6.1 and 6.2 Composition, One-to-One, Invers Functions THOMPSON

1) The domain of the composite function $(f \circ g)(x)$ is the NOT the same as g(x).

2) Find $(f \circ g)(x)$ if $f(x) = \sqrt{x+4}$ and $g(x) = \frac{2}{x}$ we plug the g into f $\sqrt{\frac{2}{x} + 4}$

3) If $H = f \circ g$ and $H(x) = \sqrt{36 - 16x^2}$, which of the following cannot be the component functions f and g?

Choose the correct answer below.

✔ $f(x) = \sqrt{36 - x^2}$; $g(x) = 16x$

4) Given $f(x) = 5x$ and $g(x) = 4x^2 + 4$, find the following expressions.

(a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

(a)	(b)	(c)	(d)
g(4) = 68	f(2) = 10	f(1) = 5	g(0) = 4
f(68) = 340	g(10) = 404	f(5) = 25	g(4) = 68

5) Given $f(x) = 2x^2 - 1$ and $g(x) = 6 - \frac{1}{2}x^2$, find the following expressions.

(a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

(a)	(b)	(c)	(d)
g(4) = -2	f(2) = 7	f(1) = 1	g(0) = 6
f(-2) = 7	$g(7) = -\frac{37}{2}$	f(1) = 1	g(6) = -12

6) For f(x) = 5x + 4 and g(x) = 8x find the following composite functions and state the domain of each.

Write the function listed first with a blank in place of the x value

(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

(a)	(b)	(c)	(d)
5(8x) + 4	8(5x+4)	5(5x+4) + 4	8(8x)
40x + 4	40x+32	25x + 24	64x

✔ B. The domain of $g \circ f$ is all real numbers. For all parts because they are lines

7) For $f(x) = 7x + 9$ and $g(x) = x^2$, find the following composite functions and state the domain of each.

(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

(a) $f \circ g$	(b) $g \circ f$	(c) $f \circ f$	(d) $g \circ g$
$7(x^2) + 9$	$(7x+9)^2$	$7(7x+9) + 9$	$(x^2)^2$
$7x^2 + 4$	$49x^2+126x+81$	$49x + 72$	x^4

B. The domain of $g \circ f$ is all real numbers. For all parts because they are parabolas

8) For $f(x) = x^2$ and $g(x) = x^2 + 7$, find the following composite functions and state the domain of each.

(a) $f \circ g$	(b) $g \circ f$	(c) $f \circ f$	(d) $g \circ g$
$(x^2+7)^2$	$(x^2)^2+7$	$(x^2)^2$	$(x^2+7)^2+7$
$x^4+14x^2 +49$	x^4+7	x^4	$x^4+14x^2 +56$

B. The domain of $g \circ f$ is all real numbers. For all parts because they are parabolas

9) $f(x) = \frac{8}{x-3}$ $g(x) = \frac{1}{x}$

(a) $f \circ g$ $= \frac{8}{\left(\frac{1}{x}\right)-3} =$ $\frac{8}{\frac{1-3x}{x}}$ =flip bottom and multiply= $\frac{8x}{1-3x}$

Domain $\{x|x \neq 0, \frac{1}{3}\}$ check both denominators (answer and original)

(b) $g \circ f$ $= \frac{1}{\left(\frac{8}{x-3}\right)} =$ flip bottom and multiply $= \frac{x-3}{8}$

Domain $\{x|x \neq 3\}$ only original has restrictions

(c) $f \circ f$ $= \frac{8}{\left(\frac{8}{x-3}\right)-3} = \frac{8}{\frac{8-3x+9}{x-3}} = \frac{8}{\frac{17-3x}{x-3}}$ =flip bottom and multiply= $\frac{8(x-3)}{17-3x}$

**bring the 8 over then distribute the solo # to the denominator Domain $\{x|x \neq 3, \frac{17}{3}\}$

(d) $g \circ g$ $= \frac{1}{\left(\frac{1}{x}\right)} =$ flip bottom and multiply $= x$

