6.8 Exponential Growth and Decay Application MATH 161 THOMPSON

1) The size of P of a certain insect population at time t (in days) obeys the function $P(t) = 300e^{0.06t}$

a) Determine the number of insects at t = 0 days. $300e^{0.06(0)} \approx$ **300**

b) What is the growth rate of the insect population? 6%

c) What is the population after 10 days? $300e^{0.06(10)} \approx$ **547 insects**

d) When will the insect population reach 510? $510 = 300e^{0.06(t)}$ *divide by 300*

$1.7 = e^{.06t} \rightarrow \frac{ln 1.7)}{.06} = \frac{\cancel{.06}}{.06} t$ **t = 8.8**

e) When will the insect population double? $600 = 300e^{0.06(t)}$ *divide by 300*

$2 = e^{.06t} \rightarrow \ln 2 = .06t$

2) Stronium 90 is a radioactive material that decays according to the function $A(t) = A_0e^{-0.0244t}$, where A_0 is the initial amount time t (in years). Assume that a scientist has a sample of 400 grams of strontium 90.

a) What is the decay rate of strontium 90? **-2.44%**

b) How much strontium 90 is left after 20 years?

we just plug in 20 for t* $400e^{-0.0244\cdot 20} \approx$ **246

c) When will only 300 grams of strontium 90 be left? $300=400e^{-0.0244t}$

300 is what we end with so we divide $\frac{300}{400} = .75$ $0.75=e^{-0.0244t}$

take ln of both sides $\ln .75 = -0.244t \rightarrow \frac{ln 0.75)}{-0.0244} = t$ **t ≈ 11.8**

d) What is the half-life of strontium 90? $200=400e^{-0.0244t}$

200 is half of 400 then we divide $\frac{200}{400} = .5$ $0.5=e^{-0.0244t}$

take ln of both sides $\ln .5 = -0.244t$ $\frac{ln 0.5)}{-0.0244} = t$ **t ≈ 28.4**

3) In a town whose population is 2700, a disease creates an epidemic. The number of people N infected days after the disease has begun is given by the function: $\mathbf{N(t)} = \frac{2700}{1+22.9e^{-0.7t}}$

a) How many are initially infected with the disease (t=0)?

$$\frac{2700}{(1+22.9e^{-0.7\cdot 0}} = 113$$

b) Find the number infected after 2 days, 5 days, 8 days, 12 days, and 16 days.

The number infected after 2 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 2}} = 406$

The number infected after 5 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 5}} = 1596$

The number infected after 8 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 8}} = 2489$

The number infected after 12 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 12}} = 2686$

The number infected after 16 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 16}} = 2699$

c) Using the model, can you say whether all 2700 people will ever be infected?

a) At the present, there are 2700 animals in the forest.

b) It will take approximately 25 years for the animal population to reach 16,000.

c) It will take approximately 23.5 years for the animal population to reach 16,000 (Round to one decimal place)

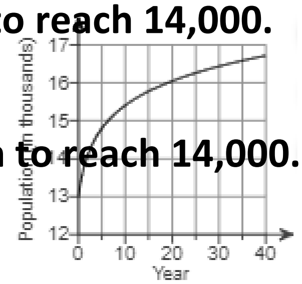
4) The population of an animal in a national forest is modeled by the formula P=12,200 + 1000·ln(t+1), where t is the number of years from the present.

a) How many animals are now in the forest?

P=12,200 + 1000·ln(0+1) = 12200

b) **It will take approximately 5 years for the animal population to reach 14,000.**

(use graph to find year at population 14)



c) **It will take approximately 5.0 years for the animal population to reach 14,000.**

14,000 = 12,200 + 1000·ln(t+1)

1800 = 1000·ln(t+1)

1.8 = ln(t+1)

$e^{1.8} = t + 1$

$e^{1.8} - 1 = t$ t = 5.0 round to one decimal

5) The sound level, L, in decibels (db), is given by the formula $L = 10 \cdot \log(I \times 10^{12})$ db, where I is the intensity of the sound in watts per square meter. The sound level is 90db. What value of I gives sound of 90db?

$90 = 10 \cdot \log(I \times 10^{12})$

$9 = \log(I \times 10^{12})$

$10^9 = I \times 10^{12}$

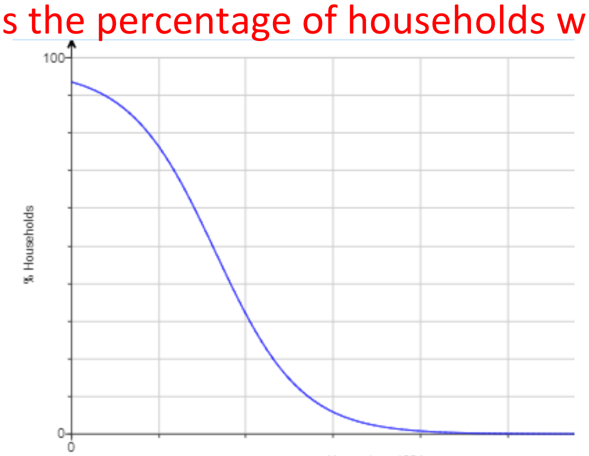
$I = \frac{10^9}{10^{12}} = .001$

6) The logistics model $\frac{96.8181}{1+0.0351e^{0.203t}}$ represents the percentage of households that do not own a personal computer t years since 1984.

