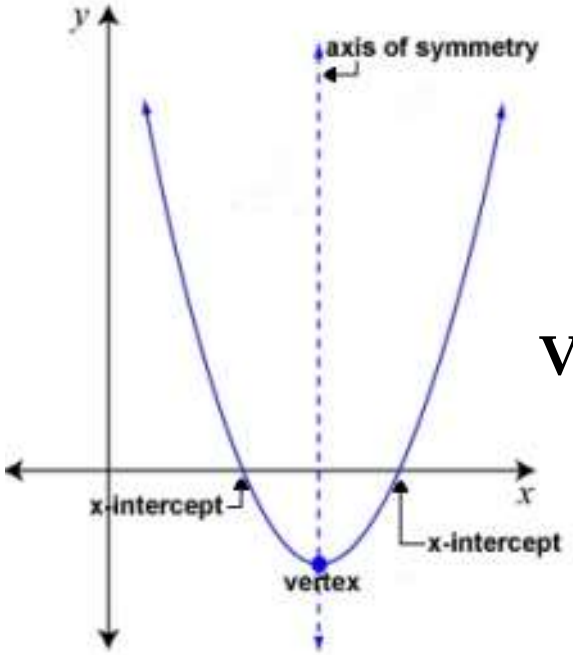
4.3 Quadratic Functions MATH 161 THOMPSON

$f(x) = ax^2+bx+c$

If a>0 then the parabola opens UP

If a<0 then the parabola opens DOWN



VERTEX

$$x = \frac{-b}{2a}$$

1 The graph of a quadratic function is called a(n) parabola.

2) The vertical line passing through the vertex of a parabola is called the axis of symmetry.

3) The x-coordinate of the vertex of $f(x) = ax^2 + bx + c$, $a \neq 0$, is $-\frac{b}{2a}$.

4) To graph $y = (x - 4)^2$, you shift the graph of $y = x^2$ to the right a distance of 4 units.

5) The y-coordinate of the vertex of $f(x) = -x^2 + 4x + 7$ is f(2)

Find the x value using vertex formula

$x = \frac{-4}{2(-1)} = 2$ therefore the y value would be f(2)

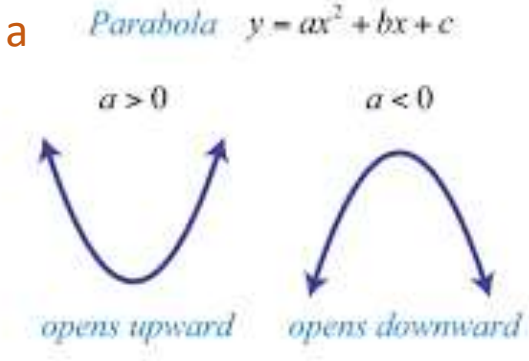
The statement is true because the x-coordinate of the vertex is 2.

6) If the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, has a maximum value at its vertex, which of the following conditions must be true?

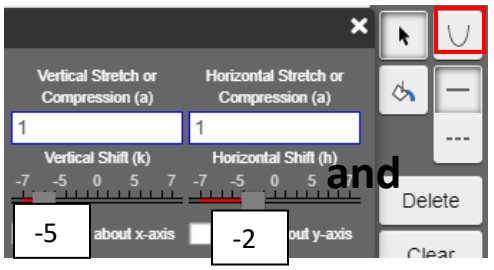
Select all that apply.

✔ A. $a < 0$

The x^2 coefficient is negative and parabola open down to have a maximum vertex

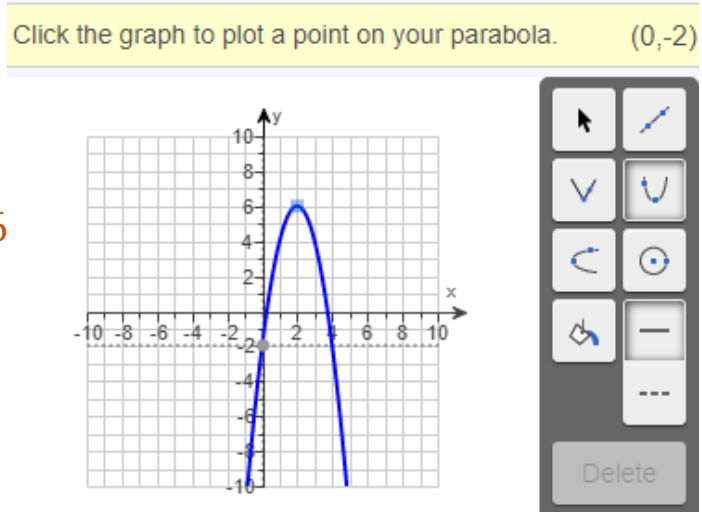


7) Graph the function $f(x) = x^2 + 4x - 1$ by starting with the graph of $y = x^2$ and using transformations (shifting, stretching/compressing, and/or reflecting).



