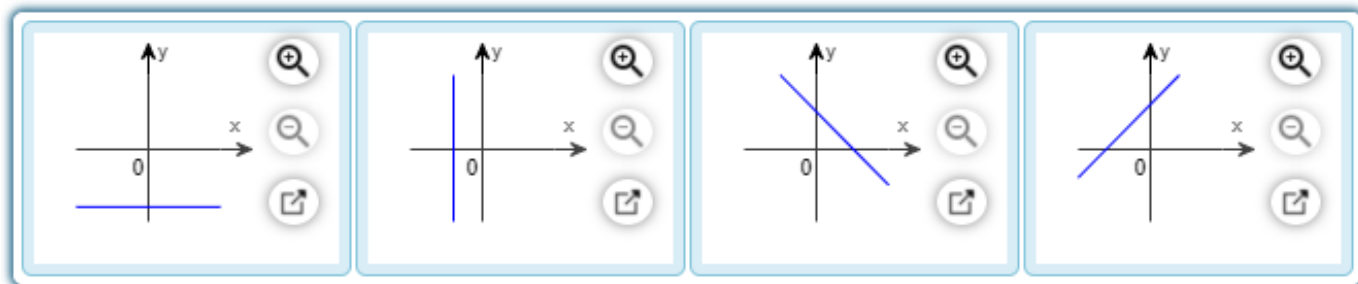
Use the graphing tool to graph the line.



Plot the point first then use slope
From $(-3, -6)$ move down 3 and
right 1 and plot your second point



7) Match each slope in the first row with the graph displaying the line with that slope in the second row.



Drag each graph above to the area that corresponds to the given slope. Items may only be used once.

Slope

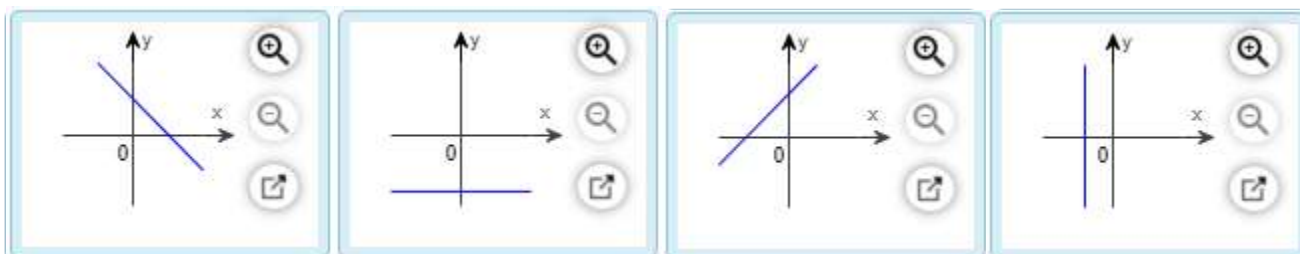
- 1

0

1

undefined

Graph



Forms for the Equation of a Line

Slope-Intercept	$y = mx + b$	m is the slope b is the y-intercept
Point-Slope	$y - y_1 = m(x - x_1)$	m is the slope (x_1, y_1) is a point on the line
Standard Form	$ax + by = c$	a is positive

- 8) A point on a line and its slope are given. Find the point-slope form of the equation of the line.

$P = (1, 2); m = 6$

Point-Slope

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 6(x - 1)$$

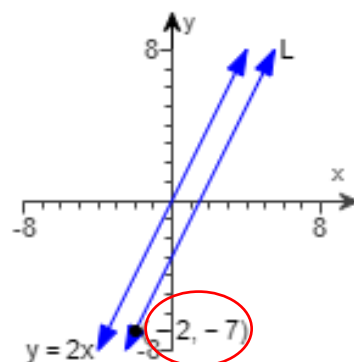
- 9) Find an equation of the line L.

L is parallel to $y = 2x$.

Parallel lines have the same slope $m = 2$

$$y - y_1 = m(x - x_1)$$

$$y + 7 = 2(x + 2)$$



- 10) Find an equation of the line L.

L is perpendicular to $y = 2x$.

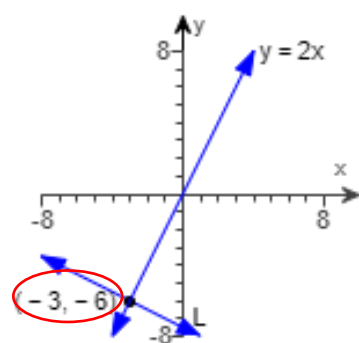
Perpendicular lines have the opposite slope

Change sign and take reciprocal $m = -\frac{1}{2}$

Since we have a point we will use point-slope form

$$y - y_1 = m(x - x_1)$$

$$y + 6 = -\frac{1}{2}(x + 3)$$



- 11) Find an equation for the line with the given properties. Express your answer using either the general form or the slope-intercept form of the equation of a line.

