6.6 Logarithmic and Exponential Equations MATH 161 THOMPSON

1.) $log_5 x = 2$ $5^2 = x$ x = 25

2.) $log_3(11x) = 3$ $11x = 3^3$ x = $\frac{27}{11}$

3.) $log_8(x + 6) = log_8 14$ **when bases equal they drop* x+6=14 x = 8

4.) $\frac{1}{3}log_2 x = 2log_2 3$ $\sqrt[3]{x} = 3^2$ $x = 9^3$ x = 729

5.) $5log_2 x = -log_2 32$ $x^5 = \frac{1}{32}$ $x = \sqrt[5]{\frac{1}{32}}$ x = $\frac{1}{2}$

exponent of (x-9)

6.) $2log_2(x-9) + log_2 2 = 5$ Condense $log_2 2(x-9)^2 = 5$ $2^5 = 32$

$2(x^2 - 18x + 81) = 32$ divide 32/2

$x^2 - 18x + 81 = 16$

$x^2 - 18x + 65 = 0$

(x-5)(x-13)=0 x = ~~5~~,**13** (x-9) has to be positive

7.) logx + log(x-15) = 2 *condense the logs (multiply since +)*

$logx(x-15) = 2$ *change to exponential form*

$x^2 - 15x - 100$

$(x-20)(x+5) = 0$ x = ~~-5~~,**20** (x-15) has to be positive

8a.) $\log(4x+4) = 1 + \log(x-9)$ $\log(4x+4) - \log(x-9) = 1$

$\frac{4x+4}{x-9} = 10^1$ cross multiply

10x-90=4x+4

x = $\frac{47}{3}$

8b.) $\log(6x+5) = 1 + \log(x-5)$

$\log(6x+5) - \log(x-5) = 1$

condense: $log\frac{6x+5}{x-5} = 1$ $\rightarrow \frac{6x+5}{x-5}=10 \rightarrow$ 10(x-5)=6x+5 $\rightarrow$10x-50=6x+5

4x = 55 $x = \frac{55}{4}$

8c.) log(3x + 6) = 1 + log(x – 7) $\rightarrow$ log (3x+6)-log(x-7)=1

condense: $log_{10}\frac{3x+6}{x-7} = 1$ $\rightarrow \frac{3x+6}{x-7} = 10 \rightarrow$ 10(x-7) = 3x+6

10x – 70 = 3x + 6 $x = \frac{76}{7}$

9.) $log_3(x+10) + log_3(x+4) = 3$ $log_3(x+10)(x+4) = 3$

(x+10)(x+4) = 27

x^2 +14x + 40 = 27

x^2 +14x + 13 = 0

(x+13)(x+1)=0 x = **-1**,~~-13~~ (-1+11) is positive

10.) $\log_2(x-5) = 3 - \log_2(x-3)$ $\rightarrow$ $\log_2(x-5)+\log_2(x-3)=3$

condense: $log_2(x-5)(x-3) = 3$ $\rightarrow$ $x^2 - 8x + 15 = 8$

$x^2 - 8x + 7 = 0$

(x-7)(x-1) x=7,1 *can't have negative* x = 7

11.) $\log_{1/7}(x^2+x) - \log_{1/7}(x^2 - x) = -1 \rightarrow$ $log_{1/7}\frac{x^2+x}{x^2-x} = -1 \rightarrow \frac{x(x+1)}{x(x-1)} = 7$

cross multiply after cancelling xs 7x-7 = x+1

6x = 8 $x = \frac{4}{3}$

12a.) $\log_a(x - 2) - \log_a(x + 7) = \log_a(x - 3) - \log_a(x + 4)$

condense: $log_a \frac{(x-2)}{(x+7)} = log_a \frac{(x-3)}{(x+4)}$ $\rightarrow$ $\frac{(x-2)}{(x+7)} = \frac{(x-3)}{(x+4)}$