Find the vertex $x = \frac{-b}{2a}$ $x = \frac{-4}{2} = -2$ **and use the x for horizontal shift and y for vertical shift** $y = (-2)^2 + 4(-2) - 1 = -5$

8) Graph the quadratic function. Give the vertex, axis of symmetry, domain, and range.



$f(x) = -2x^2 + 8x - 2$

$x = \frac{-8}{2(-2)} = 2$ $y = -2(2)^2 + 8(2) - 2 = 6$

Vertex (2,6) axis of symmetry is x= 2

D: (-∞,∞) R: (-∞,6]

First plot the vertex (2,6) then plot the y-intercept (0,-2)

9) For the quadratic function $f(x) = x^2 - 4x$, answer parts **(a)** through **(c)**.

A. The graph opens Up because x^2 term is positive

What are the coordinates of the vertex? $x = \frac{4}{2(1)} = 2$ $y = (2)^2 - 4(2) = -4$

Vertex (2,-4)

(Type an ordered pair. Use integers or fractions for any numbers in the expression. Simplify your answer.)

What is the equation of the axis of symmetry? X value of the vertex

The axis of symmetry is x= 2

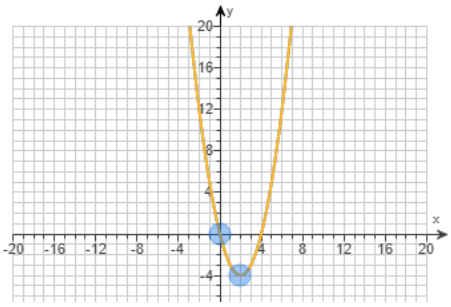
(Simplify your answer. Type an equation. Use integers or fractions for any numbers in the equation.)

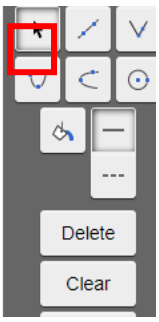
What are the intercepts? Select the correct choice below and fill in any answer boxes within your choice.

x-intercepts: factor $x^2 - 4x$ using (GCF) x(x-4)

x-intercepts are 0,4

y-intercept is 0





Plot the vertex first
then plot the y-intercept = 0

D: $(-\infty,\infty)$ R: $[-4,\infty)$ opens up

Domain: x-values (left to right), Range: y-values (bottom to top)

Increasing on $(2, \infty)$ Decreasing on $(-\infty,2)$

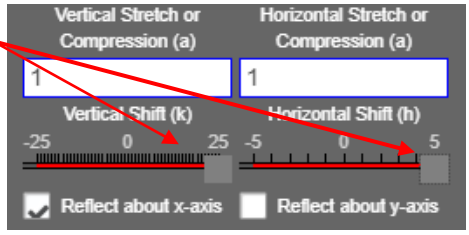
*Increasing and Decreasing is always parenthesis and only uses x value

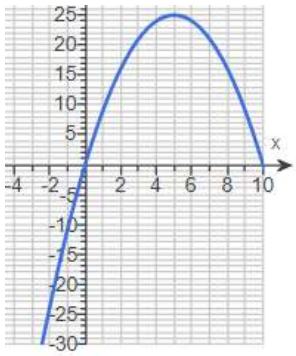
10) f(x) = $-x^2 + 10x$ Find vertex opens down

$x = \frac{-10}{2(-1)} = 5$ $y = -(5)^2 + 10(5) = 25$ Fill in vertex (h,k)

$$x = \frac{-b}{2a}$$

$vertex\ (5,25)$



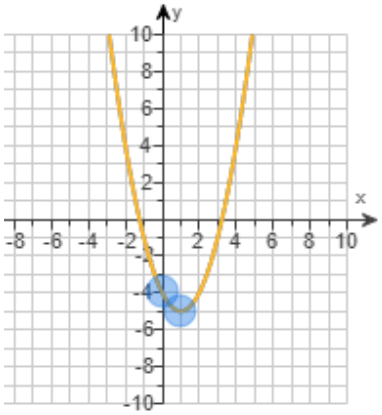


ALWAYS CHECK REFLECT ABOUT X-AXIS **FIRST**

11) Graph the parabola. Then determine the domain and range.

$f(x) = x^2 - 2x - 4$

x = 1 $y = (1)^2 - 2(1) - 4 = -5$



First plot the vertex (-1,-5)
then plot the y-intercept (0,-4)

D: $(-\infty,\infty)$ R: $[-5,\infty)$

12) For the quadratic function $f(x) = x^2 + 2x - 3$, answer parts **(a)** through **(c)**.

(a) Graph the quadratic function by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any.

Does the graph of f open up or down?

- ○ down
- ✔ up

Up because x^2 term is positive

What are the coordinates of the vertex?

The vertex of the parabola is $(-1, -4)$. $x = \frac{-2}{2(1)} = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$

(Type an ordered pair. Use integers or fractions for any numbers in the expression.)

