## 5.4 Graphs of Rational Functions MATH 161 THOMPSON

1) Find the intercepts of the graph of the equation $y = \frac{x^2 - 16}{x^2 - 9}$.

Select the correct choice below and, if necessary, fill in the an choice.

A. The intercept(s) is/are $(-4,0),(4,0),\left(0,\frac{16}{9}\right)$.

x-intercepts: solve top by factoring

y-intercept: cover up the x values

2) Which type of asymptote will never intersect the graph of a rational function?

Choose the correct answer below.

- oblique
- horizontal
- vertical
- all of these

**VERTICAL ASYMPTOTES**

Solve the bottom of the fraction

**HORIZONTAL ASYMPTOTES**

- If higher exponent of x is on top there is none
- If higher exponent of the x is on bottom then y=0
- If the exponents on higher x match then y = coefficient in front EX: $\frac{3x^2+4x}{5x^2-6}$ $y = \frac{3}{5}$

3) Identify the y-intercept of the graph of $R(x) = \frac{12(x-1)}{(x+1)(x+3)}$.

Choose the correct y-intercept below.

- 1
- 3
- −1
- −4

$$\frac{12(0-1)}{(0+1)(0+3)} = \frac{-12}{3} = -4$$

4) Follow the steps for graphing a rational function to graph the function $R(x) = \frac{x+2}{x(x+7)}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. R(x) = ☐ (Type your answer in factored form. Do not simplify.)

B. R(x) cannot be factored further.

Follow the steps for graphing a rational function to graph the function $R(x) = \frac{x+2}{x(x+7)}$.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\{x|x <$ ___ $\}$ (Type an integer or a simplified fraction.)
- B. $\{x|x \geq$ ___ $\}$ (Type an integer or a simplified fraction.)
- ✔ C. $\{x|x \neq -7,0\}$ solve the bottom

Odd exponent crosses
Even exponent touches

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. R(x) = ___ (Simplify your answer.)
- ✔ B. R(x) is already in lowest terms.

Check multiplicity (exponent) of each asymptote:
If it is odd, it approaches ∞ on one side and -∞ on the other
If it is even, it approaches either ∞ or -∞

The graph has x-intercept(s) $-2$ and no y-intercept. solve the top for x-intercept, since

The graph will cross the x-axis at $x = -2$. crosses because exponent on that term is odd

The function has two vertical asymptotes. The leftmost asymptote is $x = -7$, and the rightmost asymptote is $x = 0$.

It approaches $\infty$ on one side of the asymptote(s) at $x = -7,0$ and $-\infty$ on the other. odd exponent crosses

The function has one horizontal asymptote, $y = 0$. higher x exponent on the bottom y=0

The function has no oblique asymptote.

The graph of R intersects the horizontal or oblique asymptote at $(-2,0)$

put horizontal asymptote y=0 into the equation
0 = x – 2  x = 2  write coordinate

The graph of R is above the x-axis on the interval(s) $(-7,-2),(0,\infty)$ and below the x-axis on the interval(s) $(-\infty,-7),(-2,0)$. Use the + and – intervals you made

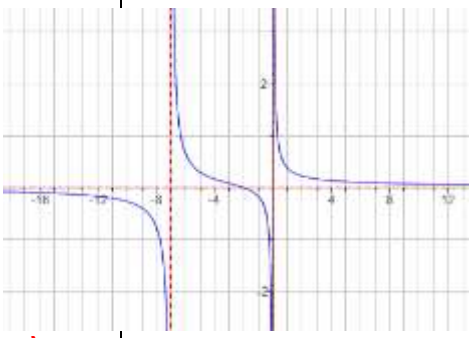
Plug in numbers left and right of vertical asymptotes **and** x-intercept

-7  -2  0

-8  -3  -1  1  check + or - intervals

-  +  -  +

graph using the intervals, crosses at x = -2 bc it is + left and – right of -2 (the x-intercept)



5) Follow the steps for graphing a rational function to graph the function $R(x) = \frac{3x+3}{4x+16}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x) = \frac{3(x+1)}{4(x+4)}$ (Type your answer in factored form. Do not simplify.)

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $\{x|x \neq -4\}$ Solve the bottom

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x) =$ (Simplify your answer.)

B. R(x) is already in lowest terms. x-int solve the top y-int set x=0

The graph has x-intercept(s) $-1$ and y-intercept $\frac{3}{16}$.

The graph will cross the x-axis at $x = -1$.

The function has one vertical asymptote, $x = -4$ crosses because the term has odd exponent

It approaches $\infty$ on one side of the asymptote(s) at $x = -4$ and $-\infty$ on the other. vertical asymptote: solve the bottom

The function has one horizontal asymptote, $y = \frac{3}{4}$

exponents match: coefficients in front

There is no oblique asymptote.

There is no point at which the graph of R intersects the horizontal or oblique asymptote.

The graph of R is above the x-axis on the interval(s) $(-\infty, -4), (-1, \infty)$ and below the x-axis on the interval(s) $(-4, -1)$.

