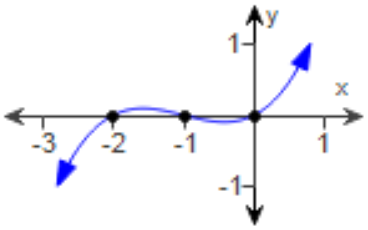
5.5 Polynomial and Rational Inequalities **MATH 161** **THOMPSON**

1) $18 - 3x \geq 6$

$-3x \geq -12$ *switch the sign when dividing by negative #*

$x \leq 4$ $\{x \mid x \leq 4\}$ or $(-\infty, 4]$

2) Use the graph of the function f to solve the inequality.



(a) $f(x) > 0$ $(-2,-1) \cup (0,\infty)$

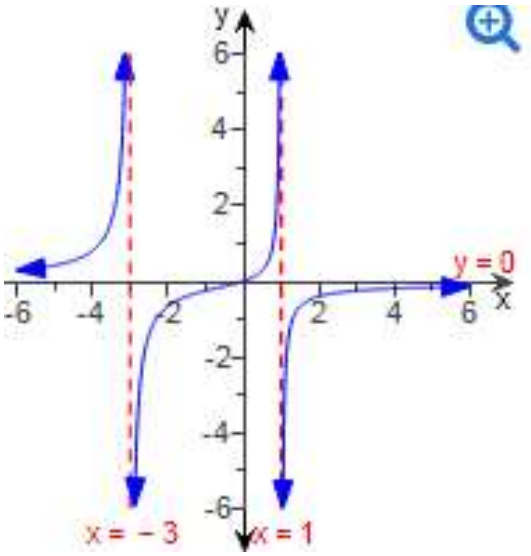
(b) $f(x) \leq 0$ $(-\infty,-2] \cup [-1,0]$

*a) *since it is >, it does not include the point on the x-axis parenthesis*

*b) *since it is ≤, it does include the point on the x-axis brackets*

3)

Use the graph of the function f to solve the inequality.

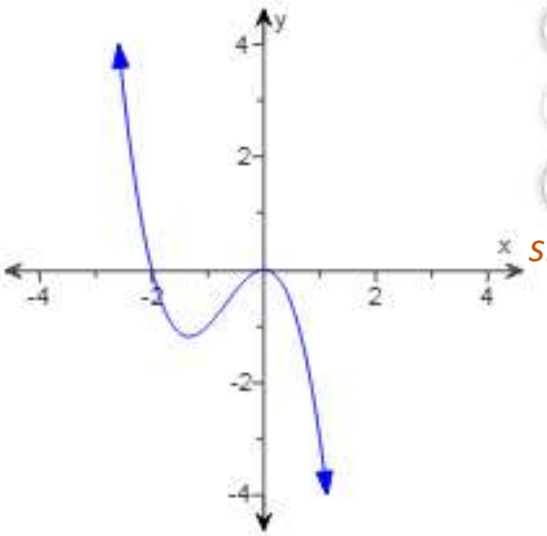


(a) $f(x) < 0$ $(-3,0) \cup (1, \infty)$

(b) $f(x) \geq 0$ $(-\infty,-3) \cup [0,1)$

4)

Solve the inequality $f(x) < 0$, where $f(x) = -x^2(x+2)$, by using the graph of the function.

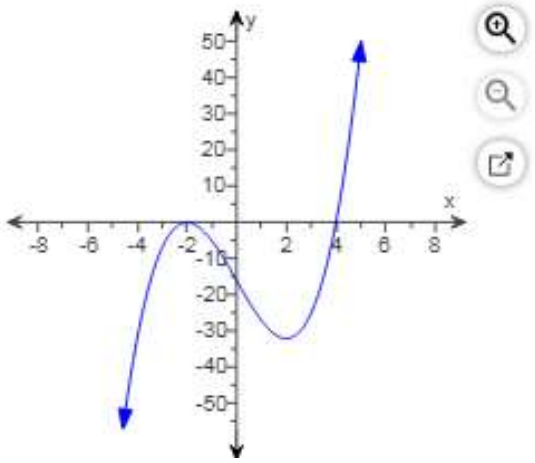


$(-2,0) \cup (0, \infty)$

since it is <, it does not include the point on the x-axis

Solve the inequality $f(x) \le 0$, where $f(x) = (x-4)(x+2)^2$, by using the graph of the function.

