5.1 Polynomial Functions **MATH 161** **THOMPSON**

Polynomial has real coefficients and exponents are non-negative.

Degree is highest exponent.

The x-intercepts are when f(x) = 0 or y = 0

The graph of a polynomial function is smooth and continuous.

The even multiplicity touches the x-axis

The odd multiplicity crosses the x-axis

If r is a solution of f(x)=0 then r is a real zero x-intercept, and x-r is a factor.

The multiplicity of a zero is the # of times it corresponding factor

END BEHAVIOR

$f(x)=x^2$ $f(x)= -x^2$

for any even function

$f(x)=x^3$ $f(x)= -x^3$

for any odd function

1) Is the expression $4x^3 - 3.6x^2 - \sqrt{2}$ a polynomial? If so, what is its degree?

Yes, degree 3 $\sqrt{2}$ is just a number

2) The x-intercepts of a function y = f(x) are the real solutions of the equation f(x) = 0.

True

3) The graph of every polynomial function is both smooth and continuous.

4) If r is a real zero of even multiplicity of a function f, then the graph of f touches the x-axis at r.

5) If r is a solution to the equation f(x) = 0, name three additional statements that can be made about f and r assuming f is a polynomial function.

If r is a solution to the equation f(x) = 0, then r is a real zero of the polynomial function f, r is an x-intercept of the graph of f, and x – r is a factor of f.

6) The graph of the function $f(x) = 2x^4 - x^3 + 9x^2 - 4x - 5$ resembles the graph of $y = 2x^4$ for large values of $|x|$.

End behavior (basic graph shape) is highest exponent term

7) The multiplicity of a real zero is the number of times its corresponding factor occurs.

8) The graph of $y = 5x^6 - 3x^4 + 2x - 9$ has at most how many turning points?

1 – highest exponent 5

9) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$f(x) = 5x + x^3$

Polynomial degree 3

The polynomial in standard form is $f(x) = x^3 + 5x$ with leading term x^3 and the constant 0

10) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$g(x) = \frac{1 - x^4}{5}$

Polynomial degree 4

The polynomial in standard form is $f(x) = -\frac{x^4}{5} + \frac{1}{5}$ with leading term $-\frac{x^4}{5}$ and the constant $\frac{1}{5}$

11) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$$f(x) = 7 - \frac{1}{x^2}$$

Is is not a polynomial because the variable z is raised to the -2 power

The polynomial is not a function

12) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$$g(x) = x^{\frac{1}{2}} - x^2 + 2$$

Is is not a polynomial because the variable z is raised to the ½ power

The polynomial is not a function

13) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it is not, tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$$F(x) = 8x^4 - \pi x^2 + \frac{1}{4}$$

Polynomial degree 4

The polynomial in standard form is f(x) = $8x^4 - \pi x^2 + ¼$ with leading term $8x^4$ and the constant ¼

14) Determine whether the following function is a polynomial function. If the function is a polynomial function, state its degree. If it tell why not. Write the polynomial in standard form. Then identify the leading term and the constant term.

$G(x) = 4(x-3)^2(x^2+3)$

Degree of 4

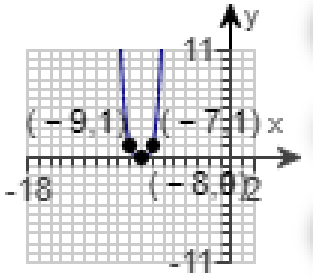
A. The polynomial in standard form is $G(x) = 4x^4 - 24x^3 + 48x^2 - 72x + 108$ with leading term $4x^4$ and constant 108.

$4(x^2-6x+9)(x^2+3)$ multiply out

$(x^2+3)(4x^2-24x+36)$ is the easiest method

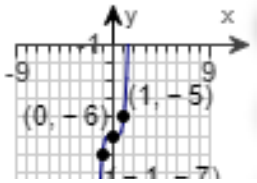
15) Use a transformation of the graph of $y = x^4$ to graph the function

$f(x) = (x+8)^4$ left 8 units



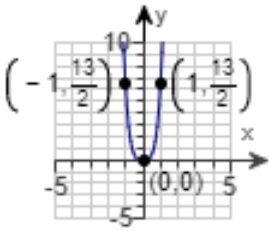
16) Use a transformation of the graph of $y = x^5$ to graph the function.