What is the equation of the axis of symmetry?

The axis of symmetry is $x = -1$.

$x^2 + 2x - 3$

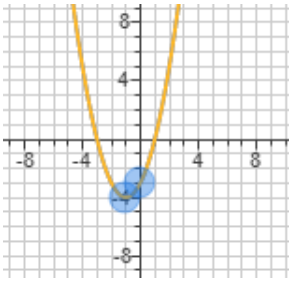
$(x+3)(x-1)$

- ✔ A. The x-intercept(s) is/are $-3, 1$.
  (Type an integer or a decimal. Use a comma to separate answers as needed.)
- ○ B. There are no x-intercepts.

What is the y-intercept? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ✔ A. The y-intercept is -3. constant by itself at the end
  (Type an integer or a decimal.)
- ○ B. There is no y-intercept.

Plot vertex first then y-intecept



D: (-∞,∞) R: [-4,∞) opens up

Domain: x-values (left to right), Range: y-values (bottom to top)

Increasing on (-1, ∞) Decreasing on (-∞,-1)

*Increasing and Decreasing is always parenthesis and only uses x value

13) A. Graph $f(x) = 2x^2 - 3x + 3$ by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any.
B. Determine the domain and the range of the function.
C. Determine where the function is increasing and where it is decreasing.

A. The graph opens up .

What are the coordinates of the vertex?

$\left(\frac{3}{4}, \frac{15}{8}\right)$ $x = \frac{3}{2(2)} = \frac{3}{4}$ $y = 2\left(\frac{3}{4}\right)^2 3\left(\frac{3}{4}\right) + 3 = \frac{15}{8}$

(Type an ordered pair. Simplify your answer.)

What is the equation of the axis of symmetry?

The axis of symmetry is $x = \frac{3}{4}$.

(Simplify your answer. Type an equation.)

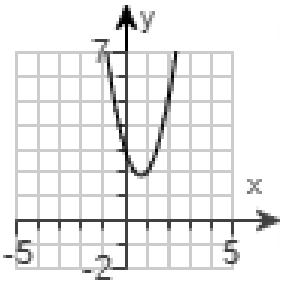
What are the intercepts? Select the correct choice below and fill in any answer boxes within your choice.

x^2 - 3x +6 slide and divide will not factor

no x-intercepts, only intercept is y=3

- ✔ A. The intercept(s) is/are (0,3).
  (Type an ordered pair. Use a comma to separate answers as needed.)
- ○ B. There are no intercepts.

Look at vertex and y-intercept to find graph



D: (-∞,∞) R: $[\frac{15}{8}, \infty)$ opens up

Domain: x-values (left to right), Range: y-values (bottom to top)

Increasing on $(\frac{3}{4}, \infty)$ Decreasing on $(-\infty, \frac{3}{4})$

*Increasing and Decreasing is always parenthesis and only uses x value

14) f(x) =4x² -24x+33 Find vertex

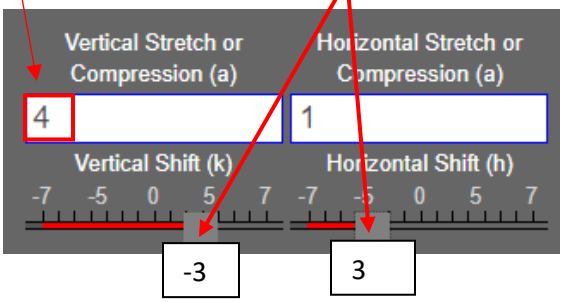
$$x = \frac{24}{2(4)} = 3 \qquad y = 4(3)^2 - 24(3) + 33$$

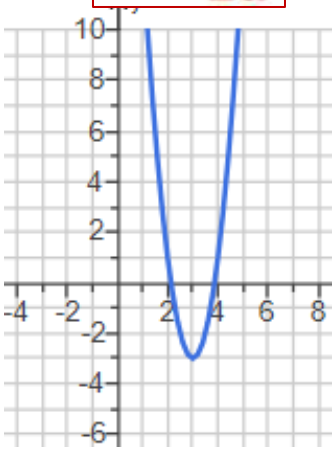
$$x = \frac{-b}{2a}$$

$$vertex\ (3, -3)$$

Always put first coefficient as vertical stretch

Use vertex as vertical and horizontal shifts





15) Determine, without graphing, whether the given quadratic function has a maximum value or a minimum value and then find the value.

$f(x) = 6x^2 + 12x$ $\quad x = \frac{-12}{2(6)} = -1 \qquad y = 6(-1)^2 + 12(-1) = -6$

The quadratic function has a minimum value. Opens up has a minimum

The value is −6. The minimum is the y-value of the vertex

16) Determine, without graphing, whether the given quadratic function has a maximum value or a minimum value and then find the value.