Slope = -2 ; containing the point $(3, -1)$

$$y - y_1 = m(x - x_1)$$

Point slope form: $y + 1 = -2(x - 3)$ in order to get slope-intercept we distribute the solve for y

$y + 1 = -2x + 6$ *get y by itself for slope-intercept form

$$y = -2x + 5$$

- 12) Find an equation for the line with the given properties. Express your answer using either the general form or the slope-intercept form of the equation of a line.

x-intercept = 1; y-intercept = -1

Write each intercept as a coordinate (1,0) and (0,-1)

$$m = \frac{-1-0}{0-1} = \frac{-1}{-1} = 1 \quad \text{Using slope and y-intercept (from problem)}$$

$$y = mx + b \quad y = x - 1$$

- 13) Find the equation of the line that contains the point (-3, -7) and has a slope that is undefined. Express the equation using either the general form or the slope-intercept form of the equation of a line.

Undefined slope is a vertical line where the line crosses x-axis $x = \underline{\hspace{1cm}}$
 Slope of zero is a horizontal line where the line crosses the y-axis $y = \underline{\hspace{1cm}}$

Undefined slope with point (-3,-7) we use the x value $x = -3$

- 14) Find an equation for the line with the given properties. Express your answer using either the general form or the slope-intercept form of the equation of a line.

Parallel to the line $x - 5y = -4$; containing the point (0,0)

Solve the equation for y to determine the slope $x - 5y = -4$

$-5y = -x - 4$ then divide all by -5

$$y = \frac{1}{5}x + \frac{4}{5}$$

Parallel lines have the same slope

*we only need the slope from that equation

slope = $\frac{1}{5}$ and y-intercept is 0 from the point

$$y = \frac{1}{5}x + 0$$

- 15) Find the equation of a line that is perpendicular to the line $x = 2$ and contains the point (-2, -4).

Perpendicular lines have the opposite slope change sign and take reciprocal
 since $x = 2$ gives an undefined slope the reciprocal will be a slope of 0 and $y =$

$$y = -4$$

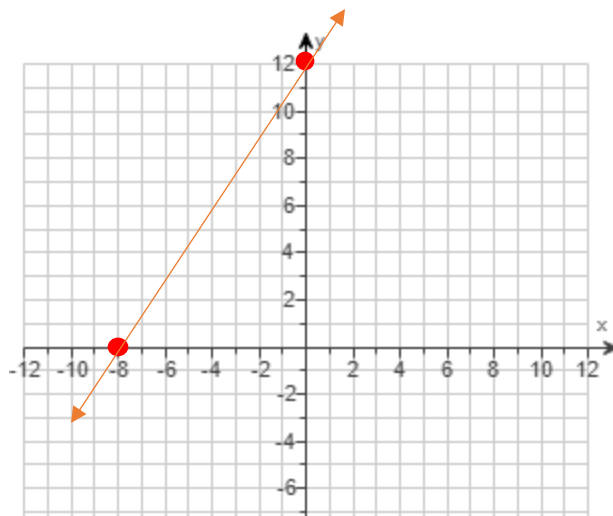
- 16) Using the given equation,
 (a) find the intercepts of its graph and
 (b) graph the equation.

$$-6x + 4y = 48$$

x-intercept, set $y=0$ $x = -8$

y-intercept, set $x=0$ $y = 12$

Plot -8 on x-axis and 12 on y-axis



- 17) A truck rental company rents a moving truck for one day by charging \$36 plus \$0.05 per mile. Write a linear equation that relates the cost C , in dollars, of renting the truck to the number x of miles driven. What is the cost of renting the truck if the truck is driven 121 miles? 416 miles?

Type the linear equation that relates the cost C , in dollars, of renting the truck to the number of x miles driven.

0.05 per mile is the slope and 36 is y-intercept

$$C = .05x + 36$$

(Use integers or decimals for any numbers in the expression. Do not include the \$ symbol in your answer.)

What is the cost of renting the truck if the truck is driven 121 miles?

$$C = \$42.05 \quad C = 0.05(121) + 36$$

What is the cost of renting the truck if the truck is driven 416 miles?

$$C = \$56.80 \quad C = 0.05(416) + 36$$

- 18) The relationship between Celsius ($^{\circ}\text{C}$) and Fahrenheit ($^{\circ}\text{F}$) degrees of measuring temperature is linear. Find a linear equation relating $^{\circ}\text{C}$ and $^{\circ}\text{F}$ if 0°C corresponds to 32°F and 30°C corresponds to 86°F . Use the equation to find the Celsius measure of 69°F .