$f(x) = x^5 - 6$ Down 6 units



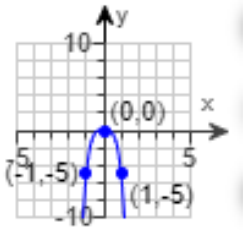
17) Use a transformation of the graph of $y = x^4$ to graph the function.

$f(x) = \frac{13}{2}x^4$ Vertical stretch of $\frac{13}{2}$



18) Use a transformation of the graph of $y = x^4$ to graph the function.

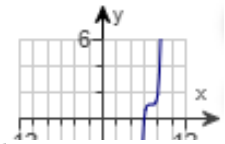
$h(x) = -5x^4$ Reflects across the x-axis<
vertical stretch of 5



19) Use a transformation of the graph of $y = x^5$ to graph the function.

$f(x) = (x-7)^5 + 1$

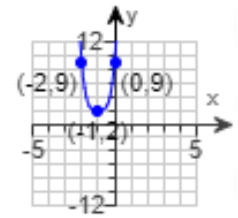
Shifts right 7 and up 1 unit



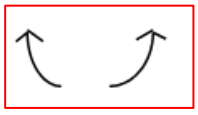
20) Use a transformation of the graph of $y = x^4$ to graph the function.

$h(x) = 7(x+1)^4 + 2$

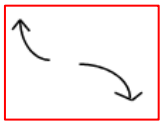
Shifts left 1 and up 2 units



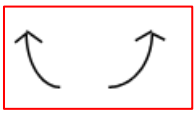
21) $f(x) = 8x^4 - 5x + 1$



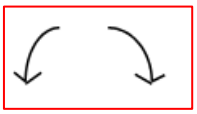
22) $f(x) = -8x^3 - 6x^2 + 2$



23) $f(x) = 9x^6 - 3x^4 + x^2 - 4$



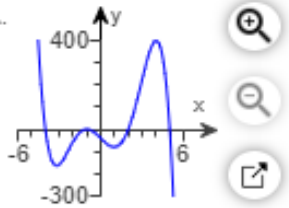
24) $f(x) = 5 + 4x - 2x^2 - 3x^{10}$



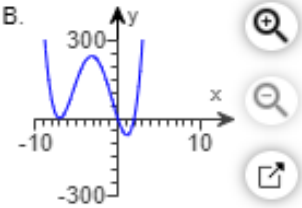
25)

The comprehensive graphs of four polynomial functions are shown in A-D.

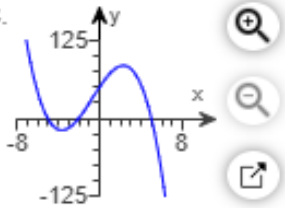
A.



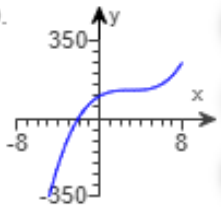
B.



C.



D.



Which of the graphs cannot be that of a cubic polynomial function?

The graph(s) in A,B cannot be the graph of a cubic polynomial function.

*because graph cannot touch more than 3 nor turn more than 2

26) Find the polynomial function of degree 3 whose zeros are -5, 3, and 7

$f(x) = (x+5)(x-3)(x-7)$

27) Find the polynomial function of degree 3 whose zeros are -1, 0, and 6

$f(x) = x(x+1)(x-6)$

28) Zeros: -3, multiplicity 1, 6, multiplicity 2

multiplicity is exponent $f(x) = (x+3)(x-6)^2$

29) Find a polynomial function with the given real zeros whose graph contains the given point.

Zeros: $-3, 0, 2, 1$
Degree: 4
Point: $\left(-\frac{1}{2}, -150\right)$

$y = ax(x+3)(x-2)(x-1)$

$-150 = a\left(-\frac{1}{2}\right)\left(-\frac{1}{2}+3\right)\left(-\frac{1}{2}-2\right)\left(-\frac{1}{2}-1\right)$

$-150 = -\frac{75}{16}a \qquad a = 32$

$f(x) = 32x(x+3)(x-2)(x-1)$

30) Construct a polynomial function f with the following characteristics.

zeros: -1, 2, and 4
degree 3
y-intercept: -16

Choose the correct answer below.