$f(x) = 3x^2 + 24x - 2$ $\quad x = \frac{-24}{2(3)} = -4 \qquad y = 3(-4)^2 + 24(-4) - 2 = -50$

Does the quadratic function f have a minimum value or a maximum value?

- ✔ The function f has a minimum value. Opens up has a minimum
- ○ The function f has a maximum value.

What is this minimum or maximum value? The minimum is the y-value of the vertex

−50

17) Determine, without graphing, whether the given quadratic function has a maximum value or a minimum value and then find the value.

$f(x) = -2x^2 + 16x - 1$ $x = \frac{-16}{2(-2)} = 4$ $y = -2(4)^2 + 16(4) - 1 = 31$

Does the quadratic function f have a minimum value or a maximum value?

- [x] The function f has a maximum value. Opens down has a maximum
- [ ] The function f has a minimum value. The max is the y-value of the vertex

What is this minimum or maximum value?

31

18) Determine, without graphing, whether the given quadratic function has a maximum value or a minimum value and then find the value.

$f(x) = 3x^2 + 12x + 1$ $x = \frac{-12}{2(3)} = -2$ $y = 3(-2)^2 + 12(-2) + 1 = -11$

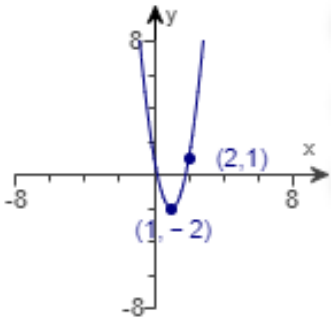
The quadratic function has a minimum value. Opens up has a minimum

The value is -11. The minimum is the y-value of the vertex

19) Determine the quadratic function f whose graph is given.

The vertex is $(1, -2)$ and the other given point is $(2,1)$.

(h,k) (x,y)



$y = a(x-h)^2 + k$ where vertex is (h,k)

$1 = a(2-1)^2 - 2$ Find a 1st

$1 = a - 2$ same

$a = 3$ then plug a, h, and k back in $y = 3(x-1)^2 - 2$

20) The graph of the function $f(x) = ax^2 + bx + c$ has its vertex at (0, 2) and passes through the point (1, 9). Find a, b, and c.

a = ☐ b = ☐ c = ☐

$y = a(x-h)^2 + k$ where vertex is (h,k)

$9 = a(1 - 0)^2 + 2$ Find a 1st

$9 = a + 2$ same

$a = 7$ then plug a, h, and k back in $y = 7(x-0)^2 + 2$

Then we have to multiply it out $y = 7(x^2) + 2$

$y = 7x^2 + 2$

$y = ax^2 + bx + c$ $a = 7$ $b = 0$ $c = 2$

21) Select **Equation Type** $y = ax^2 + bx + c$. Check the **x-intercepts** and **Vertex** boxes. Use the sliders to graph the function $f(x) = 2x^2 + 4x - 6$ and note the vertex and x-intercepts. Which of the following functions is the same function written in vertex form?

Use the interactive figure to find your answer. Use the left and right arrow keys to move along a slider as needed.
Click here to launch the interactive figure.

$$x = \frac{-b}{2a}$$

Choose the correct answer below.

$x = \frac{-4}{2(2)} = -1$ $y = 2(-1)^2 + 4(-1) - 6 = 8$

Vertex (-1,-8)

Put 1st coefficient in front and insert vertex

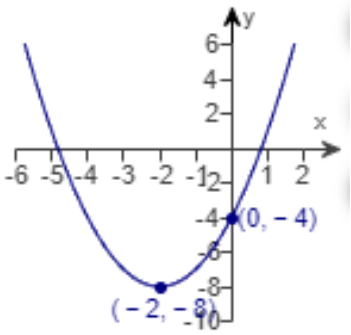
$y = a(x-h)^2 + k$

$y = 2(x+1)^2 - 8$

- ○ A. $f(x) = 2(x+1)^2 + 8$
- ○ B. $f(x) = 2(x-1)^2 - 8$
- ● C. $f(x) = 2(x+1)^2 - 8$
- ○ D. $f(x) = (x-2)^2 - 8$

22) Determine the quadratic function f whose graph is given.

The vertex is $(-2, -8)$ and the y-intercept is -4.



$y = a(x-h)^2 + k$

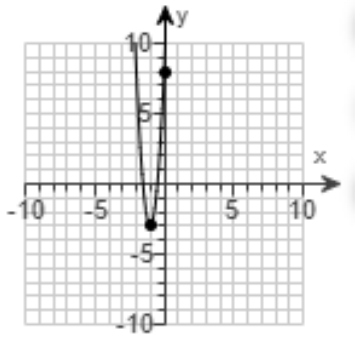
$-4 = a(0+2)^2 - 8$

$-4 = 4a - 8$

$4 = 4a$ $a = 1$

Use vertex to make equation $y = (x + 2)^2 - 8$

23) Determine the quadratic function whose graph is given.