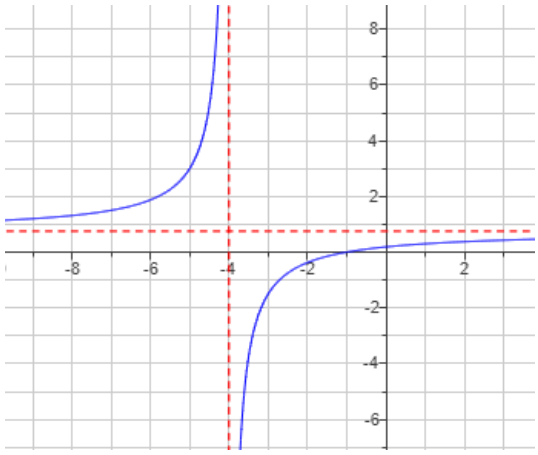
asym and x-intercept

-4 -1

-5 -2 0 check + or - intervals

\+ - +

graph using intervals and crosses at x = -1 bc – to the left and + to the right of -1



6) Follow the steps for graphing a rational function to graph the function $R(x)=\frac{5}{x^2-36}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $\{x|x \neq -6,6\}$

B. R(x) is already in lowest terms.

x-int: solve the top but no x value

The graph has y-intercept $-\frac{5}{36}$ and no x-intercept.

y-int set x = 0

The function has two vertical asymptotes. The leftmost asymptote is $x=-6$, and the rightmost asymptote is $x=6$.

It approaches $\infty$ on one side of the asymptote(s) at $x=$ $-6,6$ and $-\infty$ on the other.

green boxes top of the page

The function has one horizontal asymptote, $y=0$.

There is no oblique asymptote.

The graph of R is above the x-axis on the interval(s) $(-\infty,-6),(6,\infty)$ and below the x-axis on the interval(s) $(-6,6)$.

asym and x-intercept

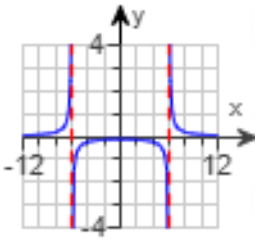
-6 6

-7 0 7

\+ - +

graph using intervals

does not cross y asymptote




7) Follow the steps for graphing a rational function to graph the function $R(x)=\frac{x^2}{x^2+x-20}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x)=\frac{x^2}{(x+5)(x-4)}$ (Type your answer in factored form. Do not simplify.)

B. R(x) is already in factored form.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $\{x|x \neq -5,4\}$

Solve the bottom

(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. R(x) is already in lowest terms.

Locate the intercept(s) of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-intercept set top = 0

y-intercept set x = 0

A. The graph has x-intercept(s) 0 and y-intercept 0.
(Simplify your answers. Type integers or fractions. Use a comma to separate answers as needed. Type each answer only once.)

Determine the behavior of the graph of R at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

B. The graph will touch but not cross the x-axis at x = 0.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

touches because of the even exponent

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(ex) to complete your choice.

B. The function has two vertical asymptotes. The leftmost asymptote is $x = -5$, and the rightmost asymptote is $x = 4$.

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

C. It approaches $\infty$ on one side of the asymptote(s) at $x = -5, 4$ and $-\infty$ on the other.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Green box on top of page

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has one horizontal asymptote, $y = 1$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

Since exponents top and bottom match, the horizontal asymptote is the coefficients out in front.

C. There is no oblique asymptote.

Determine the points, if any, at which the graph of R intersects the horizontal or oblique asymptote, if one exists. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The graph of R intersects the horizontal or oblique asymptote at (20,1).
(Simplify your answer. Type an ordered pair. Use a comma to separate answers as needed.)

Plug in horizontal asymptote y = 1 into the equation:

$1 = \frac{x^2}{x^2+x-20}$ cross multiply $x^2 + x - 20 = x^2$

x = 20 coordinate (20,1)

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

C. The graph of R is above the x-axis on the interval(s) $(-\infty,-5),(4,\infty)$ and below the x-axis on the interval(s) $(-5,0),(0,4)$.

Plug in numbers left and right of vertical asymptotes and x-intercept

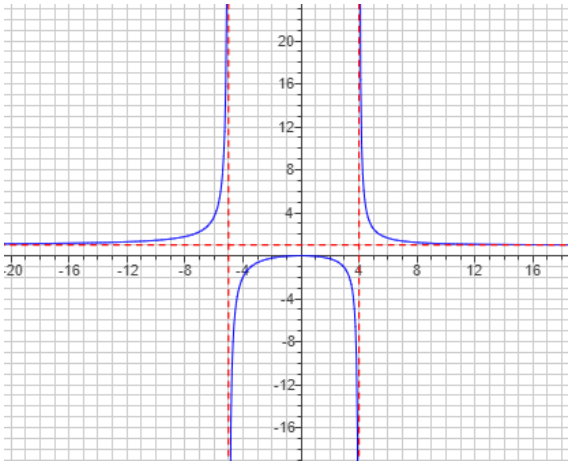
-5 0 4

-6 -1 1 5 to check + and - intervals

\+ - - +

sketch what the graph will look like using + and – intervals

does not cross the y asymptote since the sign stayed – at x=0




8) Follow the steps for graphing a rational function to graph the function $G(x) = \frac{3x}{x^2-81}$.

If needed, first write the given function as a single rational expression. Then factor the numerator and denominator of G(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $G(x) = \frac{3x}{(x-9)(x+9)}$ (Type your answer in factored form. Do not simplify.)

B. G(x) cannot be factored further.

What is the domain of G(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $\{x|x \neq -9,9\}$ Solve the bottom

B. G(x) is in lowest terms.

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-intercept set top = 0

y-intercept set x = 0

A. The graph has x-intercept(s) 0 and y-intercept 0.

Determine the behavior of the graph of G at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The graph will cross the x-axis at x = , and touch but not cross the x-axis at x = .
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

crosses because of the odd exponent on the x-intercept (top)

B. The graph will cross the x-axis at x = 0.

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has one vertical asymptote, .
(Type an equation. Use integers or fractions for any numbers in the equation.)

