

6.3 Exponential Function

MATH 161

THOMPSON

- 1) Evaluate the exponential expression.

$$(-4)^2 \quad \text{Make sure to put } (-4)^2 \text{ is calculator} = 16$$

- 2) Simplify the given expression.

$$9^{-2} = \frac{1}{81} \quad \text{calculator: } 9^{-2} \text{ then } 2^{\text{nd}} \text{ PRB}$$

- 3) Simplify the given expression.

$$7^{-4} \cdot 7^3 \quad 7^{-4} \text{ times } 7^3 \text{ } 2^{\text{nd}} \text{ PRB}$$

$$7^{-4} \cdot 7^3 = \frac{1}{7} \quad (\text{Type an integer or a simplified fraction.})$$

- 4) Simplify the expression.

$$(4^{-1})^{-3}$$

$$(4^{-1})^{-3} = 64 \quad (\text{Type an integer or a simplified fraction.})$$

- 5) Simplify the given expression.

$$\sqrt{49}$$

$$\sqrt{49} = 7$$

- 6) Simplify the given expression.

$$\sqrt{(-8)^2}$$

$$\sqrt{(-8)^2} = 8$$

- 7) Find the value of the given expression if $x = 7$ and $y = -3$.

$$\frac{5xy^{-1}}{5xy^{-1}} = \frac{5(7)}{-3} \text{ negative exponent makes a fraction}$$
$$5xy^{-1} = -\frac{35}{3} \text{ (Simplify your answer.)}$$

- 8) Find the value of the expression if $x = 3$ and $y = -4$.

$$\frac{x^2 + y^2}{x^2 + y^2} = \frac{(3)^2 + (-4)^2}{x^2 + y^2}$$
$$x^2 + y^2 = 25 \text{ (Simplify your answer.)}$$

- 9) Find the value of the expression if $x = 4$ and $y = -1$.

$$\frac{(xy)^2}{(xy)^2} = \frac{(4 \cdot -1)^2}{(xy)^2}$$
$$(xy)^2 = 16 \text{ (Simplify your answer.)}$$

- 10) Find the value of the expression if $x = 3$.

$$\frac{\sqrt{x^2}}{\sqrt{x^2}} = \frac{\sqrt{(-3)^2}}{\sqrt{x^2}}$$
$$\sqrt{x^2} = 3 \text{ (Simplify your answer.)}$$

- 11) Find the value of the expression if $x = 6$ and $y = -7$.

$$\frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}} = \frac{\sqrt{6^2 + (-7)^2}}{\sqrt{x^2 + y^2}}$$
$$\sqrt{x^2 + y^2} = \sqrt{85}$$

- 12) Find the value of the expression if $x = 8$ and $y = -1$.

$$x^y$$

Negative exponent makes a fraction

$$x^y = \frac{1}{8} \text{ (Simplify your answer.)}$$

- 13) $4^2 = 16$ (Type an integer or a simplified fraction.)

$$8^{\frac{2}{3}} = 4 \text{ (Type an integer or a simplified fraction.)}$$

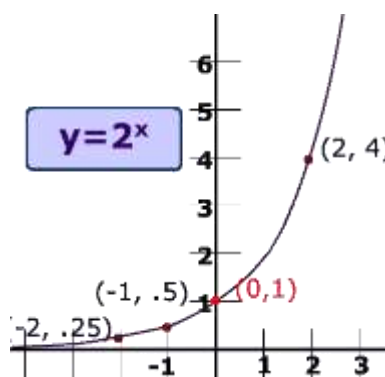
$$3^{-3} = \frac{1}{27} \text{ (Type an integer or a simplified fraction.)}$$

The exponential function $f(x) = a^x$, $a > 0$, $a \neq 1$, has no x-intercept because it is above the x-axis.

Exponential Graph →

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$



Decide whether the following statement is true or false.

14)

The domain of the exponential function $f(x) = a^x$, $a > 0$, $a \neq 1$, is the set of all real numbers.

Choose the correct answer below.

- ☐ False
☒ True

15)

Decide whether the following statement is true or false.

The graph of the exponential function $f(x) = a^x$, $a > 0$, $a \neq 1$, has no x-intercept.

Choose the correct answer below.