Domain $\{x|x \neq 0$ only original has restrictions

10) $f(x) = \frac{x}{x-4}$ $g(x) = \frac{-7}{x}$

a) Find $f^\circ g(x) = \frac{\left(\frac{-7}{x}\right)}{\left(\frac{-7}{x}\right)-4} = \frac{\frac{-7}{x}}{\frac{-7-4x}{x}}$ =flip bottom and multiply= $\frac{-7}{x} \cdot \frac{x}{-7-4x} = \frac{7}{7+4x}$

Domain $\{x|x \neq 0, -\frac{4}{7}\}$ check both denominators (answer and original)

b) Find $g^\circ f(x) = \frac{-7}{\left(\frac{x}{x-4}\right)} =$ flip bottom and multiply $= \frac{-7x+28}{x}$

Domain $\{x|x \neq 0,4\}$ check both denominators

c) Find $f^\circ f(x) = \frac{\frac{x}{x-4}}{\left(\frac{x}{x-4}\right)-4} = \frac{8}{\frac{x-4x+16}{x-3}} = \frac{\frac{x}{x-4}}{\frac{-3x+16}{x-4}}$ =flip bottom and multiply= $\frac{x}{-3x+16}$

**bring the x over then distribute the solo # to the denominator Domain $\{x|x \neq 4, \frac{16}{3}\}$

a) Find $g^\circ g(x) = \frac{-7}{\left(\frac{-7}{x}\right)} =$ flip bottom and multiply $= x$

Domain $\{x|x \neq 0$ only original has restrictions

11) $f(x) = \sqrt{x}$ $g(x) = 6x+1$

a. Find $f^\circ g(x) = \sqrt{6x+1}$ Domain $\{x|x \geq -\frac{1}{6}\}$

b. Find $g^\circ f(x) = 6\sqrt{x} + 1$ Domain $\{x|x \geq 0\}$

c. Find $f^\circ f(x) = \sqrt[4]{x}$ Domain $\{x|x \geq 0\}$

d. Find $g^\circ g(x) = 6(6x+1)+1 = 36x+7$ Domain all reals

12) $f(x) = x^2 + 6$ $g(x) = \sqrt{x-4}$

a. Find $f^\circ g(x) = (\sqrt{x-4})^2 + 6 = x + 2$ Domain $\{x|x \geq 4\}$

b. Find $g^\circ f(x) = \sqrt{x^2+6-4} = \sqrt{x^2+2}$ Domain all reals

c. Find $f^\circ f(x) = (x^2+6)^2 + 6 = x^4 + 12x^2 + 42$ Domain all reals

d. Find g°g(x) $= \sqrt{\sqrt{x-4}-4}$ Domain {x|x ≥ 20}

**Square 4 then add 4 to get 20

*ALWAYS SQUARE THE # THEN ADD IT TO ITSELF

EX : $\sqrt{\sqrt{x-8}-8} \rightarrow$ 8^2 +8 = 75 THEN Domain {x|x ≥ 72}

13) $f(x) = \frac{x-4}{x+7}$ $g(x) = \frac{x+2}{x-3}$

a) Find f°g(x) $= \frac{\left(\frac{x+2}{x-3}\right)-4}{\left(\frac{x+2}{x-3}\right)+7} = \frac{\frac{x+2-4(x-3)}{x-3}}{\frac{x+2+7(x-3)}{x-3}} = \frac{x+2-4x+12}{x+2+7x-21} = \frac{-3x+14}{8x-19}$

**bring the x+2 over then distribute the solo # to the denominator

Domain $\{x|x \neq 3, \frac{19}{8}\}$ check both denominators (original and answer)

b) Find g°f(x) $= \frac{\left(\frac{x-4}{x+7}\right)+2}{\left(\frac{x-4}{x+7}\right)-3} = \frac{\frac{x-4+2(x+7)}{x+7}}{\frac{x-4-3(x+7)}{x+7}} = \frac{x-4+2x+14}{x+-4-3x-21} = \frac{3x+10}{-2x-25}$

Domain $\{x|x \neq -7, -\frac{25}{2}\}$ check both denominators

c) Find f°f(x) $= \frac{\left(\frac{x-4}{x+7}\right)-4}{\left(\frac{x-4}{x+7}\right)+7} = \frac{\frac{x-4-4(x+7)}{x+7}}{\frac{x-4+7(x+7)}{x+7}} = \frac{x-4-4x-28}{x-4+7x+49} = \frac{-3x-32}{8x+45}$