The vertex is $(-1, -3)$.
The y-intercept is $(0, 8)$.

$y = a(x-h)^2 + k$

$8 = a(0+1)^2 - 3$

$8 = a - 3$

$11 = a$

$y = 11(x + 1)^2 - 3$

24) Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is p dollars, the revenue R (in dollars) is $R(p) = -8p^2 + 24{,}000p$. What unit price should be established for the dryer to maximize revenue? What is the maximum revenue?

ALWAYS USE VERTEX FORMULA

The unit price that should be established to maximize revenue is $\$\square$.

$$\textbf{\textit{VERTEX}}: \quad x = \frac{-24000}{2(-8)} = \$1500$$

What is the maximum revenue?

$$y = -8(1500)^2 + 24000(1500) = \$18{,}000{,}000$$

25) The marginal cost of a product can be thought of as the cost of producing one additional unit of output. For example, if the marginal cost of producing the 50th product is \$6.20, it cost \$6.20 to increase production from 49 to 50 units of output. Suppose the marginal cost C (in dollars) to produce x thousand mp3 players is given by the function $C(x) = x^2 - 160x + 8600$.

A. How many players should be produced to minimize the marginal cost?
B. What is the minimum marginal cost?

$$\textbf{\textit{VERTEX}}: \quad x = \frac{160}{2(1)} = \$80$$

A. To minimize the marginal cost, $\square$ thousand mp3 players should be produced.

What is the maximum revenue?

$$y = (80)^2 - 160(80) + 8600 = \$2200$$

☑ Show Function

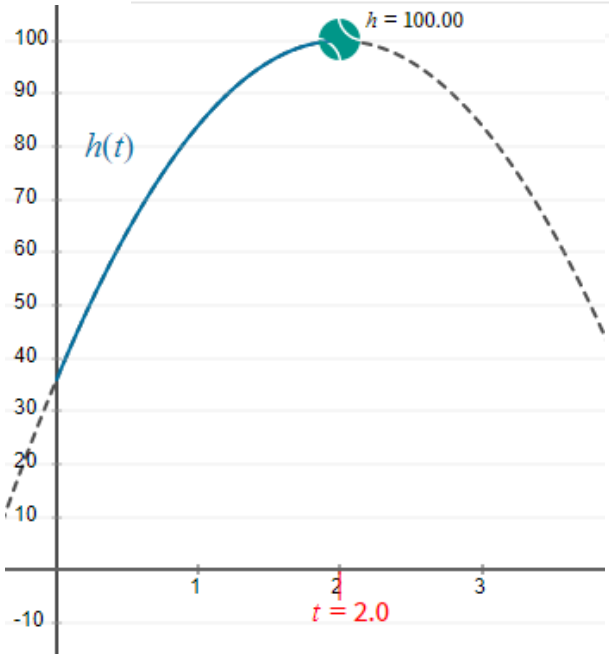
☑ Show $h(t)$

Time

t ▶ ⏮

26) A baseball is hit from a height of 36 feet at 64 feet per second. Find the height function, the maximum height, and the domain.

click here 1st



$f(t) = -16t^2 + 64t + 36$

ALGEBRAIC STEPS TO CHCEK:

VERTEX: $x = \frac{-64}{2(-16)} = 2$

Max height occurs when t=2

$y = -16(2)^2 + 64(2) + 36 = 100$

Max height of the baseball is 100 feet

27) Check the **Show h(t)** box and use the t-slider to find all the times t the baseball has a height of 64 feet.

Use the interactive figure to find your answer. Use the left and right arrow keys to move along a slider as needed.
Click here to launch the interactive figure.

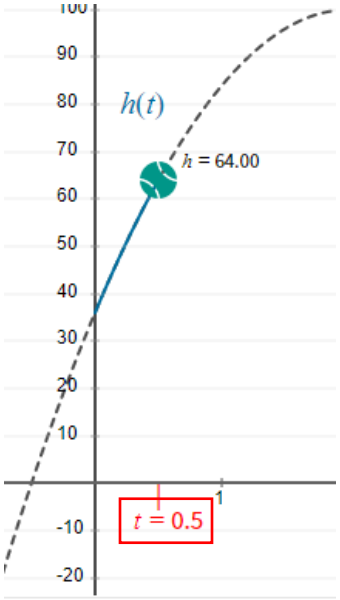
t = ☐ seconds

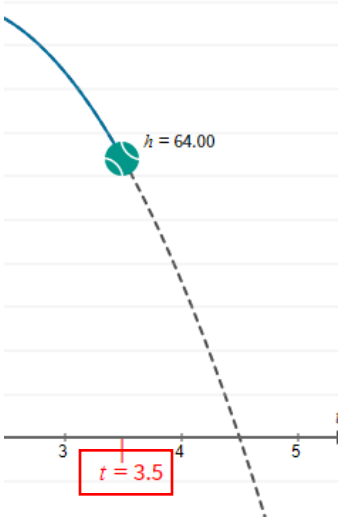
☑ Show Function

☑ Show $h(t)$

Time

t ▶ ⏮





ALGEBRAIC STEPS:

$64 = -16x^2 + 64x + 36$ set equation equal to 64

$0 = -16x^2 + 64x - 28$ move 64 to the right

$0 = -4(4x^2 - 16x + 7)$ factor out -4

$x^2 - 16x + 28 = 0$ slide and divide

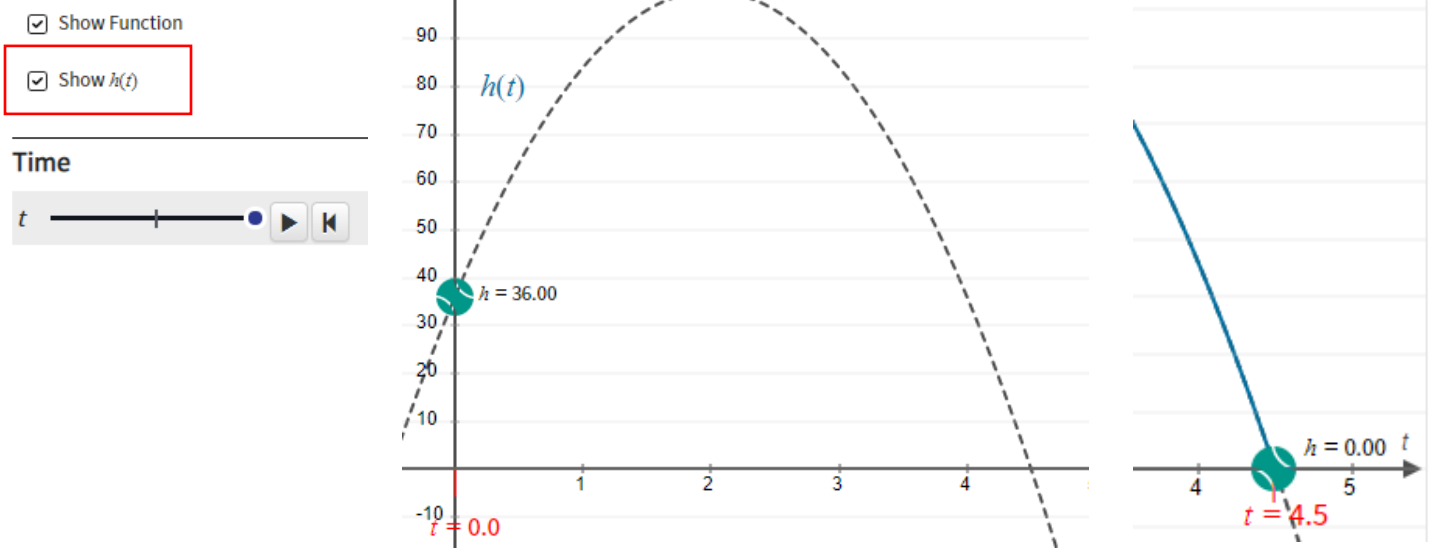
$(x-14)(x-2)=0$

4 4

x = 0.5,3.5 give answers in decimals

28)

A baseball is hit from a height of 36 feet at 64 feet per second. Find the height function, the maximum height, and the domain.



[0,4.5]

29) Find the intercepts $y = x^2 - 16$ {means ALL intercepts}

x-intercepts: $x^2 - 16 = 0$ $(x-4)(x+4)=0$ $x = -4,4$

y=intercept: $y = -16$

ordered pairs: (-4,0),(4,0),(0,-16)

EXTRA PROBLEMS

A)

A. Graph $f(x) = -x^2 - 2x$ by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any.
B. Determine the domain and the range of the function.
C. Determine where the function is increasing and where it is decreasing.

A. The graph opens Down because x^2 term is negative

What are the coordinates of the vertex? $x = \frac{2}{2(-1)} = -1$ $y = -(-1)^2 - 2(-1) = 1$

Vertex (-1,1)

(Type an ordered pair. Use integers or fractions for any numbers in the expression. Simplify your answer.)

What is the equation of the axis of symmetry? X value of the vertex

The axis of symmetry is X = -1

(Simplify your answer. Type an equation. Use integers or fractions

This shows the coordinate you on

What are the intercepts? Select the correct choice below and fill in any answer boxes within your choice.