- A. $f(x) = -2(x+1)(x-2)(x-4)$
- B. $f(x) = (x+1)(x-2)(x-4)$
- C. $f(x) = 2(x+1)(x-2)(x-4)$
- D. $f(x) = -2(x+1)^2(x-2)(x-4)$

Cross out the ones that will be a + answer

Find which one has -16 when x = 0

31) For the polynomial function below: (a) List each real zero and its multiplicity. (b) Determine whether the graph crosses or touches the x-axis at each x-intercept. (c) Determine the maximun number of turning points on the graph. (d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

$f(x) = -6(x^2+64)(x-3)^3$

$x^2 + 64$ does not factor

(a) Find any real zeros of f. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$x - 3 = 0$

$x = 3$

- A. The real zero(s) of f is/are 3.
  (Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)
- B. There are no real zeros.

exponent of the (x-3) term

The multiplicity of the zero is 3.

(b) The graph of f crosses the x-axis at the x-intercept.

ODD CROSSES

EVEN TOUCHES

(c) The maximum number of turning points on the graph is 4.
(Type a whole number.)

(d) Type the power function that the graph of f resembles for large values of $|x|$.

$y = -6(x^2)(x)^3$ add exponents $y = -6x^5$

32) For the polynomial function $f(x) = 2(x^2+1)^2(x-3)^3$ answer the following questions.
(a) List each real zero and its multiplicity.
(b) Determine whether the graph crosses or touches the x-axis at each x-intercept.
(c) Determine the maximum number of turning points on the graph.
(d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

(a) Find any real zeros of f. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

Only $x - 3 = 0$ $x = 3$

A. The real zero of f is 3 with multiplicity of 3.
(Simplify your answer. Type integers or fractions. Type each answer only once.)

(b) Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The graph touches the x-axis at .
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. The graph crosses the x-axis at 3. Same as zeros odd exponent crosses

(c) The maximum number of turning points on the graph is 6.
(Type a whole number.)

largest exponent - 1

(d) The power function that the graph of f resembles for large values of $|x|$ is $y = 2x^7$.

$2(x^2)^2x^3 = 2x^4\,x^3$

33) For the polynomial function below: (a) List each real zero and its multiplicity. (b) Determine whether the graph crosses or touches the x-axis at each x-intercept. (c) Determine the maximum number of turning points on the graph. (d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

$$f(x) = -8\left(x+\frac{1}{4}\right)^2(x+8)^3$$

Square is on outside so I do not factor

(a) Find any real zeros of f. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The real zero(s) of f is/are $-8, -\frac{1}{4}$.

$x + ¼ = 0$ $x+8 = 0$

$x = -¼$ $x = -8$

(Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

B. There are no real zeros.

exponent on the -¼ term even touches

The multiplicity of the larger zero is 2.

(b) The graph of f touches the x-axis at the larger x-intercept.

The graph of f crosses the x-axis at the smaller x-intercept.

Multiplicity -8 term is 3

(c) The maximum number of turning points on the graph is 4.
(Type a whole number.)

$x^2 x^3 = x^5$ subtract 1

(d) Type the power function that the graph of f resembles for large values of $|x|$.

$y = -8x^5$

$y = -8x^2(x^3)$ add exponents

34) For the polynomial function below: (a) List each real zero and its multiplicity. (b) Determine whether the graph crosses or touches the x-axis at each x-intercept. (c) Determine the maximun number of turning points on the graph. (d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

$f(x) = -7x^2(x^2 - 11)$

Do not forget the zero answer

(a) Find any real zeros of f. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$X^2 = 0$ $X^2 - 11 = 0$

A. The real zero(s) of f is/are $-\sqrt{11}, 0, \sqrt{11}$.
(Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

B. There are no real zeros.

The multiplicity of the largest zero is 1.
The multiplicity of the middle zero is 2.
The multiplicity of the smallest zero is 1.

$-\sqrt{11}$ has exponent of 1
0 has exponent of 2
$\sqrt{11}$ has exponent of 1

(b) The graph of f crosses the x-axis at the largest x-intercept. **ODD CROSSES**

The graph of f touches the x-axis at the middle x-intercept. **EVEN TOUCHES**

The graph of f crosses the x-axis at the smallest x-intercept.