a) Evaluate an interpret P(0) plug in t=0 $\frac{96.8181}{1+0.0351e^{0.203 \cdot 0}} = 93.5$

P(0) is the percentage of households without a personal computer in 1984

b) Graph



c) What percentage of households did not own a personal computer in 1996?

t = 1996-1984 = 12 plug in t=12 $\frac{96.8181}{1+0.0351e^{0.203\cdot 12}}$ =69.1

d) In what year will the percentage of households that do not own a personal computer reach 20%?

$$20 = \frac{96.8181}{1+0.0351e^{0.203t}}$$

$$20(1 + 0.0351e^{0.203t}) = 96.8181$$

$$20 + 0.702e^{0.203t} = 96.8181$$

-20

$$0.702e^{0.203t} = 76.8181$$

$$t = \frac{\ln\left(\frac{76.8181}{0.702}\right)}{0.203} = 23.1$$

23.1 + 1984 = **2007**

7) An employee brings a contagious disease to an office with 100 employees. The number of employees infected by the disease t days after the employees are first exposed to it is given by: $\mathrm{N} = \frac{70}{1+69e^{-0.6t}}$

The number of days until 69 employees have been infected is

$$69 = \frac{70}{1+69e^{-0.6t}}$$

$$69(1 + 69e^{-0.6t}) = 70$$

$$69 + 4761e^{-0.6t} = 70$$

-69

$$4761e^{-0.6t} = 1$$

$$t = \frac{\ln\left(\frac{1}{4761}\right)}{-0.6} = 14$$

8) The number of students infected with flue at a high school after t days is modeled by the function P(t) = $\frac{700}{1+49e^{-0.4t}}$

The school will close when 600 of the 700 students are infected.

After how many days will the school close?

$$600 = \frac{700}{1+49e^{-0.4t}}$$

$$600(1+49e^{-0.4t}) = 700$$

$$600 + 29400e^{-0.4t} = 700$$

$$-600$$

$$29400e^{-0.4t} = 100$$

$$t = \frac{\ln\left(\frac{1}{294}\right)}{-0.4} = 14$$

9) According to Newton's Law of Cooling, if a body with temperature T_1 is placed in surroundings with temperature T_0, different from that of T_1, the body will either cool or warm to temperature T(t) after t minutes,

$$T(t) = T_0 + (T_1 - T_0)e^{-kt}$$

A cup of coffee with temperature 135°F is placed in a freezer with temperature 0° F. After 5 minutes, the temperature of the coffee is 90°F. Use Newton's Law of Cooling to find the coffee's temperature after 15 minutes.

$T_0=0$ T_1-135

$$T(t) = 0 + (135 - 0)e^{-kt}$$

$$T(t) = 135e^{-kt}$$

$$90 = 135e^{-5k}$$

$$\frac{90}{135} = e^{-5k}$$

$$\frac{\ln(\frac{90}{135})}{-5} = k$$

$$T(t) = 135e^{-\frac{\ln\left(\frac{90}{135}\right)}{-5}(15)} \rightarrow 135e^{3\ln\left(\frac{90}{135}\right)} = 40°F$$

10) According to Newton's Law of Cooling, if a body with temperature T_1 is placed in surroundings with temperature T_0, different from that of T_1, the body will either cool or warm to temperature T(t) after t minutes,

$$T(t) = T_0 + (T_1 - T_0)e^{-kt}$$

A chilled jello salad with temperature 49°F is taken from a refrigerator and placed in a 68° F room. After 10 minutes, the temperature of the salad is 54°F. Use Newton's Law of Cooling to find the coffee's temperature after 20 minutes.

$T_0 = 68 \qquad T_1 = 48$

$T(t) = 68 + (49 - 68)e^{-kt}$

$T(t) = 68 - 19e^{-kt}$

since temp warms up to 54 after 5 min $\qquad 54 = 68 - 19e^{-5k}$

$-14 = -19e^{-kt}$

$$\frac{14}{19} = e^{-10k}$$

$$\frac{\ln(\frac{14}{19})}{-10} = k$$

$$T(t) = 68 - 19e^{-\frac{\ln\left(\frac{14}{19}\right)}{-10}(20)} \rightarrow 68 - 19e^{2\ln\left(\frac{14}{19}\right)} = 58$$

11) Stars have been classified into magnitude according to their brightness. Stars in the first six magnitudes are visible to the naked eye, and those of higher magnitudes are visible only through a telescope. The magnitude, m, of the faintest start that is visible with a telescope having lens diameter, d, in inches, is modeled by m=8.8 + 5.1 log d. What is the highest magnitude of a star that is visible with a 230-inch telescope?