- ☒ True
☐ False

16) Which of the following is the range of the exponential function $f(x) = a^x$, $a > 0$ and $a \neq 1$?

Choose the correct answer below.

- ☐ $(-\infty, \infty)$
☒ $(0, \infty)$
☐ $(-\infty, 0)$
☐ $(-\infty, 0) \cup (0, \infty)$

17) Determine the function is linear, exponential, or neither?

To check linear: check slope on two set of points

$$\frac{1-11}{0-1} = \frac{-10}{-1} = 10$$

$$\frac{11-121}{1-2} = \frac{-110}{-1} = 110 \text{ no}$$

x	f(x)
-1	1/11
0	1
1	11
2	121
3	1331

To check exponential:

$$\frac{f(x+1)}{f(x)} = a \text{ where } f(x) = Na^x \text{ and } f(x) = N$$

$$\frac{f(0)}{f(-1)} = \frac{1}{1/11} = 11$$

$$\frac{f(1)}{f(0)} = \frac{11}{1} = 11 \text{ yes, therefore: } f(0) = 1 \text{ then } f(x) = 11^x$$

*HINT when $x = -1$ and y is a fraction and $x = 0$ is always 1 then check $x = 1$ for $y =$ the same number as bottom of the first fraction.

18) Determine the function is linear, exponential, or neither?

To check linear: check slope on two set of points

$$\frac{13-156}{0-1} = \frac{-143}{-1} = 143$$

x	f(x)
-1	13/12
0	13
1	156
2	1872
3	22464

$$\frac{156-1872}{1-2} = \frac{-1716}{-1} = 1716 \text{ no}$$

To check exponential:

$$\frac{f(x+1)}{f(x)} = a \text{ where } f(x) = Na^x \text{ and } f(x) = N$$

$$\frac{f(0)}{f(-1)} = \frac{13}{13/12} = 12$$

$$\frac{f(1)}{f(0)} = \frac{165}{13} = 12 \text{ yes, therefore: } f(0) = 13 \text{ then } f(x) = 13 \cdot 12^x$$

*HINT when $x = -1$ and y is a fraction and $x = 0$ is always 1 then check $x = 1$ for $y =$ the same number as bottom of the first fraction. The top number if not 1 goes in front of the exponential function. $y = N/a$

- 19) Determine whether the function given by the table is linear, exponential, or neither. If the function is linear, find a linear function that models the data; if it is exponential, find an exponential function that models the data.

x	f(x)
-1	2
0	5
1	8
2	11
3	14

Select the correct choice below and fill in any answer boxes within your choice.

- ☒ A. The function is linear. A linear function that models the data is $f(x) = 3x + 5$.
(Simplify your answer.)

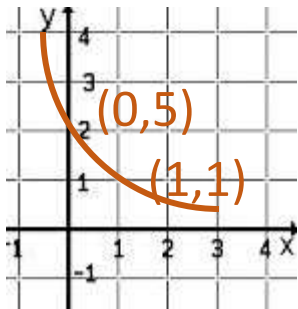
To check linear: check slope on two set of points

$$\frac{5-2}{0-(-1)} = 3$$

$$\frac{8-5}{1-0} = 3 \text{ linear}$$

$$5 = 3(0) + b \text{ then } b = 5$$

20) Match the graph to a function:



A. $y = 5^x$

B. $y = 5^{-x}$

C. $y = -5^x$

D. $y = -5^{-x}$

E. $y = 5^x - 1$

F. $y = 5^{-x} - 1$

G. $y = 5^{1-x}$

H. $y = 1 - 5^{-x}$

*Reflects across the y-axis and shifts right 1

Plug in $x=0$ then $y = 5$ and $x = 1$ then $y = 1$ G

21) Match the graph to one of the following functions.

A. $y = 7^x$

B. $y = 7^{-x}$

C. $y = -7^x$

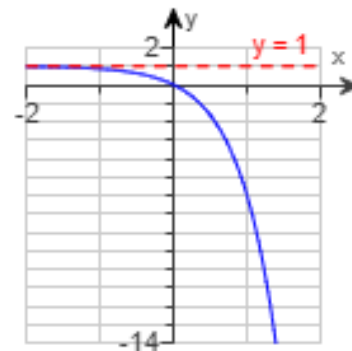
D. $y = -7^{-x}$

E. $y = 7^x - 1$

F. $y = 7^{x-1}$

G. $y = 7^{1-x}$

H. $y = 1 - 7^x$



*Reflects across the x-axis and shifts up 1

Plug in $x=0$ then $y = 0$ and $x = 1$ then $y = -6$ H

22) Match the graph to one of the following functions.