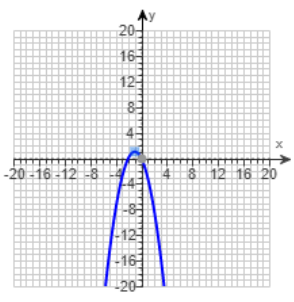
x-intercepts: factor the original (GCF) –x(x+2)

Click the graph to plot a point on your parabola. (0,0)

x-intercepts are 0,-2

y-intercept is 0

Plot the vertex first
then plot the y-intercept



D: (-∞,∞) R: (-∞,1] opens down

Domain: x-values (left to right), Range: y-values (bottom to top)

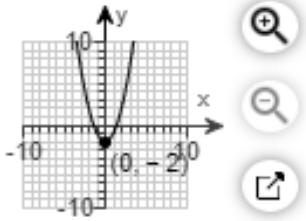
Increasing on (2, ∞) Decreasing on (-∞,2)

*Increasing and Decreasing is always parenthesis and only uses x value

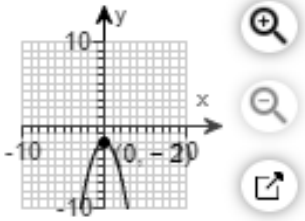
B) $x = \frac{-b}{2a}$ **y = f(x)**

all positive x-coordinate of vertex will have negative b term

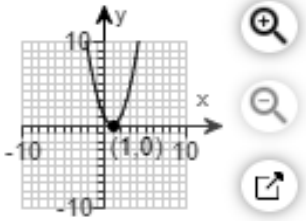
Drag each function given above into the area below the appropriate graph, depending on which function is represente



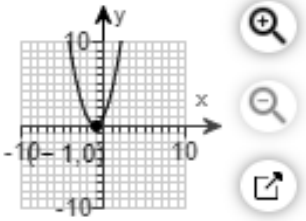
$f(x) = x^2 - 2$



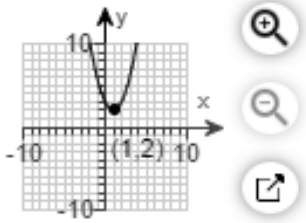
$f(x) = -x^2 - 2$



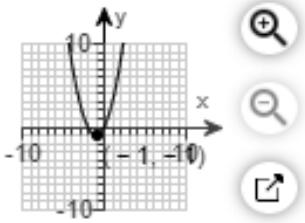
$f(x) = x^2 - 2x + 1$



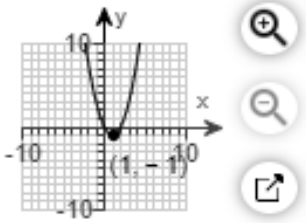
$f(x) = x^2 + 2x + 1$



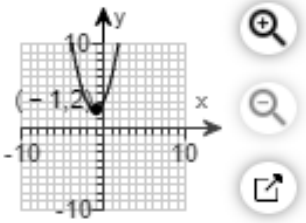
$f(x) = x^2 - 2x + 3$



$f(x) = x^2 + 2x$



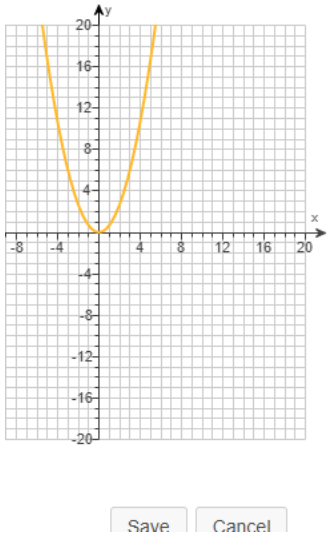
$f(x) = x^2 - 2x$

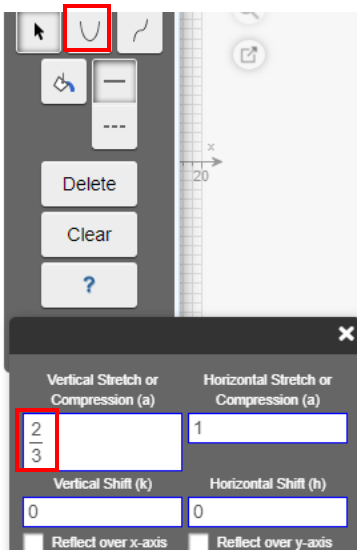


$f(x) = x^2 + 2x + 3$

C) Graph the following function by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflection).

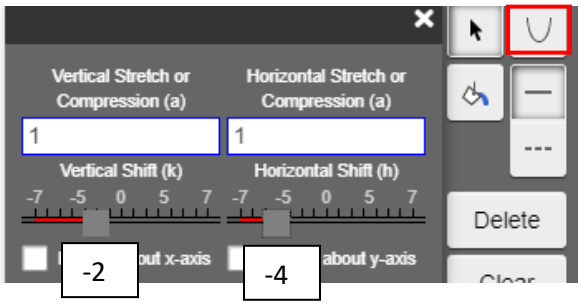
$$f(x) = \frac{2}{3}x^2$$





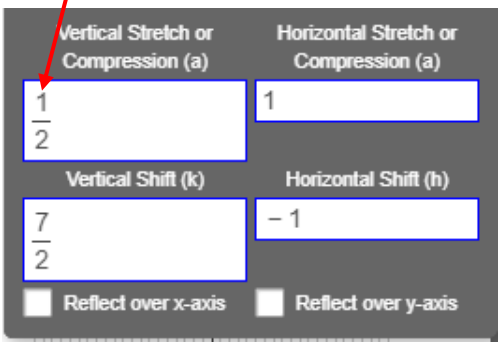
D) Graph the function $f(x) = (x + 4)^2 - 2$ by starting with the graph of $y = x^2$ and using transformations (shifting, stretching/compressing, and/or reflecting).