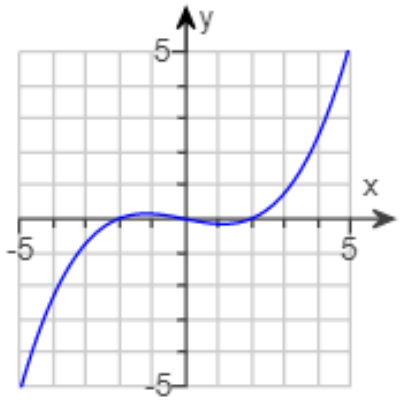
(c) The maximum number of turning points on the graph is 3. Highest exponent - 1

(d) Type the power function that the graph of f resembles for large values of $|x|$.

$y = -7x^4$

$y = -7x^2(x^2)$

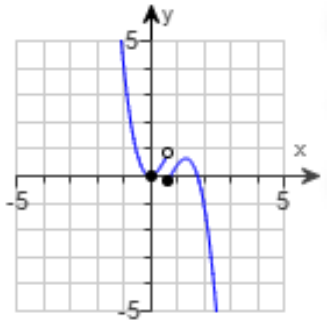
35) Determine whether the graph could be the graph of a polynomial function. If it could be, list the real zeros and state the least degree the polynomial can have.



crosses x - axis

A. The graph shows a polynomial function. The real zero(s) is/are $-2,0,2$. The least degree the polynomial can have is 3.

36) Determine whether the graph could be the graph of a polynomial function. If it could be, list the real zeros and state the least degree the polynomial can have.

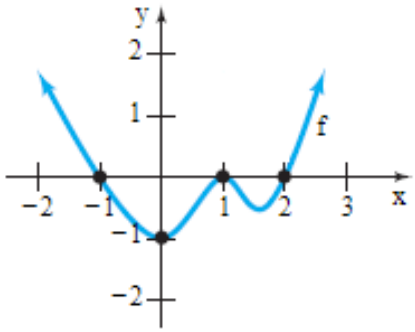


Select the correct choice below and fill in any answer boxes within your choice.

A. The graph shows a polynomial function. The real zero(s) is/are ____. The least degree the polynomial can have is ____.
(Use a comma to separate answers as needed. Round to the nearest integer as needed.)

B. The graph does not show a polynomial function.

37) Construct a polynomial function that might have the given graph.



crosses at -1 (x+1)

touches at 1 $(x-1)^2$

crosses at 2 (x-2)

is up so the number out front is positive

Which of the following is a polynomial function that might have the given graph?

A. $f(x) = -\frac{1}{2}(x+1)(x-1)^2(x-2)$

B. $f(x) = -\frac{1}{2}(x+1)(x-1)(x-2)$

C. $f(x) = \frac{1}{2}(x+1)(x-1)(x-2)^2$

D. $f(x) = \frac{1}{2}(x+1)^2(x-1)(x-2)^2$

E. $f(x) = \frac{1}{2}(x+1)(x-1)(x-2)$

F. ✔ $f(x) = \frac{1}{2}(x+1)(x-1)^2(x-2)$

38) Analyze the polynomial function $f(x) = x^2(x-6)$ using parts (a) through (e).

(a) Determine the end behavior of the graph of the function.

The graph of f behaves like $y = x^3$ for large values of $|x|$.

$x^2(x)$

(b) Find the x- and y-intercepts of the graph of the function.

$x-6 = 0$

The x-intercept(s) is/are 0,6 .

$x = 0$ $x = 6$

(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

The y-intercept is 0 .

$y = 0\ (0-6) = 0$

(Simplify your answer. Type an integer or a fraction.)

(c) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

same as x-intercept

The zero(s) of f is/are 0,6 .

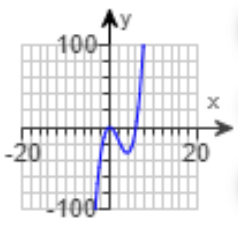
exponent even x-int

The lesser zero of the function is of multiplicity 2 , so the graph of f touches the x-axis at x = 0 . The greater zero of the function is of multiplicity 1 , so the graph of f crosses the x-axis at x = 6 .

(d) Determine the maximum number of turning points on the graph of the function. 2 exponent -1

ODD CROSSES **EVEN TOUCHES**

Touches at x = 0 Crosses at x = 6



39) Analyze the polynomial function $f(x) = (x+4)^2(3-x)$ using parts (a) through (e).

(a) Determine the end behavior of the graph of the function.