B. The function has two vertical asymptotes. The leftmost asymptote is $x = -9$ and the rightmost asymptote is $x = 9$.

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. It approaches $\infty$ on one side of the asymptote(s) at x = and $-\infty$ on the other. It approaches either $\infty$ or $-\infty$ on both sides of the asymptote(s) at x = .
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

B. It approaches $\infty$ on one side of the asymptote(s) at x = −9,9 and $-\infty$ on the other.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Green box top of the page

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(ex) to complete your choice.

A. The function has one horizontal asymptote, $y = 0$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

Since exponents on bottom is larger than horizontal asymptote is y=0

C. There is no oblique asymptote.

Determine the points, if any, at which the graph of G intersects the horizontal or oblique asymptote, if one exists. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The graph of G intersects the horizontal or oblique asymptote at $(0,0)$.
(Simplify your answer. Type an ordered pair. Use a comma to separate answers as needed.)

Plug in horizontal asymptote y = 0 into the equation:

$0 = 3x$ x = 0 coordinate (0,0)

Plug in numbers left and right of vertical asymptotes and x-intercept

-9 0 9

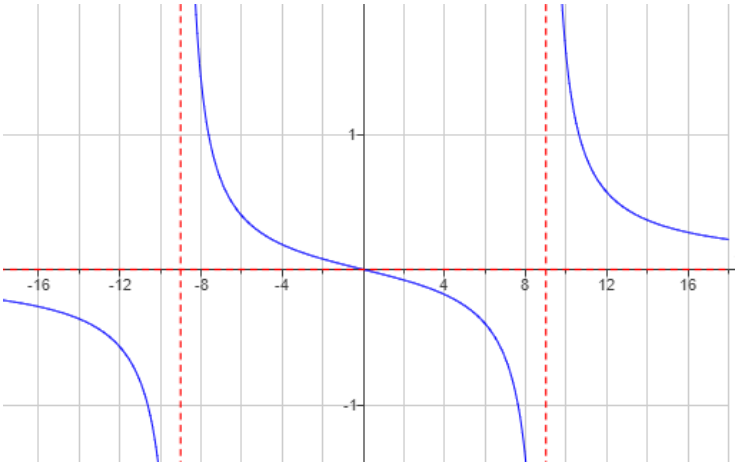
-10 -1 1 10 to check + and - intervals

\- + - +

crosses the y asymptote bc + left and – right of x =0 which is the same as using the algebraic steps in the green above.

Use the real zeros of the numerator and denominator of G to divide the x-axis into intervals. Determine where the graph of G is above or below the x-axis by choosing a number in each interval and evaluating G there. Select the correct choice below and fill in the answer box(es) to complete your choice.

A. The graph of G is above the x-axis on the interval(s) $(-9,0),(9,\infty)$ and below the x-axis on the interval(s) $(-\infty,-9),(0,9)$.



9) Follow the steps for graphing a rational function to graph the function $R(x) = \dfrac{8}{(x-4)(x^2-49)}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x) = \dfrac{8}{(x-4)(x-7)(x+7)}$ (Do not simplify.)

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $\{x|x < \square\}$ (Type an integer or a simplified fraction.)

B. $\{x|x \geq \square\}$ (Type an integer or a simplified fraction.)

C. $\{x|x \neq -7,4,7\}$
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

D. The domain is the set of all real numbers.

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x) = \square$ (Simplify your answer.)

B. R(x) is already in lowest terms.

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

$\dfrac{8}{(0-4)(0-49)}$

A. The graph has y-intercept $\square$ and x-intercept(s) $\square$.
(Simplify your answers. Type integers or fractions. Use a comma to separate answers as needed. Type each answer only once.)

B. The graph has y-intercept $\dfrac{2}{49}$ and no x-intercept.

Determine the behavior of the graph of R at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

D. There is no x-intercept.

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has three vertical asymptotes. The leftmost asymptote is $x = -7$, the middle asymptote is $x = 4$, and the rightmost asymptote is $x = 7$.

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. It approaches $\infty$ on one side of the asymptote(s) at $x =$ ____ and $-\infty$ on the other. It approaches either $\infty$ or $-\infty$ on both sides of the asymptote(s) at $x =$ ____.
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

all odd exponents

B. It approaches $\infty$ on one side of the asymptote(s) at $x = -7,4,7$ and $-\infty$ on the other.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

Since exponents on bottom is larger than horizontal asymptote is y=0

A. The function has one horizontal asymptote, $y = 0$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

C. The function has no oblique asymptote.

Determine the points, if any, at which the graph of R intersects the horizontal or oblique asymptote, if one exists. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

There is no x-intercept because no x-value on top

A. The graph of R intersects the horizontal or oblique asymptote at ____.
(Simplify your answer. Type an ordered pair. Use a comma to separate answers as needed.)