A. $y = 7^x$

B. $y = 7^{-x}$

C. $y = -7^x$

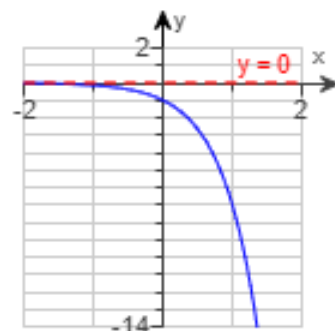
D. $y = -7^{-x}$

E. $y = 7^x - 1$

F. $y = 7^{x-1}$

G. $y = 7^{1-x}$

H. $y = 1 - 7^x$

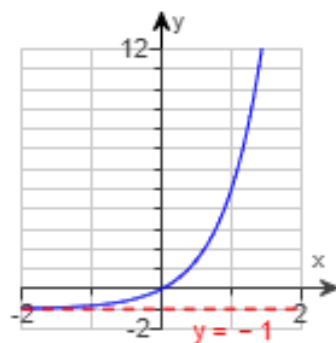


*Reflects across the x-axis

Plug in $x=0$ then $y = -1$ and $x = 1$ then $y = -7$ C

23) Match the graph to one of the following functions.

- | | |
|------------------|------------------|
| A. $y = 6^x$ | B. $y = 6^{-x}$ |
| C. $y = -6^x$ | D. $y = -6^{-x}$ |
| E. $y = 6^x - 1$ | F. $y = 6^{x-1}$ |
| G. $y = 6^{1-x}$ | H. $y = 1 - 6^x$ |

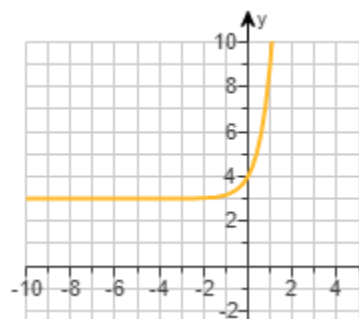
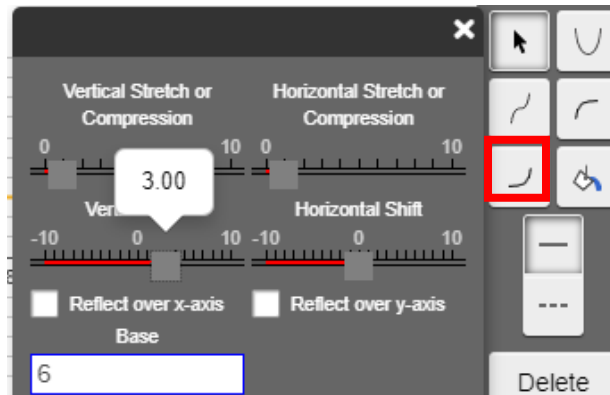


*Shifts down 1

Plug in $x=0$ then $y = 0$ and $x = 1$ then $y = 5$ E

24) Use transformations to graph the function.
Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$f(x) = 6^x + 3$ shifts up 3



What is the domain of $f(x) = 6^x + 3$?

$(-\infty, \infty)$
(Type your answer in interval notation.)

What is the range of $f(x) = 6^x + 3$?

$(3, \infty)$
(Type your answer in interval notation.)

Vertical shift up 3

The function has one horizontal asymptote, $y = 3$. Up 3
(Type an equation.)

What is the y-intercept of $f(x) = 6^x + 3$?

4 (Type an integer or a simplified fraction.)