The graph of f behaves like $y = -x^3$ for large values of $|x|$. - x x^2

(b) Find the x- and y-intercepts of the graph of the function.

The x-intercept(s) is/are $-4, 3$. x+4 = 0 3 – x = 0

(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed. Type each answer only once.)

The y-intercept is 48. $(0+4)^2$ (3-0)

(Simplify your answer. Type an integer or a fraction.)

(c) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

same as x-intercepts

The zero(s) of f is/are $-4, 3$.

exponent even x-int

The lesser zero of the function is of multiplicity 2, so the graph of f touches the x-axis at $x = -4$.

The greater zero of the function is of multiplicity 1, so the graph of f crosses the x-axis at $x = 3$.

(d) Determine the maximum number of turning points on the graph of the function.

2 (Type a whole number.) $x^2(-x)$ $y = -x^3$ exponent - 1

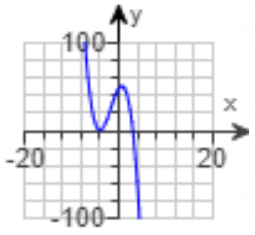
(e) Use the above information to draw a complete graph of the function. Choose the correct graph below.

$y = -x^3$

Reflects across x-axis

touches at x = -4

crosses t x = 3



40) Analyze the polynomial function $f(x) = x^3 + x^2 - 6x$. Answer parts **(a)** through **(e)**. [Hint: First factor the polynomial.]

(a) Determine the end behavior of the graph of the function.

The graph of f behaves like $y = x^3$ for large values of $|x|$. $x(x^2 + x - 6)$

(b) Find the x- and y-intercepts of the graph of the function. $x(x+3)(x-2)=0$

The x-intercept(s) is/are $0, -3, 2$.

(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

The y-intercept is 0.

(c) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

same as x-intercepts

The zero(s) of f is/are $0, -3, 2$.

(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

exponent odd x- int

The smallest zero is of multiplicity 1, so the graph of f crosses the x-axis at $x = -3$. The middle zero is of multiplicity 1, so the graph of f crosses the x-axis at $x = 0$. The largest zero is of multiplicity 1, so the graph of f crosses the x-axis at $x = 2$.

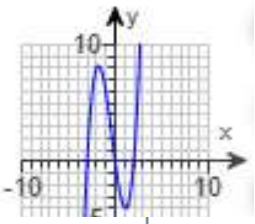
(d) Determine the maximum number of turning points on the graph of the function.

highest exponent -1

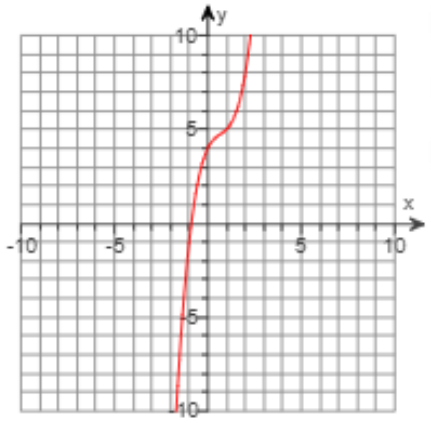
The graph of the function will have at most 2 turning points.

cube function

crosses at -3,0,2



41) Use end behavior and analysis of the dominating term to determine which function matches the graph.



Which function matches the graph?

- A. $f(x) = -2x^4 + x^3 - 2x^2 + 2x + 4$
- B. $f(x) = -x^3 - 2x^2 + 2x + 4$
- C. $f(x) = 2x^4 + x^3 - 2x^2 + 2x + 4$
- D. $f(x) = x^3 - 2x^2 + 2x + 4$

Ex) For the polynomial function below: (a) List each real zero and its multiplicity. (b) Determine whether the graph crosses or touches the x-axis at each x-intercept. (c) Determine the maximun number of turning points on the graph. (d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

$$f(x) = -6(x^2 + 64)(x - 3)^3$$

$x^2 + 64$ does not factor

(a) Find any real zeros of f. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The real zero(s) of f is/are 3.
(Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

B. There are no real zeros.

The multiplicity of the zero is 3.

(b) The graph of f crosses the x-axis at the x-intercept.

(c) The maximum number of turning points on the graph is 4.
(Type a whole number.)

One less than the degree

(d) Type the power function that the graph of f resembles for large values of $|x|$.

$y = -6x^5$