B. The graph of R intersects the horizontal or oblique asymptote at infinitely many points.

C. There is no point at which the graph of R intersects the horizontal or oblique asymptote.

Plug in horizontal asymptote y = 0 into the equation:

$$0 = \frac{8}{(x-4)(x^2-49)}$$ cross multiply 0 = 8 is undefined

therefore, there is not point that intersects the horizontal asymptote

Plug in numbers left and right of vertical asymptotes

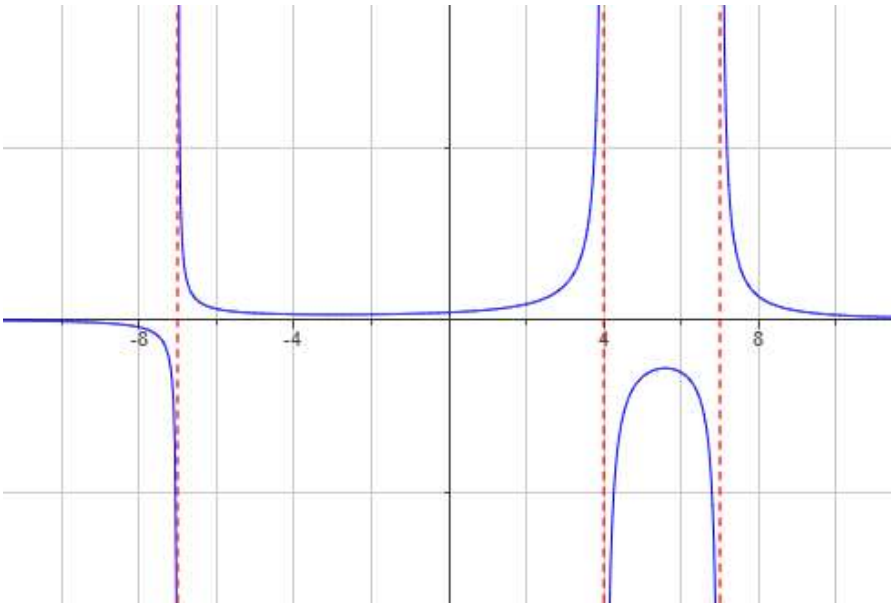
-7 4 7

-8 0 5 8 to + and - intervals

\- + - +

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

A. The graph of R is above the x-axis on the interval(s) $(-7,4),(7,\infty)$ and below the x-axis on the interval(s) $(-\infty,-7),(4,7)$.




10) Follow the steps for graphing a rational function to graph the function $R(x) = \frac{x(x-5)^2}{(x+6)^3}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $R(x) =$ ☐ (Type your answer in factored form. Do not simplify.)
- B. R(x) is already in factored form.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\{x|x \neq -6\}$
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- B. R(x) is already in lowest terms.

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-int: solve the top  y-int: set x=0

Determine the behavior of the graph of R at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-intercept of 0 had exponent of 1 crosses, 5 has exponent of 2 touches

- A. The graph will cross the x-axis at x = 0, and touch but not cross the x-axis at x = 5.
  (Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has three vertical asymptotes. The leftmost asymptote is ____, the middle asymptote is ____, and the rightmost asymptote is ____.
(Type equations. Use integers or fractions for any numbers in the equations.)

B. The one vertical asymptote is $x = -6$.

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

C. It approaches $\infty$ on one side of the asymptote(s) at $x = -6$ and $-\infty$ on the other. odd exponent
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has two horizontal asymptotes. The top asymptote is ____, and the bottom asymptote is ____.
(Type equations. Use integers or fractions for any numbers in the equations.) $\frac{x^3}{x^3}$

B. The function has one horizontal asymptote, $y = 1$. coefficient is 1

C. The function has no oblique asymptote.

C. There is no point at which the graph of R intersects the horizontal or oblique asymptote.

Determine the points, if any, at which the graph of R intersects the horizontal or oblique asymptote, if one exists. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Plug in horizontal asymptote y = 1 into the equation:

$$1 = \frac{x^3-10x^2+25x}{x^2+x-20}$$ cannot be factored so none

Plug in numbers left and right of vertical asymptotes and x-int

-6 0 5

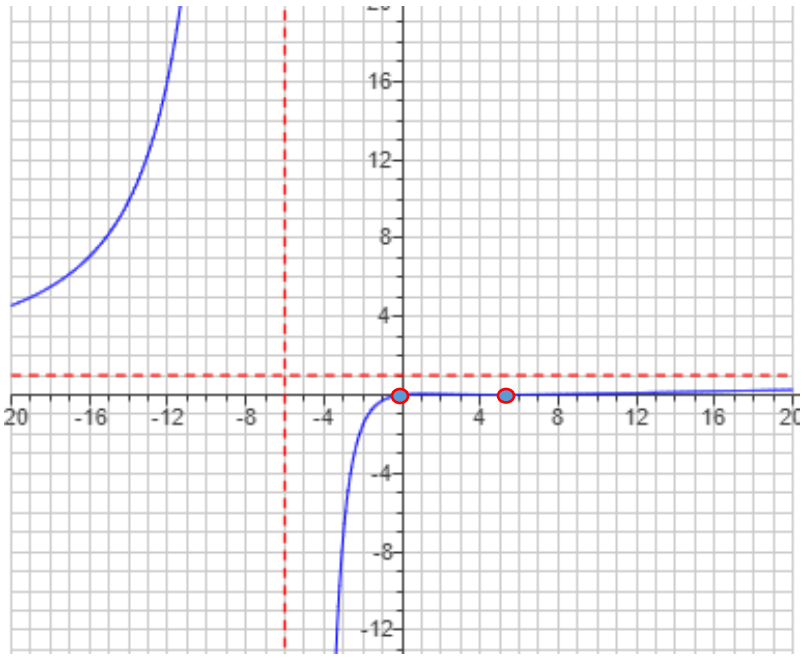
-7 -1 1 6 to check + and - intervals

\+ - + +

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

Crosses at 0 and touches at 5 because of the exponents which makes extra intervals at those points

A. The graph of R is above the x-axis on the interval(s) $(-\infty,-6),(0,5),(5,\infty)$ and below the x-axis on the interval(s) $(-6,0)$.