Plug 0 in for x

- 25) Use transformations to graph the function. Determine its domain, range, and horizontal asymptote.

$$f(x) = 5^x - 4 \quad \text{Down 4}$$

What is the domain of $f(x) = 5^x - 4$?

$$(-\infty, \infty)$$

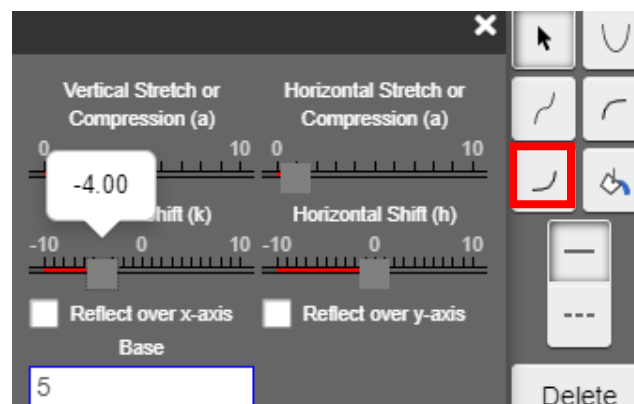
(Type your answer in interval notation.)

What is the range of $f(x) = 5^x - 4$?

$$(-4, \infty)$$

(Type your answer in interval notation.)

Vertical down up 4



What line is the horizontal asymptote of $f(x) = 5^x - 4$?

Down 4

$$y = -4 \quad \text{(Type an equation.)}$$

- 26) Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$$f(x) = 5^{x-1} \quad \text{Right 1}$$

What is the domain of $f(x) = 5^{x-1}$?

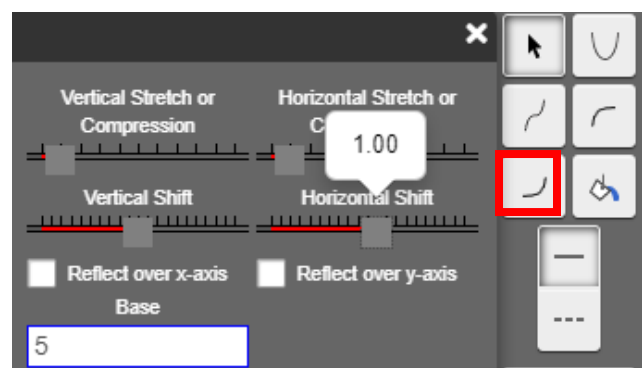
$$(-\infty, \infty)$$

(Type your answer in interval notation.)

What is the range of $f(x) = 5^{x-1}$?

$$(0, \infty)$$

(Type your answer in interval notation.)



The function has one horizontal asymptote, $y = 0$.

What is the y-intercept of $f(x) = 5^{x-1}$?

$$\frac{1}{5} \quad \text{Plug 0 in for x } 5^{-1}$$

- 27) Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$$f(x) = 6 \cdot \left(\frac{1}{4}\right)^x$$

Vertical stretch of 6

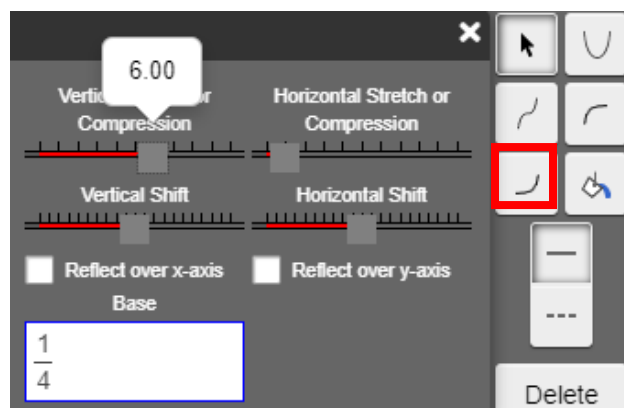
What is the domain of $f(x) = 6 \cdot \left(\frac{1}{4}\right)^x$?

$(-\infty, \infty)$

(Type your answer in interval notation.)

What is the range of $f(x) = 6 \cdot \left(\frac{1}{4}\right)^x$?

$(0, \infty)$



The function has one horizontal asymptote, $y = 0$.

What is the y-intercept of $f(x) = 6 \cdot \left(\frac{1}{4}\right)^x$?

6 (Type an integer or a simplified fraction.)

Plug 0 in for x

- 28) Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$$f(x) = 3^{-x} - 2$$

Reflects across y down 2

What is the domain of $f(x) = 3^{-x} - 2$?