$m = 8.8 + 5.1 \log 230$

$m = 20.8$

EXTRA EXAMPLES:

A) The population of a colony of mosquitoes obeys the law of uninhibited growth. If there are 1000 mosquitoes initially and there are 1700 after 1 day? How many are there after 3 days? How long is in until there are 30,000 mosquitoes? **k replaces r in the formula and find k first*

Find **rate (k)** first: $1700 = 1000e^{k(1)}$

divide by 1000 $1.7 = e^{k}$

$\ln 1.7 = k$

3 days $\rightarrow 1000e^{(3\cdot\ln 1.7)}$

$\approx$ **4913 mosquitoes**

$3000 = 1000e^{\ln 1.7t}$

divide by 1000 $30 = e^{\ln 1.7t}$

take ln of both sides $\ln 30 = \ln 1.7t$ $t = \frac{\ln 30}{\ln 1.7}$ **t ≈ 6.4 days**

B) The population of a southern city follows the exponential law. If the population doubled in size over 21 months and the current population is 20,000, what will the population be 3 years from now?

Find rate (k) first: $2 = e^{k(21)}$

$\frac{\ln 2}{21} = k$

3 years is 36 months $\rightarrow 20{,}000e^{\wedge}\left(36\left(\frac{\ln 2}{21}\right)\right)$

$\approx$ **69627**

C) The hal-life of radium is 1690 years. If 20 grams are present now, how much will be present in 670 years?

Find rate (k) first: $10 = 20e^{k(1690)}$

$\ln 0.5 = e^{1690k}$

$k = \frac{\ln 0.5}{1690}$

$20e^{\wedge}\left(670\left(\frac{\ln 0.5}{1690}\right)\right)$ $\approx$ **15.195 grams**

D) The half-life of carbon-14 is 5600 years. If a piece of charcoal made from the wood of a tree shows only 67% of the carbon-14 expected in living matter, when did the tree die?

Find rate (k) first: ln0.5 = e^{5600k}

$$k = \frac{ln0.5}{5600}$$

$$.67 = e^{(\frac{ln0.5}{5600})t}$$

$$\ln.67 = \frac{ln0.5}{5600}t$$

*multiply by reciprocal

$$\frac{(5600 \cdot ln0.67)}{ln0.5} = t \approx \mathbf{3235}$$

E) Reacting with water in an acidic solution at a particular temperature, compound A decomposes into compound B and C according to the law of uninhibited decay. An initial amount of 0.60 M compound A decomposes to 0.57 in 30 minutes. How much of compound A w ill remain after 2 hours? How long will it take until 0.10 M of compound A remains?

Find rate (k) first: 0.57 = $0.60e^{30k}$

$$\ln\left(\frac{0.57}{0.60}\right) = 30k$$

$$k = \frac{\ln(\frac{0.57}{0.60})}{30}$$

*initial amount goes with e

*time is in minutes

(2hours = 120 min)

*reduce $\frac{120}{30} = 4$

2 hours $\rightarrow 0.60e^{120(\frac{\ln(\frac{0.57}{0.60})}{30})}$

$$0.60e^{\wedge}\left(4\ln(\frac{0.57}{0.60})\right) \approx \mathbf{0.49}$$

0.10 remains $\rightarrow 0.10 = 0.60e^{t(\frac{\ln(\frac{0.57}{0.60})}{30})}$

$$\ln\left(\frac{0.10}{.060}\right) = t(\frac{\ln\left(\frac{0.57}{0.60}\right)}{30}) \rightarrow \frac{\left(30 \cdot \ln(\frac{0.10}{0.60})\right)}{\ln(\frac{0.57}{0.60})} = t \quad \approx \mathbf{1048.0}$$

*multiply by reciprocal

F) After the release of radioactive material into the atmosphere from a nuclear power plant in a country in 1980, the hay in that country was contaminated by a radioactive isotope remains, how long did the farmers need to wait to use this hay?

Find rate (k) first: $\ln 0.5 = e^{5k}$ $\qquad .09 = e^{\left(\frac{ln0.5}{5}\right)t}$

$k = \frac{ln0.5}{5}$ $\qquad \ln .09 = \frac{ln0.5}{5}t$ **multiply by reciprocal*

$\frac{(5 \cdot ln0.09)}{ln0.5} = t \approx$ **17.4**

G) The population of an animal in a national forest is modeled by the formula P=12,200 + 1000·ln(t+1), where t is the number of years from the present.

a) How many animals are now in the forest?

P=12,200 + 1000·ln(0+1) = 12200

b) It will take approximately <u>5</u> years for the animal population to reach 14,000.

(use graph to find year at population 14)

c) It will take approximately <u>5.0</u> years for the animal population to reach 14,000.

14,000 = 12,200 + 1000·ln(t+1)

1800 = 1000·ln(t+1)

1.8 = ln(t+1)

$e^{1.8} = t + 1$

$e^{1.8} - 1 = t$ $\qquad t = 5.0$ round to one decimal