11) Follow the steps for graphing a rational function to graph the function $R(x)=\dfrac{x^2+5x-36}{x^2-x-12}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x)=\dfrac{(x+9)(x-4)}{(x+3)(x-4)}$ (Type your answer in factored form. Do not simplify.)

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

C. $\{x|x\neq -3,4\}$ only use both x-values in bottom for domain

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Cancel out common factors

A. $R(x)=\dfrac{x+9}{x+3}$ (Simplify your answer.)

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-int: solve the top  y-int set x=0

A. The graph has x-intercept(s) $-9$ and y-intercept $3$.
(Simplify your answers. Type integers or fractions. Use a comma to separate answers as needed. Type each answer only once.)

Determine the behavior of the graph of R at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The graph will cross the x-axis at x = [ ] and touch but not cross the x-axis at x = [ ].
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

B. The graph will cross the x-axis at x = −9.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

solve the top odd exponent so it crosses

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

solve the bottom

C. The function has one vertical asymptote, x = −3.
(Type an equation. Use integers or fractions for any numbers in the equation.)

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. It approaches ∞ on one side of the asymptote(s) at x = [ ] and −∞ on the other. It approaches either ∞ or −∞ on both sides of the asymptote(s) at x = [ ].
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

B. It approaches ∞ on one side of the asymptote(s) at x = −3 and −∞ on the other.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

Exponents top and bottom match

so it's the coefficient

A. The function has one horizontal asymptote, y = 1.
(Type an equation. Use integers or fractions for any numbers in the equation.)

C. The function has no oblique asymptote.

Determine the points, if any, at which the graph of R intersects the horizontal or oblique asymptote, if one exists. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

C. There is no point at which the graph of R intersects the horizontal or oblique asymptote.

Plug in horizontal asymptote y = 1 into the equation:

$1 = \frac{x+9}{x+3}$ $\quad x + 3 = x + 9$ no solution

Plug in numbers left and right of vertical asymptotes and x-int

-9 -3 4

-10 -4 0 5 to check + and - intervals

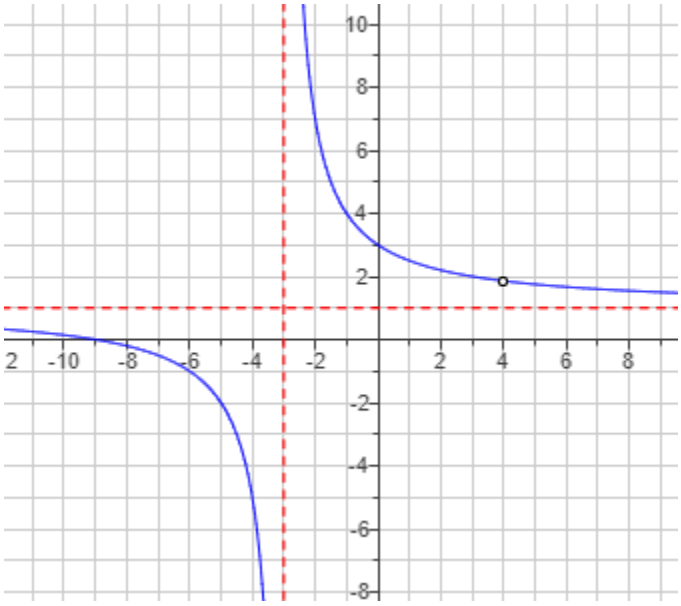
\+ - + +

does not include x=4 because the cancelled term is a whole in the graph, use intervals to choose graph and the whole makes an extra interval

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

A. The graph of R is below the x-axis on the interval(s) .
(Type your answer in interval notation. Use a comma to separate answers as needed.)

B. The graph of R is above the x-axis on the interval(s) $(-\infty,-9),(-3,4),(4,\infty)$ and below the x-axis on the interval(s) $(-9,-3)$.



12) Follow the steps for graphing a rational function to graph the function $R(x) = \frac{16x^2 - 16x - 5}{4x^2 - 13x + 10}$.

If needed, first write the given function as a single rational expression. Then factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Factor using slide and divide

A. $R(x) = \frac{(4x+1)(4x-5)}{(x-2)(4x-5)}$ (Type your answer in factored form. Do not simplify.)

B. R(x) is already in factored form.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

only use both x-values in bottom for domain

A. $\{x|x \geq \square\}$ (Type an integer or a simplified fraction.)

B. $\left\{x|x \neq 2, \frac{5}{4}\right\}$

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Cancel out common factors

A. $R(x) = \frac{4x+1}{x-2}$ (Simplify your answer.)

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-int solve top  y-int set x=0

A. The graph has x-intercept(s) $-\frac{1}{4}$ and y-intercept $-\frac{1}{2}$.

Determine the behavior of the graph of R at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The graph will cross the x-axis at x = $\square$, and touch but not cross the x-axis at x = $\square$.
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

B. The graph will cross the x-axis at $x = -\frac{1}{4}$.

solve the top

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

solve the bottom

C. The function has one vertical asymptote, $x = 2$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. It approaches $\infty$ on one side of the asymptote(s) at x = ____ and $-\infty$ on the other. It approaches either $\infty$ or $-\infty$ on both sides of the asymptote(s) at x = ____.
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

Solve the bottom and odd exponent

B. It approaches $\infty$ on one side of the asymptote(s) at x = 2 and $-\infty$ on the other.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has two horizontal asymptotes. The top asymptote is ____ and the bottom asymptote is ____.
(Type equations. Use integers or fractions for any numbers in the equations.)