Domain $\{x|x \neq -7, -\frac{45}{8}\}$ check both denominators

d) Find g°g(x) $= \frac{\left(\frac{x+2}{x-3}\right)+2}{\left(\frac{x+2}{x-3}\right)-3} = \frac{\frac{x+2+2(x-3)}{x-3}}{\frac{x+2-3(x-3)}{x-3}} = \frac{x+2+2x-6}{x+2-3x+9} = \frac{3x-4}{-2x+11}$

Domain $\{x|x \neq 3, \frac{11}{2}\}$ check both denominators

14) If $f(x) = 4x^3 - 4x^2 + 2x - 1$ and $g(x) = 3$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Find f°g(x)

$4(3)^3 - 4(3)^2 + 2(3) - 1 = 77$

Find g°f(x)

3 (has no x to substitute in for)

15) If every horizontal line intersects the graph of a function at no more than one point, f is a one-to-one function.

16) If f is a one-to-one function and $f(5) = 2$, then $f^{-1}(2) =$ __5____

17) If f^{-1} denotes the inverse of a function f, then the graphs of f and f^{-1} are symmetric with respect to the line _$y = x$_.

18) If the domain of a one-to-one function f is $[8, \infty)$, the range of its inverse, f^{-1}, is $[8, \infty)$.

19) If $(-1, 3)$ is a point on the graph of a one-to-one function f, which of the following points is on the graph of f^{-1}?

Choose the correct answer below.

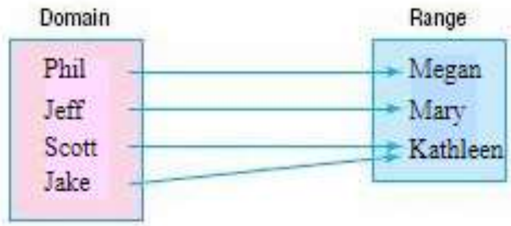
- ○ $(1, -3)$
- ○ $(-3, 1)$
- ✔ $(3, -1)$
- ○ $(-1, -3)$

20) Suppose f is a one-to-one function with a domain of $\{x | x \neq 4\}$ and a range $\left\{y \middle| y \neq \frac{3}{4}\right\}$. Which of the following is the domain of f^{-1}?

Choose the correct answer below.

- ○ $\left\{x \middle| x \neq 4, x \neq \frac{3}{4}\right\}$
- ✔ $\left\{x \middle| x \neq \frac{3}{4}\right\}$
- ○ $\{x | x \neq 4\}$
- ○ all real numbers

21) For the function on the right, determine whether the function is one-to-one.



Is the function one-to-one?

X can't to the same y

- Yes
- ✔ No

22) *With ordered pairs, one-to-one function can't have duplicating y values:*

For the following function, determine whether the function is one-to-one.

$\{(4,6), (3,9), (-8,14), (1,-8)\}$

Is the function one-to-one?

- No
- ✔ Yes

23) For the following function, determine whether the function is one-to-one.

$\{(4,6), (3,6), (-8,3), (6,-5)\}$

Is the function one-to-one?

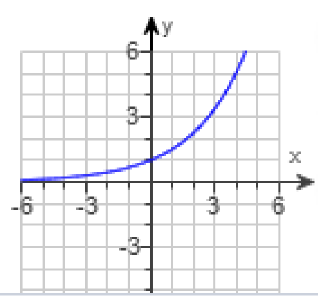
- ✔ No
- Yes

24) The graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.

Is f one-to-one?

Vertical line test

- ✔ Yes
- No

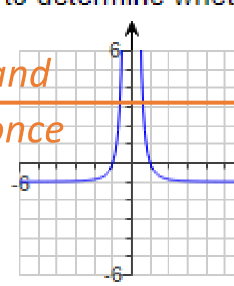


25) The graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.

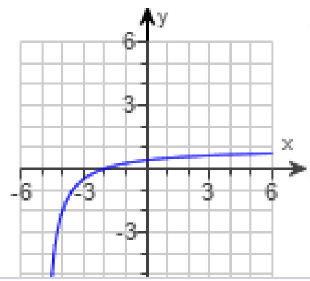
Is f one-to-one?