Two points are $(0,32)$ and $(30,86)$ $m = \frac{86-32}{30-0} = \frac{54}{30} = \frac{9}{5}$

$$y - y_1 = m(x - x_1)$$

Use the point $(0,32)$ $^{\circ}\text{F} - 32 = \frac{9}{5} (^{\circ}\text{C} - 0)$ multiply left by reciprocal

$$\frac{5}{9} (^{\circ}\text{F} - 32) = ^{\circ}\text{C} \text{ *then solve for C}$$

The temperature of 71°F written as degrees Celsius is $\frac{5}{9}(71^{\circ} - 32) = 21.7^{\circ}\text{C}$

19) Find the slope of the line joining the points (5,4) and (8,3).

$$m = \frac{3-4}{8-5} = \frac{-1}{3} = -\frac{1}{3}$$

20) For the graph of the linear function $f(x) = mx + b$, m is the slope and b is y-intercept

21) If the slope m of the graph of a linear function is positive the function is increasing over its domain.

22) What is the only type of function that has a constant average rate of change?

Choose the correct answer below.

- ☐ A. step function
- ☒ B. linear function
- ☐ C. absolute value function
- ☐ D. quadratic function

23) A car has 12,500 miles on its odometer. Say the car is driven an average of 40 miles per day. Choose the model that expresses the number of miles N that will be on its odometer after x days.

Choose the correct answer below.

- ☒ A. $N(x) = 40x + 12,500$

24)

A linear function is given. Complete parts (a)-(d).

$$f(x) = 4x + 3$$

(a) Determine the slope and y-intercept of the function.

The slope is 4.

(Type an integer or a simplified fraction.)

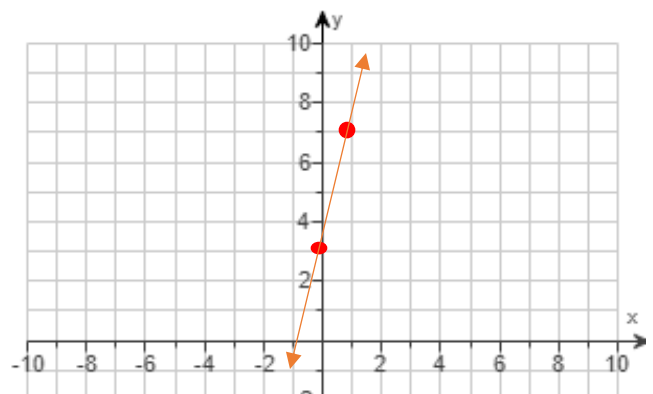
The y-intercept is 3.

(Type an integer or a simplified fraction.)

Plot 3 on y-axis then use slope, up 4 right 1

Average rate of change is the slope: 4

Increasing $(-\infty, \infty)$ bc it is always rising; doesn't decrease or stay constant



25)

A linear function is given. Complete parts (a)-(d).

$$f(x) = -2x + 8$$

(a) Determine the slope and y-intercept of the function.

The slope is .

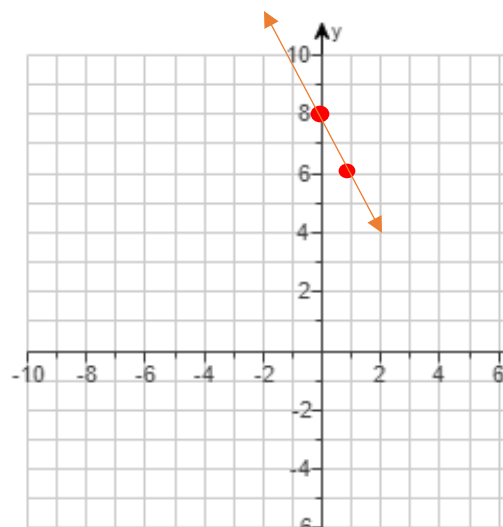
(Type an integer or a simplified fraction.)

The y-intercept is .

(Type an integer or a simplified fraction.)

(b) Use the slope and y-intercept to graph the linear function.

Use the graphing tool to graph the function. Use the slope and y-intercept when drawing the line.



Plot 8 on y-axis then use slope, down 2 right 1

Average rate of change is the slope: 4

Decreasing $(-\infty, \infty)$ bc it is always dropping; doesn't increase or stay constant

26)

Determine whether the given function is linear or nonlinear. If it is linear, determine the slope.

x	y = f(x)
1	8
2	13
3	18
4	23
5	28

Check the slope on two sets of points

$$m = \frac{8-13}{1-2} = \frac{-5}{-1} = 5 \quad m = \frac{18-23}{3-4} = \frac{-5}{-1} = 5$$

Is the function a linear function?