B. The function has one horizontal asymptote, y = 4.
(Type an equation. Use integers or fractions for any numbers in the equation.)

Exponents equal, coefficient in front

C. The function has no oblique asymptote.

C. There is no point at which the graph of R intersects the horizontal or oblique asymptote.

Plug in horizontal asymptote y = 1 into the equation:

$4 = \frac{4x+1}{x-2}$ $\quad 4x + 1 = 4x - 8$ no solution

Plug in numbers left and right of vertical asymptotes and x-int

$-\frac{1}{4}$ $\frac{5}{4}$ 2

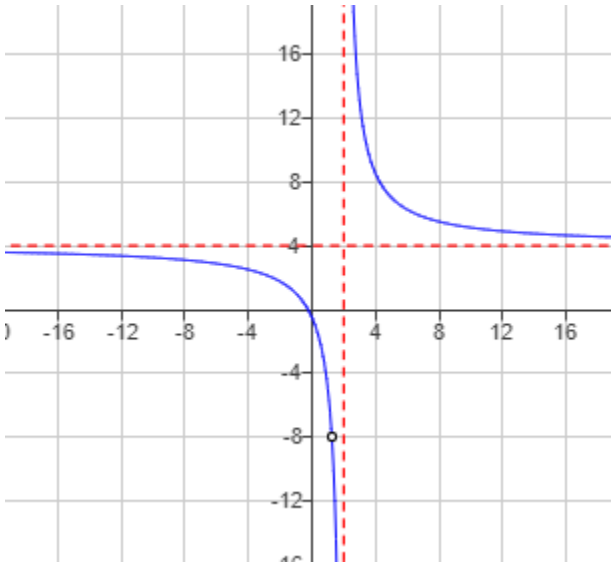
-1 0 $1\frac{1}{2}$ 3 to check + and - intervals

\+ - - +

does not include $x=\frac{5}{4}$ because the cancelled term is a whole in the graph, use intervals to choose graph and the whole makes an extra interval

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

A. The graph of R is above the x-axis on the interval(s) $\left(-\infty,-\frac{1}{4}\right),(2,\infty)$ and below the x-axis on the interval(s) $\left(-\frac{1}{4},\frac{5}{4}\right),\left(\frac{5}{4},2\right)$.




Another 12)

Follow the steps for graphing a rational function to graph the function $R(x)=\frac{6x^2-5x-4}{3x^2-13x+12}$.

If needed, first write the given function as a single rational expression. Then factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Factor using slide and divide

A. $R(x)=\frac{(2x+1)(3x-4)}{(x-3)(3x-4)}$ (Type your answer in factored form. Do not simplify.)

B. R(x) is already in factored form.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

C. $\left\{x \mid x \neq 3, \frac{4}{3}\right\}$

(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

D. The domain is the set of all real numbers.

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

cancel out common factors

A. $R(x) = \frac{2x+1}{x-3}$ (Simplify your answer.)

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-int solve top  y-int set x=0

A. The graph has x-intercept(s) $-\frac{1}{2}$ and y-intercept $-\frac{1}{3}$.

C. The graph will cross the x-axis at $x = -\frac{1}{2}$.

solve the top

C. The function has one vertical asymptote, $x = 3$.

solve the bottom

C. It approaches $\infty$ on one side of the asymptote(s) at $x = 3$ and $-\infty$ on the other.

(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

exponents equal, coefficient in front

A. The function has one horizontal asymptote, $y = 2$.

(Type an equation. Use integers or fractions for any numbers in the equation.)

C. There is no point at which the graph of R intersects the horizontal or oblique asymptote.

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

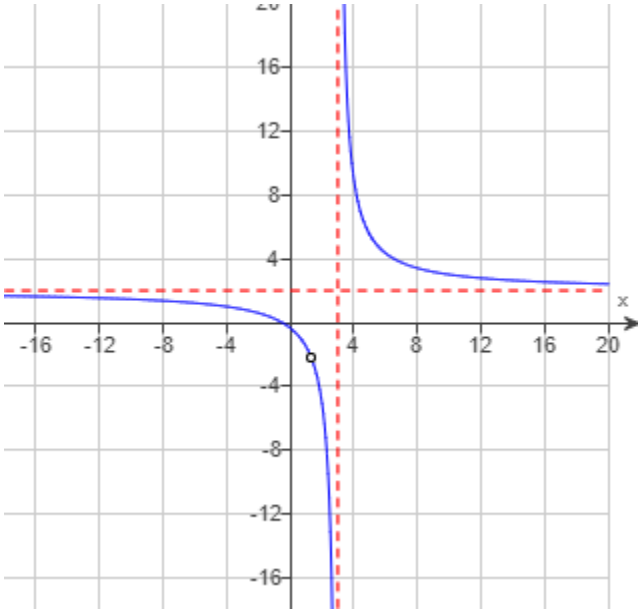
C. The graph of R is above the x-axis on the interval(s) $\left(-\infty, -\frac{1}{2}\right), (3, \infty)$ and below the x-axis on the interval(s) $\left(-\frac{1}{2}, \frac{4}{3}\right), \left(\frac{4}{3}, 3\right)$.