Use the graphing tool to graph the function.



E) $f(x) = \frac{1}{2}x^2 + x + 4$ Find vertex form by completing the square

$$x = \frac{-1}{2(1/2)} = -1 \qquad y = (1/2)(-1)^2 + (-1) + 4 = 3\frac{1}{2} = \frac{7}{2}$$



x is horizontal

y is vertical

F)

For the quadratic function $f(x) = -x^2 + 2x - 1$, answer parts **(a)** through **(c)**.

A. The graph opens Down because x^2 term is negative

What are the coordinates of the vertex? $x = \frac{-2}{2(-1)} = 1$ $y = -(1)^2 + 2(1) - 1 = 0$

Vertex (1,0)

(Type an ordered pair. Use integers or fractions for any numbers in the expression. Simplify your answer.)

What is the equation of the axis of symmetry? X value of the vertex

The axis of symmetry is X = 1

(Simplify your answer. Type an equation. Use integers or fractions for any numbers in the equation.)

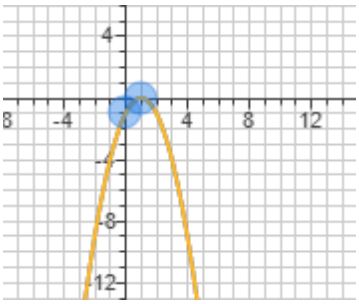
What are the intercepts? Select the correct choice below and fill in any answer boxes within your choice.

$-(x^2 - 2x + 1)$

$(x-1)(x-1)$

x-intercept is1

y-intercept is -1



Plot the vertex first

then plot the y-intercept

D: (-∞,∞) R: (-∞,0] opens down

Domain: x-values (left to right), Range: y-values (bottom to top)

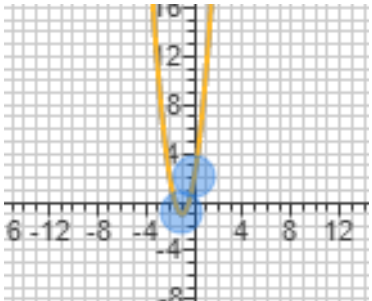
Increasing on (-∞,1) Decreasing on (1, ∞)

*Increasing and Decreasing is always parenthesis and only uses x value

G) For the quadratic function $f(x) = 3x^2 + 6x + 2$, answer parts **(a)** through **(c)**. Verify the results using a graphing utility.

$x = \frac{-6}{2(3)} = -1$ $y = 3(-1)^2 + 6(-1) + 2 = -1$ vertex (-1,-1)

y-intercept is 2 (0,2)



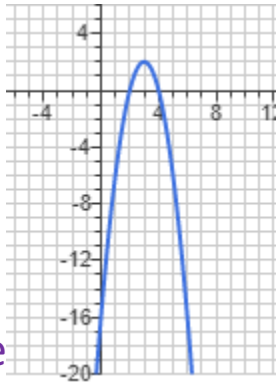
Plot the vertex first
then plot the y-intercept

D: (-∞,∞) R: [-1,-∞) opens up
Domain: x-values (left to right), Range: y-values (bottom to top)
Increasing on (-1, ∞) Decreasing on (-∞,-1)
*Increasing and Decreasing is always parenthesis and only uses x value

H) For the function $g(x) = -2(x-3)^2 + 2$, answer parts **(a)** through **(c)**.

(a) Graph $g(x) = -2(x-3)^2 + 2$. Use the graphing tool to graph the function.

VERTEX (3,2)



Plot x = 3 bc it makes the ()=0
and x = 2 bc it makes the () = 1

(b) Determine the domain and range of the function.

The domain is $(-\infty,\infty)$.
(Type your answer in interval notation.)

Parabolas: domain is always all real number

The range is $(-\infty,2]$.
(Type your answer in interval notation.)

Range always used the y value closed interval on the #

(c) Determine where the function is increasing and where it is decreasing.

Determine where the function is increasing. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Moving up from - ∞ to the x-value

A. The function is increasing on the interval $(-\infty,3)$.
(Type your answer in interval notation.)

Determine where the function is decreasing. Select the correct choice below and, if necessary, answer box to complete your choice.

Moving down from x-value to ∞

A. The function is decreasing on the interval $(3,\infty)$.

I) Decide whether the following statement is true or false.

The graph of $f(x) = 4x^2 + 5x - 7$ opens up.

Positive in front of x^2 term opens up

Choose the correct answer below

- False
- True