I can draw a horizontal line and touch the graph more than once

- Yes
- ✔ No



26) The graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.



Is f one-to-one?

Yes

27) $f(x) = -8x - 8$ $g(x) = -\frac{1}{8}(x + 8)$

a) Find f(g(x)) $-8\left(-\frac{1}{8}(x + 8)\right) - 8$

$-8\left(-\frac{1}{8}x - 1\right) - 8$

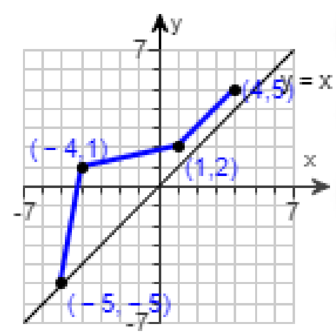
$x + 8 - 8$ = x

b) Find g(f(x)) $-\frac{1}{8}((-8x - 8) + 8)$

$-\frac{1}{8}(-8x)$ = x

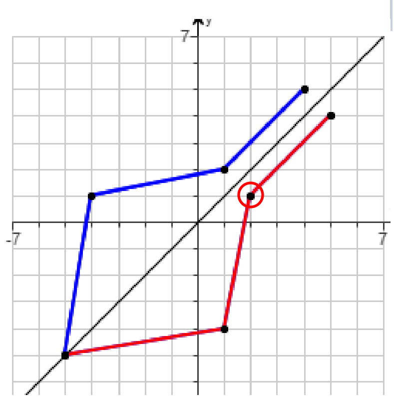
If both equal x then they are inverses of each other; therefore, YES

28) The graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1}. For convenience (and as a hint), the graph of y = x is also given.

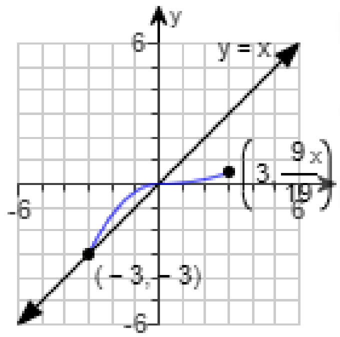


Pick a point and switch the x and y

(1,2) to (2,1)

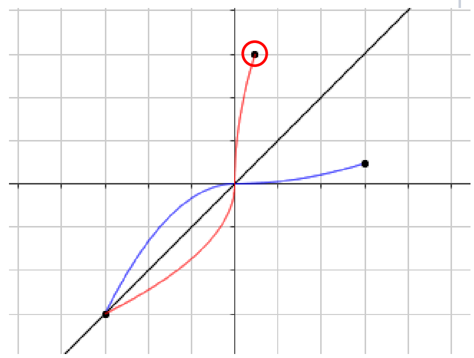


29) The graph of a one-to-one function is shown to the right. Draw the graph of the inverse function f^{-1}.

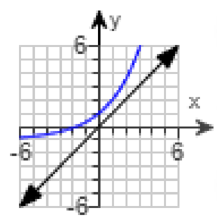


Pick a point and switch the x and y

$(3,\frac{9}{19})$ to $(\frac{9}{19},3)$

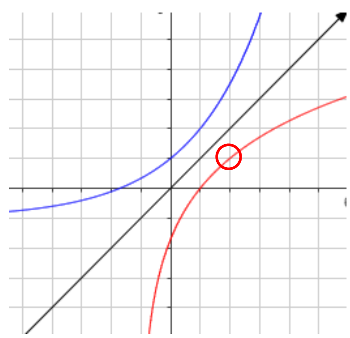


30) The graph of a one-to-one function is shown to the right. Draw the graph of the inverse function f^{-1}.



Pick a point and switch the x and y

(1,2) to (2,1)



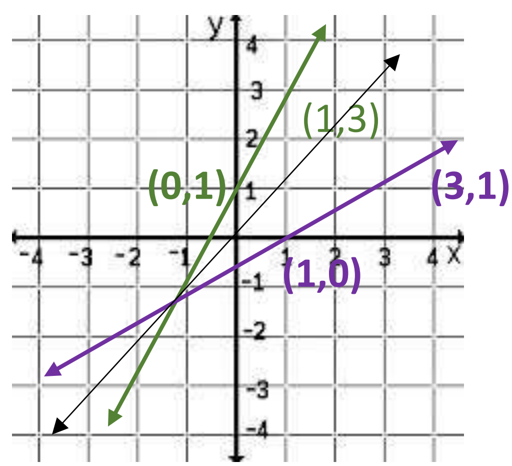
31) Find the inverse of f(x) =2x+1

$x = 2y + 1$

$\frac{x-1}{2} = y'$ $\quad$ $f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$

Domain and Range are all reals

Can't plot fractions on the inverse function find points on the original line and switch the coordinates for the inverse line.