Plug in numbers left and right of vertical asymptotes and x-int

$-\frac{1}{2}$ $\frac{4}{3}$ 3

-1 0 2 4 to check + and - intervals

\+ - - + does not include x=$\frac{4}{3}$ because the cancelled term is a whole in the graph which will make an extra interval




13) Follow the steps for graphing a rational function to graph the function $H(x)=\frac{3x-18}{36-x^2}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of H(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

]

A. $H(x)=\frac{3(x-6)}{(6-x)(6+x)}$ (Type your answer in factored form. Do not

What is the domain of H(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

B. $\{x|x \neq -6,6\}$

Write H(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $H(x)=\frac{-3}{(x+6)}$ (Simplify your answer.)

C. The graph has y-intercept $-\frac{1}{2}$ and no x-intercept.

D. There is no x-intercept.

A. The function has one vertical asymptote, $x=-6$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

C. It approaches $\infty$ on one side of the asymptote(s) at $x=-6$ and $-\infty$ on the other.

A. The function has one horizontal asymptote, $y=0$.

C. There is no oblique asymptote.

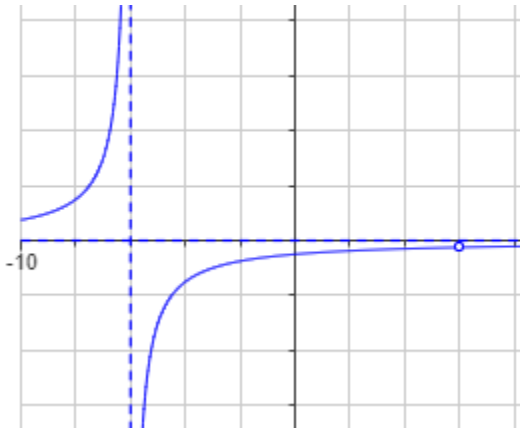
C. There is no point at which the graph of H intersects the horizontal or oblique asymptote.

A. The graph of H is above the x-axis on the interval(s) $(-\infty,-6)$ and below the x-axis on the interval(s) $(-6,6),(6,\infty)$.

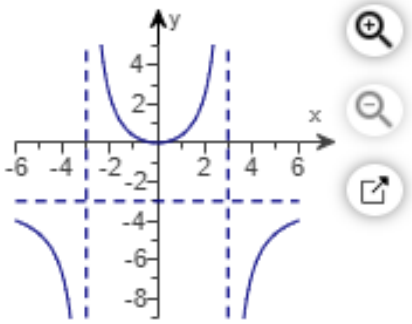
Plug in numbers left and right of vertical asymptotes (-6,6)

-7, 0, 7 to check positive or negative

\+ - - does not include x=6 because the cancelled term is a whole in the graph which will make and extra interval



14) Which rational function has the given graph?




$X^2 - 9$ on bottom
plug in zero and find which on is positive

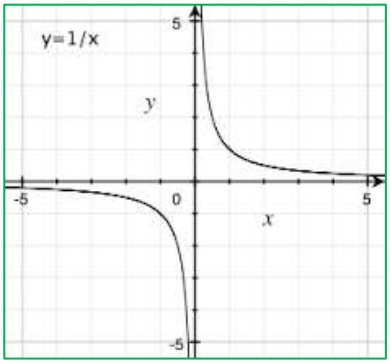
A. $R(x) = \frac{3x^2}{x^2 - 9}$

B. $R(x) = -\frac{3x^2}{x^2 + 9}$

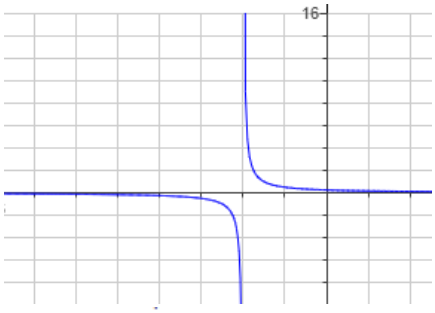
C. $R(x) = \frac{3x^2}{x^2 + 9}$

D. $R(x) = -\frac{3x^2}{x^2 - 9}$

15) Explain how the graph of the function $f(x) = \frac{1}{x+4}$ can be obtained from the graph of $y = \frac{1}{x}$. Then graph f and give the **(a)** domain and **(b)** range. Determine the intervals of the domain for which the function is **(c)** increasing or **(d)** decreasing.




To obtain the graph of f, shift the graph of $y = \frac{1}{x}$ to the left 4 unit(s).




**(a)** The domain of f(x) is $(-\infty, -4) \cup (-4, \infty)$.
(Type your answer in interval notation.)

**(b)** The range of f(x) is $(-\infty, 0) \cup (0, \infty)$.
(Type your answer in interval notation.)

Use union symbol for domain and range

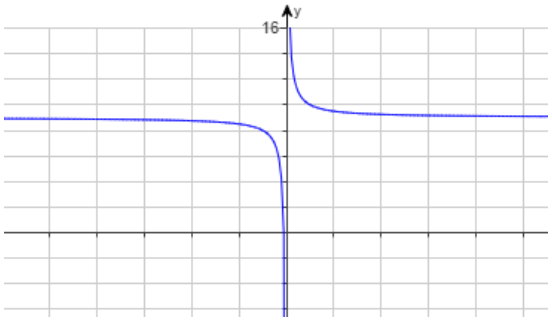
B. There is no interval of the domain for which the function is increasing.

A. The function is decreasing on $(-\infty, -4), (-4, \infty)$.

Use comas for increasing and decreasing

16) Explain how the graph of the function $f(x)=\frac{1}{x}+9$ can be obtained from the graph of $y=\frac{1}{x}$. Then graph f and give the **(a)** domain and **(b)** range. Determine the intervals of the domain for which the function is **(c)** increasing or **(d)** decreasing.

