

## 6.1 and 6.2 Composition, One-to-One, Invers Functions THOMPSON

1) The domain of the composite function  $(f \circ g)(x)$  is the **NOT** the same as  $g(x)$ .

2) Find  $(f \circ g)(x)$  if  $f(x) = \sqrt{x+4}$  and  $g(x) = \frac{2}{x}$  we plug the g into f  $\sqrt{\frac{2}{x} + 4}$

3) If  $H = f \circ g$  and  $H(x) = \sqrt{36 - 16x^2}$ , which of the following cannot be the component functions  $f$  and  $g$ ?

Choose the correct answer below.

☒  $f(x) = \sqrt{36 - x^2}$ ;  $g(x) = 16x$

4) Given  $f(x) = 5x$  and  $g(x) = 4x^2 + 4$ , find the following expressions.

(a)  $(f \circ g)(4)$  (b)  $(g \circ f)(2)$  (c)  $(f \circ f)(1)$  (d)  $(g \circ g)(0)$

$$g(4) = 68$$

$$f(2) = 10$$

$$f(1) = 5$$

$$g(0) = 4$$

$$f(68) = 340$$

$$g(10) = 404$$

$$f(5) = 25$$

$$g(4) = 68$$

5) Given  $f(x) = 2x^2 - 1$  and  $g(x) = 6 - \frac{1}{2}x^2$ , find the following expressions.

(a)  $(f \circ g)(4)$  (b)  $(g \circ f)(2)$  (c)  $(f \circ f)(1)$  (d)  $(g \circ g)(0)$

$$g(4) = -2$$

$$f(2) = 7$$

$$f(1) = 1$$

$$g(0) = 6$$

$$f(-2) = 7$$

$$g(7) = -\frac{37}{2}$$

$$f(1) = 1$$

$$g(6) = -12$$

6) For  $f(x) = 5x + 4$  and  $g(x) = 8x$  find the following composite functions and state the domain of each.

**Write the function listed first with a blank in place of the x value**

(a)  $f \circ g$

(b)  $g \circ f$

(c)  $f \circ f$

(d)  $g \circ g$

$$5(8x) + 4$$

$$8(5x+4)$$

$$5(5x+4) + 4$$

$$8(8x)$$

$$40x + 4$$

$$40x+32$$

$$25x + 24$$

$$64x$$

☒ B. The domain of  $g \circ f$  is all real numbers. For all parts because they are lines

7) For  $f(x) = 7x + 9$  and  $g(x) = x^2$ , find the following composite functions and state the domain of each.

(a)  $f \circ g$       (b)  $g \circ f$       (c)  $f \circ f$       (d)  $g \circ g$

$7(x^2) + 9$        $(7x+9)^2$        $7(7x+9) + 9$        $(x^2)^2$

$7x^2 + 9$        $49x^2 + 126x + 81$        $49x + 72$        $x^4$

✔ B. The domain of  $g \circ f$  is all real numbers. For all parts because they are parabolas

8) For  $f(x) = x^2$  and  $g(x) = x^2 + 7$ , find the following composite functions and state the domain of each.

(a)  $f \circ g$       (b)  $g \circ f$       (c)  $f \circ f$       (d)  $g \circ g$

$(x^2+7)^2$        $(x^2)^2+7$        $(x^2)^2$        $(x^2+7)^2+7$

$x^4+14x^2+49$        $x^4+7$        $x^4$        $x^4+14x^2+56$

✔ B. The domain of  $g \circ f$  is all real numbers. For all parts because they are parabolas

9)  $f(x) = \frac{8}{x-3}$        $g(x) = \frac{1}{x}$

(a)  $f \circ g = \frac{8}{\left(\frac{1}{x}\right)-3} = \frac{8}{\frac{1-3x}{x}} = \text{flip bottom and multiply} = \frac{8x}{1-3x}$

Domain  $\{x|x \neq 0, \frac{1}{3}\}$  check both denominators (answer and original)

(b)  $g \circ f = \frac{1}{\left(\frac{8}{x-3}\right)} = \text{flip bottom and multiply} = \frac{x-3}{8}$

Domain  $\{x|x \neq 3\}$  only original has restrictions

(c)  $f \circ f = \frac{8}{\left(\frac{8}{x-3}\right)-3} = \frac{8}{\frac{8-3x+9}{x-3}} = \frac{8}{\frac{17-3x}{x-3}} = \text{flip bottom and multiply} = \frac{8(x-3)}{17-3x}$