$(x-2)(x+4) = (x+7)(x-3)$

$x^2 + 2x - 8 = x^2 - 4x - 21$

$-2x = -13$ $x = \frac{13}{2}$

12b.) $\log_a(x - 1) - \log_a(x + 2) = \log_a(x - 4) - \log_a(x + 2)$

condense: $log_a \frac{(x-1)}{(x+2)} = log_a \frac{(x-4)}{(x+2)}$ $\rightarrow$ $\frac{(x-1)}{(x+2)} = \frac{(x-4)}{(x+2)}$

$(x-1)(x+2) = (x+2)(x-4)$

$x^2 + x - 2 = x^2 - 2x - 8$

$3x = -6$ $x = -2$ *but log can't be negative*

NO SOLUTION

13.) $(log_2 x)^2 - 5(log_2 x) = 6$

Use u substitution: $u^2 - 5u - 6 = 0$ $u = log_2 x$

$(x-6)(x+1) = 0$

$x = 6, -1$ therefore: $log_2 x = 6$ $log_2 x = -1$

$2^6 = 64$ $2^{-1} = \frac{1}{2}$

14.) $7^{x-3} = 49 \rightarrow$ same base on both sides

$7^{x-3} = 7^2$ $x-3 = 2$ $x = 5$

15.) $8^x = 13 \rightarrow$ add $\log_8$ to both sides

$log_8 8^x = log_8 13$ $x = log_8 13$ ** base on bottom*

$x = \frac{log 13}{log 8}$ first answer $= 1.233$ **round to 3 decimal places*

You can always use ln..... $\frac{ln 13}{ln 8} = 1.233$

16.) $4^{-x} = 3.6 \rightarrow \quad -log_4 3.6$

You can always use ln..... $\frac{ln3.6}{ln4} = -0.924$

$x = -\frac{log3.6}{log4}$ first answer ≈ -0.924 **round to 3 decimal places*

17.) $5(3^{9x}) = 8 \rightarrow$

$3^{9x} = \frac{8}{5}$ *change to log form* $\log_3 (\frac{8}{5}) = 9x$ *change base formula*

$9x = \frac{\ln(\frac{8}{5})}{ln3} \rightarrow x = \frac{\ln(\frac{8}{5})}{(9\ln(3))} \approx 0.048$ **use parentheses in calculator*