YES, same slopes. SLOPE is 5

27)

Determine whether the given function is linear or nonlinear. If it is linear, determine the slope.

x	y = f(x)
-2	-12
-1	-7
0	-5
1	-4
2	-6

Is the function a linear function?

☐ Yes

☒ No

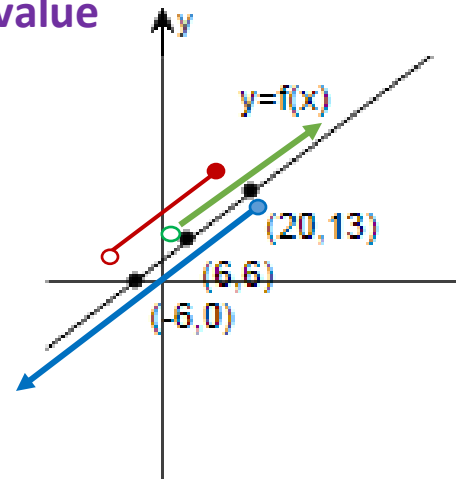
Select the correct choice below and fill in any answer boxes within your choice.

☐ A. The slope of the function is .

☒ B. The function is not linear.

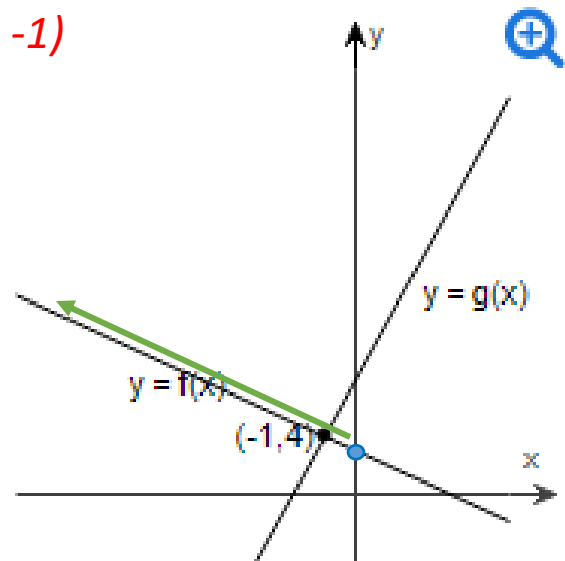
28) In parts (a)-(f), use the given figure. $f(x) = y$ value

- (a) Solve $f(x) = 6$. Find x where $y = 6$ $x = 6$
 (b) Solve $f(x) = 13$. Find x where $y = 13$ $x = 20$
 (c) Solve $f(x) = 0$. Find x where $y = 0$ $x = -6$
 (d) Solve $f(x) > 6$. Find x where $y > 6$ $(6, \infty)$
 (e) Solve $f(x) \leq 13$. Find x where $y \leq 13$ $(-\infty, 20]$
 (f) Solve $0 < f(x) < 13$. Find x where $0 < y < 13$ $(-6, 20)$



28) In parts (a) and (b), use the given figure.

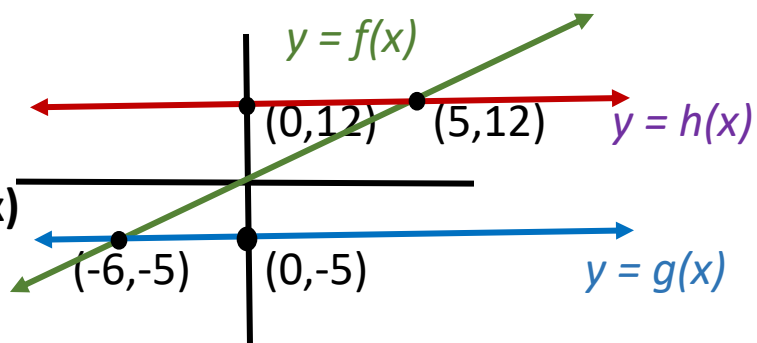
- (a) Solve the equation: $f(x) = g(x)$. x value where they cross $x = -1$
 (b) Solve the inequality $f(x) > g(x)$. where $f(x)$ is above $g(x)$ {left}
 using the x values always $(-\infty, -1)$



30)

a) Solve $f(x) = g(x)$

b) Solve $g(x) \leq f(x) < h(x)$



Solve $f(x) = g(x)$ means find the x value where they intersect

$$\underline{x = -6}$$

Solve $g(x) \leq f(x) < h(x)$ means find the interval where $f(x)$ is between $g(x)$ and $h(x)$ including the left not the right

$$\underline{[-6, 5)} \quad \text{*Use only x values for intervals}$$