Domain  $\{x|x \neq 3, \frac{17}{3}\}$

(d)  $g \circ g = \frac{1}{\left(\frac{1}{x}\right)} = \text{flip bottom and multiply} = x$

Domain  $\{x|x \neq 0\}$  only original has restrictions

10)  $f(x) = \frac{x}{x-4}$        $g(x) = \frac{-7}{x}$

a) Find  $f \circ g(x) = \frac{\left(\frac{-7}{x}\right)}{\left(\frac{-7}{x}\right)-4} = \frac{\frac{-7}{x}}{\frac{-7-4x}{x}} = \text{flip bottom and multiply} = \frac{7}{7+4x}$  *drop all negatives*

Domain  $\{x|x \neq 0, -\frac{4}{7}\}$  check both denominators (answer and original)

b) Find  $g \circ f(x) = \frac{-7}{\left(\frac{x}{x-4}\right)} = \text{flip bottom and multiply} = \frac{-7x+28}{x}$

Domain  $\{x|x \neq 0, 4\}$  check both denominators

c) Find  $f \circ f(x) = \frac{\frac{x}{x-4}}{\left(\frac{x}{x-4}\right)-4} = \frac{\frac{x}{x-4}}{\frac{x-4x+16}{x-4}} = \frac{\frac{x}{x-4}}{\frac{-3x+16}{x-4}} = \text{flip bottom and multiply} = \frac{x}{-3x+16}$

Domain  $\{x|x \neq 4, \frac{16}{3}\}$

a) Find  $g \circ g(x) = \frac{-7}{\left(\frac{-7}{x}\right)} = \text{flip bottom and multiply} = x$

Domain  $\{x|x \neq 0\}$  only original has restrictions

11)  $f(x) = \sqrt{x}$        $g(x) = 6x+1$

a. Find  $f \circ g(x) = \sqrt{6x+1}$       Domain  $\{x|x \geq -\frac{1}{6}\}$

b. Find  $g \circ f(x) = 6\sqrt{x} + 1$       Domain  $\{x|x \geq 0\}$

c. Find  $f \circ f(x) = \sqrt[4]{x}$       Domain  $\{x|x \geq 0\}$

d. Find  $g \circ g(x) = 6(6x+1) + 1 = 36x + 7$  Domain all reals

12)  $f(x) = x^2 + 6$        $g(x) = \sqrt{x-4}$

a. Find  $f \circ g(x) = (\sqrt{x-4})^2 + 6 = x + 2$       Domain  $\{x|x \geq 4\}$

b. Find  $g \circ f(x) = \sqrt{x^2 + 6 - 4} = \sqrt{x^2 + 2}$       Domain all reals

c. Find  $f \circ f(x) = (x^2 + 6)^2 + 6 = x^4 + 12x^2 + 42$       Domain all reals

d. Find  $g \circ g(x) = \sqrt{\sqrt{x-4}-4}$

Domain  $\{x | x \geq 20\}$

**\*\*Square 4 then add 4 to get 20**

**\*ALWAYS SQUARE THE # THEN ADD IT TO ITSELF**

EX :  $\sqrt{\sqrt{x-8}-8} \rightarrow 8^2 + 8 = 72$  THEN Domain  $\{x | x \geq 72\}$

13)  $f(x) = \frac{x-4}{x+7}$        $g(x) = \frac{x+2}{x-3}$

a) Find  $f \circ g(x) = \frac{\left(\frac{x+2}{x-3}\right)-4}{\left(\frac{x+2}{x-3}\right)+7} = \frac{\frac{x+2-4(x-3)}{x-3}}{\frac{x+2+7(x-3)}{x-3}} = \frac{x+2-4x+12}{x+2+7x-21} = \frac{-3x+14}{8x-19}$

**\*\*bring the x+2 over then distribute the 7 with the denominator**

Domain  $\{x | x \neq 3, \frac{19}{8}\}$  check both denominators (original and answer)

b) Find  $g \circ f(x) = \frac{\left(\frac{x-4}{x+7}\right)+2}{\left(\frac{x-4}{x+7}\right)-3} = \frac{\frac{x-4+2(x+7)}{x+7}}{\frac{x-4-3(x+7)}{x+7}} = \frac{x-4+2x+14}{x-4-3x-21} = \frac{3x+10}{-2x-25}$

Domain  $\{x | x \neq -7, -\frac{25}{2}\}$  check both denominators

c) Find  $f \circ f(x) = \frac{\left(\frac{x-4}{x+7}\right)-4}{\left(\frac{x-4}{x+7}\right)+7} = \frac{\frac{x-4-4(x+7)}{x+7}}{\frac{x-4+7(x+7)}{x+7}} = \frac{x-4-4x-28}{x-4+7x+49} = \frac{-3x-32}{8x+45}$