To obtain the graph of f, shift the graph of $y=\frac{1}{x}$ up 9 unit(s).




**(a)** The domain of f(x) is $(-\infty,0)\cup(0,\infty)$.
(Type your answer in interval notation.)

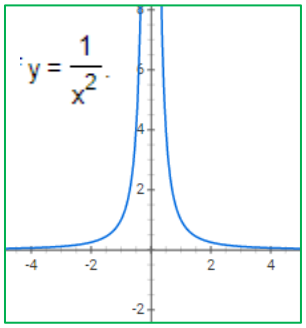
**(b)** The range of f(x) is $(-\infty,9)\cup(9,\infty)$.
(Type your answer in interval notation.)

B. There is no interval of the domain for which the function is increasing.

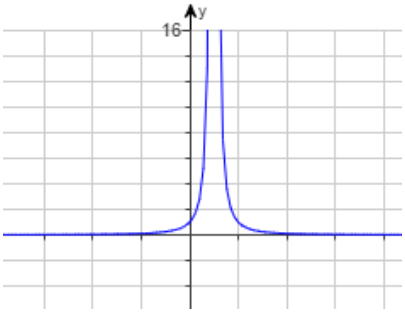
A. The function is decreasing on $(-\infty,0),(0,\infty)$.

17)

Explain how the graph of the function $f(x)=\frac{1}{(x-1)^2}$ can be obtained from the graph of $y=\frac{1}{x^2}$. Then graph f and give the **(a)** domain and **(b)** range. Determine the intervals of the domain for which the function is **(c)** increasing or **(d)** decreasing.




To obtain the graph of f, shift the graph of $y=\frac{1}{x^2}$ to the right 1 unit(s).




**(a)** The domain of f(x) is $(-\infty,1)\cup(1,\infty)$.
(Type your answer in interval notation.)

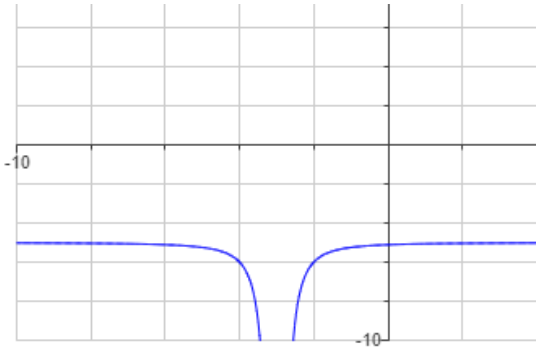
**(b)** The range of f(x) is $(0,\infty)$.
(Type your answer in interval notation.)

A. The function is increasing on $(-\infty,1)$.

A. The function is decreasing on $(1,\infty)$.

18) Explain how the graph of the function $f(x)=\frac{-1}{(x+3)^2}-5$ can be obtained from the graph of $y=\frac{1}{x^2}$. Then graph f and give the **(a)** domain and **(b)** range. Determine the intervals of the domain for which the function is **(c)** increasing or **(d)** decreasing.

To obtain the graph of f, shift the graph of $y=\frac{1}{x^2}$ to the left 3 units, reflect across the x-axis, and shift 5 units down.




**(a)** The domain of f(x) is $(-\infty,-3)\cup(-3,\infty)$.
(Type your answer in interval notation.)

**(b)** The range of f(x) is $(-\infty,-5)$.
(Type your answer in interval notation.)

A. The function is increasing on $(-3,\infty)$.

A. The function is decreasing on $(-\infty,-3)$.

19) Match the rational function with the appropriate description below.

$$f(x)=\frac{x-1}{x+5}$$

Choose the correct attribute of the rational function.

- A. The vertical asymptote is $x=-5$.
- B. The x-intercept is 5.
- C. The vertical asymptote is $x=1$.
- D. The y-intercept is $-1$.

20) Identify the appropriate description for the following rational function.

$$f(x)=\frac{1}{x+3}$$

- A. The x-intercept is 3.
- B. There is a "hole" in its graph at $x=-3$.
- C. The x-axis is its horizontal asymptote, and the y-axis is its vertical asymptote.
- D. The horizontal asymptote is $y=3$.
- E. The vertical asymptote is $x=-1$.
- F. The graph has an oblique asymptote.
- G. The y-intercept is 1.
- H. The x-axis is its horizontal asymptote, and the y-axis is not its vertical asymptote.

21) Match the rational function with the appropriate description below.

$$f(x)=\frac{x^2-16}{x-4}$$

Choose the correct attribute of the rational function.

- A. The vertical asymptote is $x=4$.
- B. There is a "hole" in the graph at $x=4$.

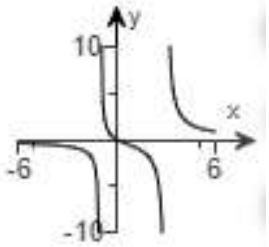
22) Which of the following functions have a graph that does not have a vertical asymptote?

Select all that apply.

- ☐ A. $f(x) = \frac{3}{x^2}$
- ☐ B. $f(x) = \frac{1}{x^2 - 2}$
- ☐ C. $f(x) = \frac{3x + 1}{x - 7}$
- ☑ D. $f(x) = \frac{1}{x^2 + 2}$

23) Sketch the graph of the rational function.

$$f(x) = \frac{3x}{x^2 - 2x - 3}$$




24) Graph the rational function.

$$f(x) = \frac{-2x}{x^2 - 25}$$