Don't forget to plot the y = x line plot (0,0) and (1,1) to get the line

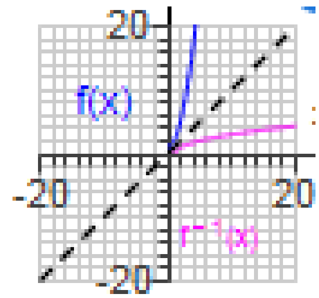
32) The function $f(x) = x^2 + 1$, $x \geq 0$ is one-to-one.
(a) Find the inverse of f and check the answer.
(b) Find the domain and the range of f and f^{-1}.
(c) Graph f, f^{-1}, and y = x on the same coordinate axes.

$X = y^2 + 1$

$x - 1 = y^2$

$\sqrt{x-1} = y'$ $\quad$ ***$f^{-1}(x) = \sqrt{x-1}$***

CHOOSE THE GRAPH



Domain of f is $x \geq 0$

Range of f is $y \geq 1$

Domain of f is $x \geq 1$

Range of f is $y \geq 0$ $\quad$ **the domain and range switch*

33) Find the inverse of $f(x) = \frac{3x}{x+5}$

Switch x and y then solve for y. $\quad x = \frac{3y}{y+5} \qquad x(y+5) = 3y$

$$xy + 5x = 3y$$
$$xy - 3y = -5x$$

**factor out the y* $\quad y(x-3) = -5x$

$$f^{-1} = \frac{-5x}{x-3}$$

Domain of f is {x|x≠-5} $\quad$ *Range of f is {y|y≠3}*

Domain and Range of function and its inverse are opposite, THEREFORE…

*Domain of **f^{-1}** is {x|x≠3}* $\quad$ *Range of **f^{-1}** is {y|y≠-5}*

34) The domain of a one-to-one function f is $[5,\infty)$, and its range is $[-9,\infty)$. State the domain and the range of f^{-1}.

What is the domain of f^{-1}?

The domain of f^{-1} is $[-9,\infty)$.
(Type your answer in interval notation.)

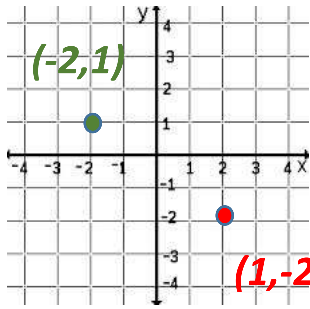
Switch domain and range

What is the range of f^{-1}?

The range of f^{-1} is $[5,\infty)$.
(Type your answer in interval notation.)

35) If f is in quadrant II, what quadrant is f^{-1} in?

Put a point in quadrant II then switch x and y and see what quadrant the inverse coordinate is in.



(1,-2) ***Quadrant IV***

****Quadrants I and III inverse coordinates do not move!***

Ex) ***Find the inverse of*** $\boldsymbol{f(x) = \frac{7x+9}{4x-7}}$

Switch x and y then solve for y. $x = \frac{7y+9}{4y-7}$ $x(4y-7) = 7y+9$

$$4xy - 7x = 7y + 9$$

$$4xy - 7y = 7x + 9$$

**factor out the y* $y(4x-7) = 7x+9$

$$f^{-1} = \frac{7x+9}{4x-7}$$

Domain of f is $\{x \mid x \neq \frac{7}{4}\}$ *Range of f is* $\{y \mid y \neq \frac{7}{4}\}$

Domain and Range of function and its inverse are opposite, THEREFORE…

Domain of $\boldsymbol{f^{-1}}$ *is* $\{x \mid x \neq \frac{7}{4}\}$ *Range of* $\boldsymbol{f^{-1}}$ *is* $\{y \mid y \neq \frac{7}{4}\}$