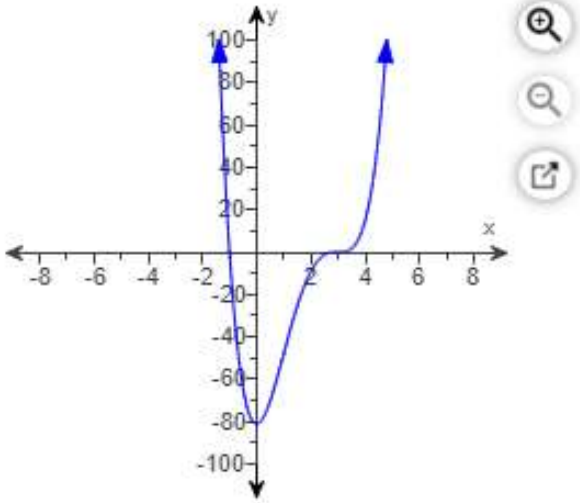
5)



$(-\infty,4]$

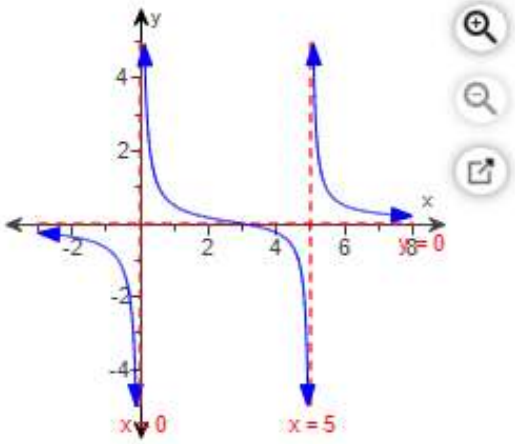
since it is $\leq$, it includes x-axis

6) Solve the inequality $f(x) \ge 0$, where $f(x) = 3(x+1)(x-3)^3$, by using the graph of the function.



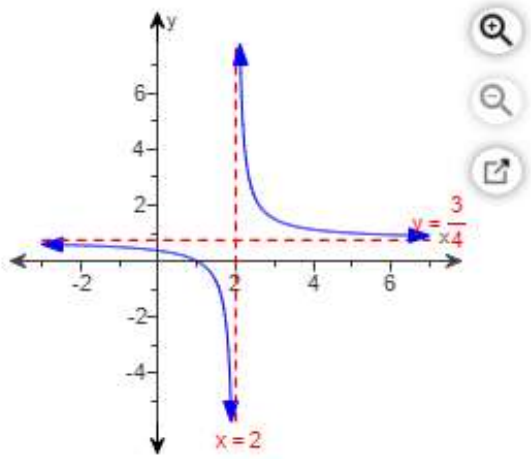
$(-\infty,-1] \cup [3, \infty)$

7) Solve the inequality $R(x) < 0$, where $R(x) = \dfrac{x-3}{x(x-5)}$, by using the graph of the function.



$(-\infty,-3) \cup (3,5)$

8) Solve the inequality $R(x) \le 0$, where $R(x) = \frac{3x-3}{4x-8}$, by using the graph of the function.



[1,2)

9) Solve the inequality algebraically.

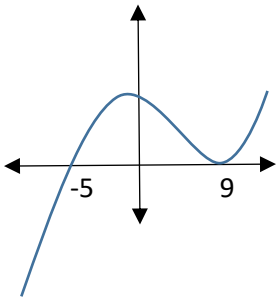
$(x-9)^2(x+5) < 0$

Intercepts are -5 and 9

List the intervals and sign in each interval. Complete the following table.
(Type your answers in interval notation. Use ascending order.)