$(-\infty, \infty)$

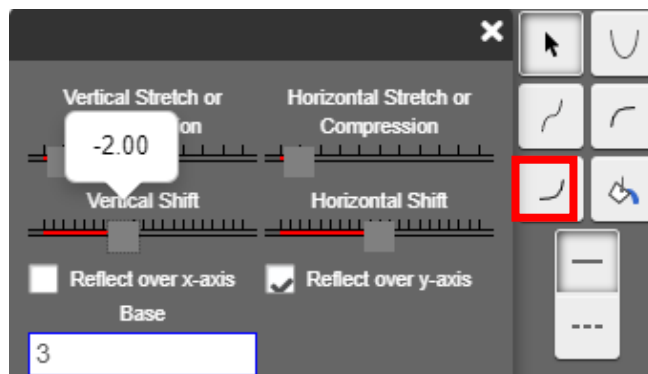
(Type your answer in interval notation.)

What is the range of $f(x) = 3^{-x} - 2$?

$(-2, \infty)$

(Type your answer in interval notation.)

Vertical down up 2



The function has one horizontal asymptote, $y = -2$.
(Type an equation.) Down 2

What is the y-intercept of $f(x) = 3^{-x} - 2$?

-1 (Type an integer or a simplified fraction.) Plug 0 in for x

- 29) Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$$f(x) = 1 + 3^{x-2} \rightarrow f(x) = 3^{x-2} + 1$$

Right 2, up 1

What is the domain of $f(x) = 1 + 3^{x-2}$?

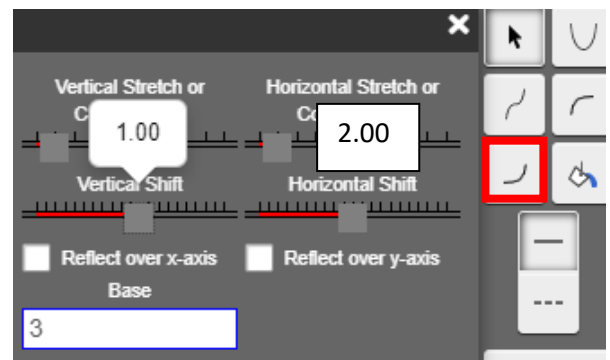
$(-\infty, \infty)$

(Type your answer in interval notation.)

What is the range of $f(x) = 1 + 3^{x-2}$?

$(1, \infty)$

Vertical shift up 1



The function has one horizontal asymptote, $y = 1$. Up 1
(Type an equation.)

What is the y-intercept of $f(x) = 1 + 3^{x-2}$?

$\frac{10}{9}$ (Type an integer or a simplified fraction.) $1 + 3^{-2} = 1 + \frac{1}{9}$

- 30) Begin with the graph of $y = e^x$ and use transformations to graph the following function. Determine the domain, range, and horizontal asymptote of the function.

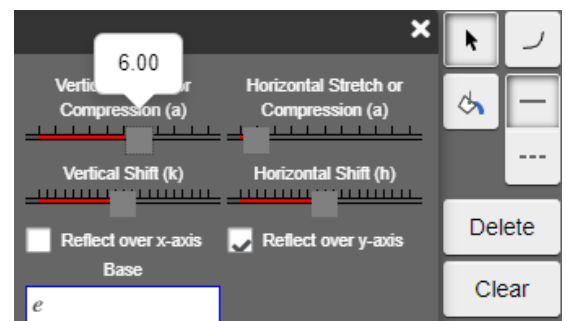
$$f(x) = 6e^{-x}$$

Reflects across the y
vertical stretch of 6

The domain of $f(x) = 6e^{-x}$ is $(-\infty, \infty)$.
(Type your answer in interval notation.)

The range of $f(x) = 6e^{-x}$ is $(0, \infty)$.
(Type your answer in interval notation.)

The horizontal asymptote of $f(x) = 6e^{-x}$ is
 $y = 0$.