divide by 9 – move it to the bottom

18.) $2^{1-5x} = 5^x \rightarrow$ TAKE *ln* OF BOTH SIDE

$= ln2^{1-5x} = ln5^x$ *exponent moves out in front then distribute*

(1-5x)ln2 = xln5

ln2 = xln5+5xln2 *move x to the right to keep it positive*

ln2= x ln5 +5xln2

ln2=x(ln5+5ln2) = *factor out the x*

$x = \frac{ln2}{ln5+5ln2}$ x≈ 0.137

19.) $\pi^{1-5x} = e^{2x} \rightarrow$ TAKE *ln* OF BOTH SIDE

$= ln\pi^{1-5x} = e^{2x}$ *exponent moves out in front then distribute*

(1-5x)lnπ = 2x

lnπ – 5xlnπ = 2x *move x to the right to keep it positive*

lnπ = 5xlnπ + 2x

lnπ = x(5lnπ+2x) *factor out the x*

$x = \frac{ln\pi}{5ln\pi+2}$ x≈ 0.148

20.) $2^{2x} + 2^x - 56=0$ → let $u = 2^x$

$u^2 + u - 56 = 0$

$(u + 8)(u - 7) = 0$ $u = -8,7$

$2^x = -8$
no solution

$2^x = 7$

$x = \log_2 7$ *base on bottom*

$x= \frac{\log 7}{\log 2} \approx 2.807$

You can always use ln..... $\frac{\ln 7}{\ln 2} = 2.807$

21.) $3^{2x} + 3^{x+1} - 54=0$ → let $u = 3^x$

$u^2 + 3u - 54 = 0$

$(u + 9)(u - 6) = 0$ $u = -9,6$

$3^x = -9$
no solution

$3^x = 6$

$x = \log_3 6$ *base on bottom*

$x= \frac{\log 6}{\log 3} \approx 1.631$

You can always use ln..... $\frac{\ln 6}{\ln 3} = 1.631$

22.) $4^x + 2^{x+1} - 15=0$ → let $u = 2^x$

$u^2 + 2u - 15 = 0$

$(u + 5)(u - 3) = 0$ $u = -5,3$

$2^x = -5$
no solution

$2^x = 3$

$x = \log_2 3$ *base on bottom*

$x= \frac{\log 3}{\log 2} \approx 1.585$

You can always use ln..... $\frac{\ln 3}{\ln 2} = 1.585$

23.) $36^x - 8 \cdot 6^x = -16$ → let $u = 6^x$

$u^2 - 8u - 16 = 0$

$(u - 4)(u - 4) = 0$ $u = 4$ $6^x = 4$

$x = \log_6 4$ *base on bottom*

$x= \frac{\log 4}{\log 6} \approx 0.774$

You can always use ln..... $\frac{\ln 4}{\ln 6} = 0.774$

24.) $4^x - 10 \cdot 4^{-x} = 3$ multiply all by 4^x **cancels the middle one*

$(4^x)^2 - 10 \cdot 1 = 3 \cdot 4^x$ → let $u = 4^x$

$u^2 - 3u - 10 = 0$

$(u - 5)(u + 2) = 0$ $u = -2$

$4^x = 5$

$x = \log_4 5$ *base on bottom*

$x= \frac{\log 5}{\log 4} \approx 1.161$

You can always use ln..... $\frac{\ln 5}{\ln 4} = 1.161$

25.) $(\sqrt[5]{7})^{3-2x} = 7^{x^2}$ *get the base the same to cancel*

$7^{\frac{1}{5}(3-2x)} = 7^{x^2}$ *set exponents equal and multiply the right side by 5*

$3-2x = 5x^2$

$5x^2+2x-3=0$ *slide and divide* $x^2+2x-15=0$

$(x+5)(x-3)=0$ $x = -1, \frac{3}{5}$

26.) The population of a certain country in 1998 was 281 million people. In addition, the population of the country was growing at a rate of 0.8% per year. Assuming that this growth rate continues, the model $P(t) = 281(1.008)^{t-1998}$ represents the population P (in millions of people) in year t.
According to this model, when will the population of the country reach 391 million people?

$391 = 281(1.008)^{t-1998}$

$\frac{391}{281} = (1.008)^{t-1998}$ *take ln of both sides

$\ln\left(\frac{391}{281}\right) = (t-1998)\ln 1.008$ *divide by (ln1.008) first then add 1998

$\frac{\left(\ln\left(\frac{391}{281}\right)\right)}{\ln 1.008} + 1998 = t$ $t = 2039$

Ex.) $\ln x + \ln(x+2) = 4$ $\ln x + \ln(x+2) = 4$

$\ln(x)(x+2) = 4$

$x^2 + 2x = e^4$

$x^2 + 2x - e^4 = 0$

$\frac{-2\pm\sqrt{4-4(1)(-e^4)}}{2} =$ *factor out the 4 under radical*

$\frac{-2\pm\sqrt{4(1+e^4)}}{2}$ *take square root of the 4*

$\frac{-2\pm2\sqrt{1+e^4}}{2} = -1 \pm \sqrt{1+e^4}$

Ex.) $ln x + ln(x+4) = 4$ $\qquad ln x + ln(x+4) = 4$

ln(x)(x+4) = 4

$x^2 + 4x = e^4$

$x^2 + 4x - e^4 = 0$

$\frac{-4 \pm \sqrt{16 - 4(1)(-e^4)}}{2} =$ *factor out the 4 under radical*

$\frac{-4 \pm \sqrt{4(4+e^4)}}{2}$ *take square root of the 4*

$\frac{-4 \pm 2\sqrt{4+e^4}}{2} = -2 \pm \sqrt{4+e^4}$