Domain  $\{x | x \neq -7, -\frac{45}{8}\}$  check both denominators

d) Find  $g \circ g(x) = \frac{\left(\frac{x+2}{x-3}\right)+2}{\left(\frac{x+2}{x-3}\right)-3} = \frac{\frac{x+2+2(x-3)}{x-3}}{\frac{x+2-3(x-3)}{x-3}} = \frac{x+2+2x-6}{x+2-3x+9} = \frac{3x-4}{-2x+11}$

Domain  $\{x | x \neq 3, \frac{11}{2}\}$  check both denominators

- 14) If  $f(x) = 4x^3 - 4x^2 + 2x - 1$  and  $g(x) = 3$ , find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

Find  $f \circ g(x)$

$$4(3)^3 - 4(3)^2 + 2(3) - 1 = 77$$

Find  $g \circ f(x)$

3 (has no x to substitute in for)

- 15) In a certain city, deforestation occurs at the rate of about 5.1% per year. If  $x$  is the total forested area of the city at the start of 2000, then  $f(x) = 0.864x$  is the amount of forested land in the city at the start of 2001. Find and interpret  $(f \circ f)(x)$ . Find and interpret  $(f \circ f \circ f)(x)$ .

$$0.864(0.864x)$$

.....

$$(f \circ f)(x) = .746x \text{ (Type an integer or decimal rounded to three decimal places as needed.)}$$

The function  $(f \circ f)(x)$  is the amount of forested land in the city at the start of 2002.

$$(f \circ f \circ f)(x) = .645x \text{ (Type an integer or decimal rounded to three decimal places as needed.)}$$

$$0.746(0.864x)$$

The function  $(f \circ f \circ f)(x)$  is the amount of forested land in the city at the start of 2003.

- 16) The regular price of a computer is  $x$  dollars. Let  $f(x) = x - 150$  and  $g(x) = 0.8x$ .

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a. Describe what the functions  $f$  and  $g$  model in terms of the price of the computer.

The function  $f$  gives the price of the computer after a \$150 discount .

The function  $g$  gives the price of the computer after a 20% discount .

b. Find  $(f \circ g)(x)$  and describe what this models in terms of the price of the computer.

$$(f \circ g)(x) = .8x - 150$$


The function  $(f \circ g)(x)$  models the price of the computer after first a 20% discount and then a \$150 discount .

c. Find  $(g \circ f)(x)$  and describe what this models in terms of the price of the computer.

$$(g \circ f)(x) = .8x - 120 \quad 0.8(x-150)$$

The function  $(g \circ f)(x)$  models the price of the computer after first a \$150 discount and then a 20% discount .

d. Which composite function models the greater discount on the computer,  $f \circ g$  or  $g \circ f$ ? Why?

-  A.  $f \circ g$  models the greater discount, since the 20% discount is taken on the regular price first.

- 17) A stone is thrown into a pond, creating a circular ripple that spreads over the pond in such a way that the radius is increasing at a rate of 2.6 ft/sec.

Complete parts a through c.

- a) Find a function for the radius in terms of  $t$ .

$$r(t) = 2.6t$$

(Use integers or decimals for any numbers in the expression.)

- b) Find a function  $A(r)$  for the area of the ripple in terms of the radius  $r$ .

$$A(r) = \pi r^2$$

(Type an exact answer, using  $\pi$  as needed.)

- c) Find  $(A \circ r)(t)$ .

$$\pi(2.6t)^2$$

$$(A \circ r)(t) = 6.76\pi t^2$$

(Simplify your answer. Type an exact answer in terms of  $\pi$ . Use integers or decimals for any numbers in the expression.)

Explain the meaning of the previous function.

Choose the correct answer below.



- A. The function gives the area of the ripple in terms of  $t$ .

- 18) A manufacturer of tools, selling rechargeable drills to a chain of home improvement stores, charges \$4 more per drill than its manufacturing cost,  $m$ . The chain then sells each drill for 170% of the price that it paid the manufacturer. Find a function  $P(m)$  for the price at the home improvement stores.

$$1.7(m+4)$$

The function for the price at the home improvement stores is given by

$$P(m) = 1.7m + 6.8$$

(Simplify your answer. Use integers or decimals for any numbers in the expression. Do not include the \$ symbol in your answer.)