31) Solve $4^{-x} = 256$ get the same base

$$4^{-x} = 4^4$$

set exponents equal and solve $-x = 4$ $x = -4$

32) Solve $\left(\frac{3}{5}\right)^x = \frac{27}{125}$ get the same base

$$\left(\frac{3}{5}\right)^x = \left(\frac{3}{5}\right)^3$$

set exponents equal $x = 3$

33) Solve $5^{4x+1} = 25$ get the same base

$$5^{4x+1} = 5^2$$

set exponents equal and solve $4x + 1 = 2$

$$4x = 1 \quad x = \frac{1}{4}$$

34) Solve $8^{x^2-5} = 64^{2x}$ get the same base

$$8^{x^2-5} = 8^{4x}$$

set exponents equal and solve $x^2 - 5 = 4x$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) \quad x = -1, 5$$

35) Solve $49^x \cdot 7^{x^2} = 2401^2$ get the same base

$$7^{2x} \cdot 7^{x^2} = 7^8$$

set exponents equal and solve $2x + x^2 = 8$

$$x^2 + 2x - 8 = 0$$

$$(x-2)(x+4) \quad x = -4, 2$$

36) Solve $e^x = e^{3x+18}$

set exponents equal and solve $x = 3x + 18$

$$-2x = 18 \quad x = -9$$

37) Solve $e^{x^2} = e^{8x} \cdot \frac{1}{e^{15}}$ get the same base

$$e^{x^2} = e^{8x} \cdot e^{-15}$$

set exponents equal and solve $x^2 = 8x - 15$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) \quad x = 3, 5$$

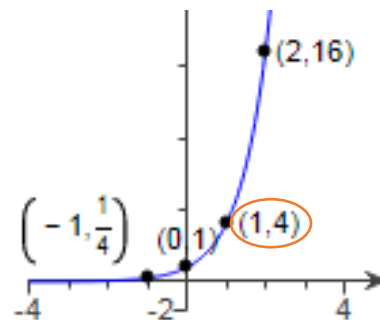
38) If $6^x = 7$ then $6^{-2x} = ?$

*The -2 means to make it a fraction and square it therefore, $x = \frac{1}{49}$

39) Give the function of the graph:

$$f(x) = 4^x$$

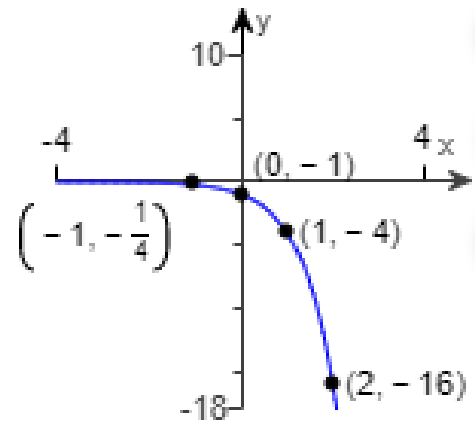
If I plug 1 in for x, $y = 4$



40) Determine the exponential function whose graph is given.

If $x = 0$ and $y = -1$
means negative in front

If $x = 1$ and $y = 4$
means 4 is the root



41) Suppose that $g(x) = 2^x + 2$

a) What is $g(-1)$? When $x = -1$, what is the point on the graph?

$$2^{-1} + 2 = \frac{5}{2} \quad (-1, \frac{5}{2})$$

b) If $g(x) = 10$, what is x ? Give point on the graph.

$$2^x + 2 = 10$$

$$2^x = 8 \quad 2^x = 2^3 \quad x = 3 \quad (3, 10)$$

42) If a single pane of glass obliterates 9% of the light passing through it, then the percent p of the light that passes through n successive panes is given approximately by the following function:

$$p(h) = 100(0.91)^n$$

(a) What percent of light will pass through 40 panes?

$$p(5) = 2\% \quad 100(0.99)^5$$

(b) What percent of light will pass through 50 panes?

$$p(10) = 1\% \quad 100(0.99)^{50}$$

43) The percentage of patients P who have survived t years after initial diagnosis of a certain disease is modeled by the function:
 $P(t) = 100(0.5)^t$

- (a) According to the model, **50%** of the patients survive 1 year after initial diagnosis. *Plug 1 in for t*
- (b) According to the model, **12.5%** of the patients survive 3 year after initial diagnosis. *Plug 3 in for t*
- (c) As each year passes **50%** of the previous year's survivors have survived.

44) $D(h) = 9e^{-0.55h}$ can be used to find the number of milligrams D of a drug that is in a patient's bloodstream h hours after the drug has been administered. How many milligrams will be present after 1 hour? After 11 hours?

$$9e^{-0.55(1)} = \mathbf{5.19}$$

$$9e^{-0.55(11)} = \mathbf{0.021}$$