Interval	$(-\infty,-5)$	$(-5,9)$	$(9,\infty)$
Sign	Negative	Positive	Positive

*easiest to make a graph of x^3 that touches at 9 crosses at -5



Use this graph to fill in the positive or negative in the chart

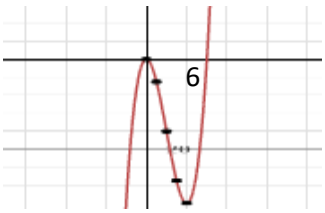
We are looking for below the x-axis since it is < 0

$(-\infty,-5)$

10) Solve the inequality $x^3-6x^2 > 0$ *factor first*

$x^2(x-6) > 0$ *x-intercepts are 0 and 6*

*easiest to make a graph of x^3 that touches at 0 and crosses at 6



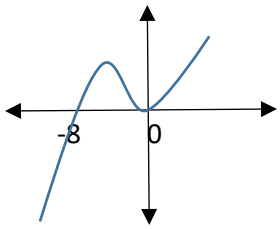
We are looking for above the x-axis since it is > 0

$(6,\infty)$

11) Solve the inequality $3x^3>-24x^2$ $3x^3+24x^2 > 0$ *factor first*

$3x^2(x+8) > 0$ *x-intercepts are 0 and -8*

*easiest to make a graph of x^3 that touches at 0 and crosses at -8



Interval	$(-\infty,-8)$	$(-8,0)$	$(0,\infty)$
Sign	Negative	Positive	Positive

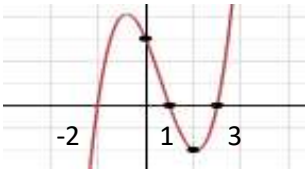
We are looking for above the x-axis since it is > 0

open circle at 0

$(-8,0) \cup (0,\infty)$

12) Solve the inequality $(x-3)(x-1)(x+2)\geq 0$ *x-intercepts are -2, 1 and 3*

*easiest to make a graph of x^3 that crosses at -2, 1, 3



We are looking for above the x-axis since it is ≥ 0

$[-2, 1]\cup[3, \infty)$

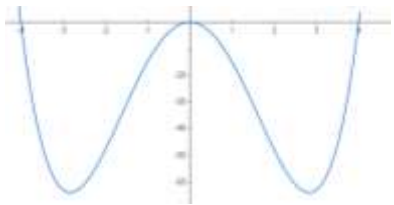
13) Solve the inequality $x^4>16x^2$ *solve first then factor*

$x^4-16x^2 > 0$

$x^2(x^2-16)>0$

$x^2(x+4)(x-4)>0$ *x-intercepts are -4, 0 and 4*

*easiest to look at the graph of x^4 and touches at 0 crosses at -4 and 4



We are looking for above the x-axis since it is > 0

$(-\infty,-4)\cup(4, \infty)$

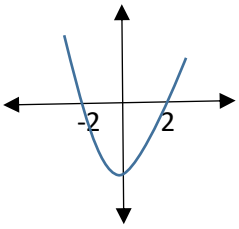
14) Solve the inequality $x^4>16$ *solve first then factor*

$x^4-16> 0$

$(x^2-4)(x^2+4)>0$

$(x-2)(x+2)(x^2+4)>0$ *x-intercepts are -2, 2*

*easiest to look at the graph of x^4 and crosses at -2 and 2



We are looking for above the x-axis since it is > 0

$(-\infty,-2)\cup(2,\infty)$

IF IT IS TOO COMPLICATED TO GRAPH WE MAKE A NUMBER LINE

$\geq$ *means* ●

$>$ *means* ○

all values on bottom are ○

15) Solve the inequality $\frac{x+6}{x-9} < 0$ *we check all the critical points on a number line x = -6, 9

check #s -7 0 10

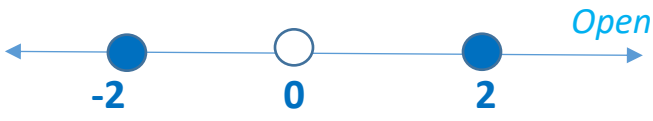
\+ - +

we want all negative intervals

(-6,9)

16) Solve the inequality $\frac{(x-2)(x+2)}{x} \leq 0$

*we check all the critical points on a number line x = -2, 0, 2



check #s -3 -1 1 3

\- + - +

Interval	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
Sign	Negative	Positive	Negative	Positive

we include the points and use negative intervals

$(-\infty,-2]\cup[0,2]$

17) Solve the inequality $\frac{(x-5)^2}{x^2-9} \geq 0$

*we check all the critical points on a number line x = -3, 3, 5

Open at -3, 3 because bottom of fraction

check #s -4 0 4 6

\+ - + + *we want all positive intervals*

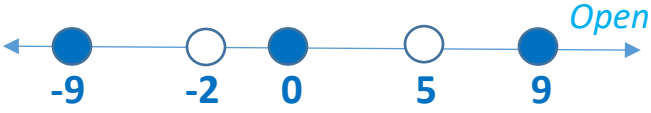
since 5 is solid and already included we don't need to make a new interval

$(-\infty,-3)\cup(3,\infty)$

18) Solve the inequality $\frac{x^2(9+x)(x-9)}{(x+2)(x-5)} \geq 0$

*we check all the critical points on a number line x = -9, -2, 0, 5, 9

Open at -3, 3 because bottom of fraction



check #s -10 -3 -1 2 6 10

\+ - + + - + *we want all positive intervals*

since 5 is open we have to make a new interval since it is NOT included

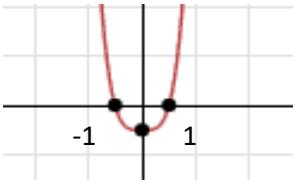
$(-\infty,-9]\cup(-2,5)\cup[9,\infty)$

19) Solve the inequality $\sqrt{x^4-1}$ *square root means x ≥0*

$(x^2-1)(x^2+1)$

$(x-1)(x+1)(x^2+1)$

*easiest to look at the graph of x^4 and crosses at -1,1



We are looking for above and include the x-axis since it is ≥0

$(-\infty,-1]\cup[1,\infty)$

20) Solve the inequality $\sqrt{\frac{x-8}{x+4}}$

square root means $x \geq 0$ but bottom cannot = 0 so bottom $x > 0$ and top $x \geq 0$

*we check all the critical points on a number line $x = -4, 8$

Open at zero because bottom of fraction

check #s -5 0 9

\+ - + *we want all positive intervals*

$(-\infty,-4) \cup [8, \infty)$

EXTRA EXAMPLES:

EX) Solve the inequality $9x - 9 \geq -4x^2$

$4x^2 + 9x - 9 \geq 0$ *use slide and divide*

$x^2 + 9x - 36$

$(x + 12)(x - 3)$ *divide by 4* $x = -3, \frac{3}{4}$

*we check all the critical points on a number line $x = 3, -\frac{3}{4}$

Open at zero because bottom of fraction

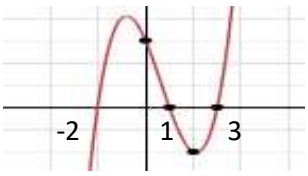
check #s -4 0 1

\+ - + *we want all positive intervals*

$(-\infty,-3] \cup [\frac{3}{4}, \infty)$

EX) Solve the inequality (x-3)(x-1)(x+2)> 0

*easiest to look at the graph of x^3 and crosses at -2, 1, 3



We are looking for above the x-axis since it is > 0

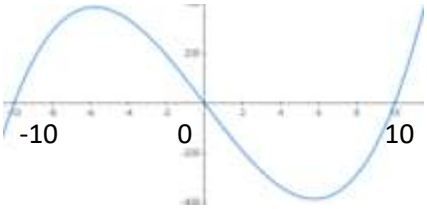
$(-2, 1) \cup (3, \infty)$

EX) Solve the inequality $x^3-100x \leq 0$

$$x(x^2-100) \leq 0$$

$$x(x-10)(x+10) \leq 0$$

*easiest to look at the graph of x^3 and crosses at -10, 0, 10



We are looking for below and include the x-axis since it is < 0

$(-\infty,-10]\cup[0,10]$