One-to-one Functions **MATH 161** **THOMPSON**

1) If every horizontal line intersects the graph of a function at no more than one point, f is a <u>one-to-one</u> function.

2) If f is a one-to-one function and $f(5) = 2$, then $f^{-1}(2) =$ 5

3) If f^{-1} denotes the inverse of a function f, then the graphs of f and f^{-1} are symmetric with respect to the line y = x.

4) If the domain of a one-to-one function f is $[8,\infty)$, the range of its inverse, f^{-1}, is $[8,\infty)$.

Switch the coordinates for inverse

5) If $(-1,3)$ is a point on the graph of a one-to-one function f, which of the following points is on the graph of f^{-1}?

Choose the correct answer below.

Switch the coordinates for inverse

- $(1,-3)$
- $(-3,1)$
- ✔ $(3,-1)$
- $(-1,-3)$

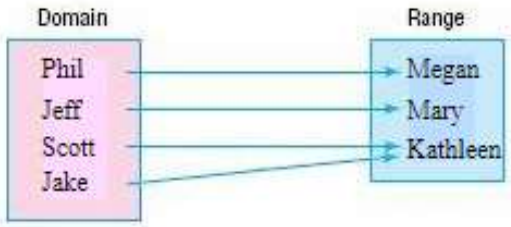
6) Suppose f is a one-to-one function with a domain of $\{x|x \neq 4\}$ and a range $\left\{y\middle|y \neq \frac{3}{4}\right\}$. Which of the following is the domain of f^{-1}?

Choose the correct answer below.

Switch domain and range for inverse

- $\left\{x\middle|x \neq 4, x \neq \frac{3}{4}\right\}$
- ✔ $\left\{x\middle|x \neq \frac{3}{4}\right\}$
- $\{x|x \neq 4\}$
- all real numbers

7) For the function on the right, determine whether the function is one-to-one.



Is the function one-to-one?

X can't to the same y

- ○ Yes
- ✔ No

8) *With ordered pairs, one-to-one function can't have duplicating y values:*

For the following function, determine whether the function is one-to-one.

$\{(4,6), (3,9), (-8,14), (1,-8)\}$

Is the function one-to-one?

- ○ No
- ✔ Yes

9) For the following function, determine whether the function is one-to-one.

$\{(4,6), (3,6), (-8,3), (6,-5)\}$

Is the function one-to-one?

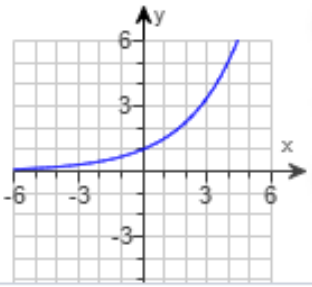
- ✔ No
- ○ Yes

10) The graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.

Is f one-to-one?

Vertical line test

- ✔ Yes
- ○ No

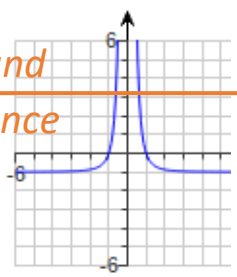


11) The graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.

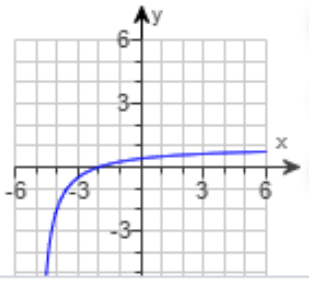
Is f one-to-one?

I can draw a horizontal line and touch the graph more than once

- ○ Yes
- ✔ No



12) The graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.



Is f one-to-one?

Yes

13) $f(x) = -8x - 8$ $\quad g(x) = -\frac{1}{8}(x+8)$

a) Find f(g(x)) $-8\left(-\frac{1}{8}(x+8)\right) - 8$

$-8\left(-\frac{1}{8}x - 1\right) - 8$

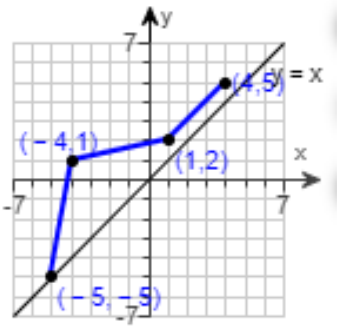
$x + 8 - 8$ = x

b) Find g(f(x)) $-\frac{1}{8}((-8x-8)+8)$

$-\frac{1}{8}(-8x)$ = x

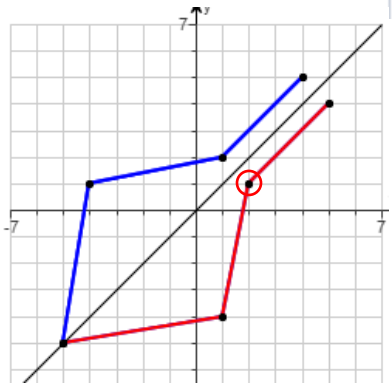
If both equal x then they are inverses of each other; therefore, YES

14) The graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1}. For convenience (and as a hint), the graph of y = x is also given.

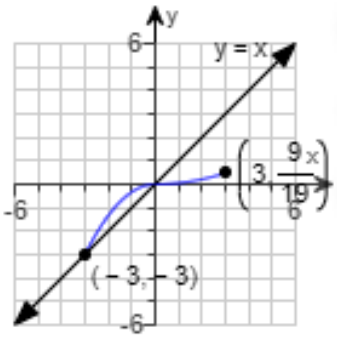


Pick a point on the graph then switch the x and y

(1,2) to (2,1)

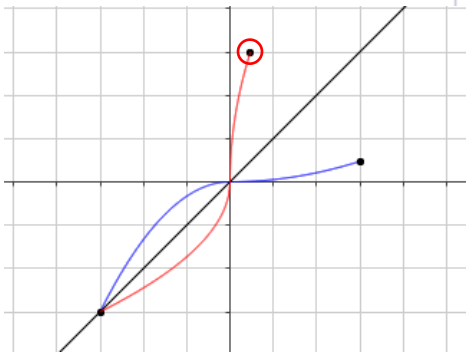


15) The graph of a one-to-one function is shown to the right. Draw the graph of the inverse function f^{-1}.

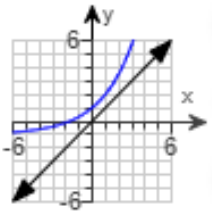


Pick a point on the graph then switch the x and y

$(3,\frac{9}{19})$ to $(\frac{9}{19},3)$

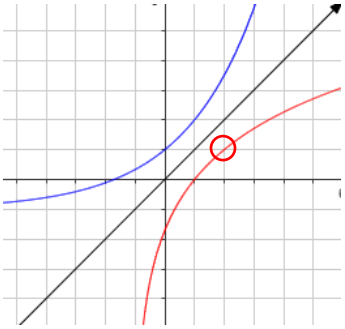


16) The graph of a one-to-one function is shown to the right. Draw the graph of the inverse function f^{-1}.



Pick a point on the graph then switch the x and y

(1,2) to (2,1)

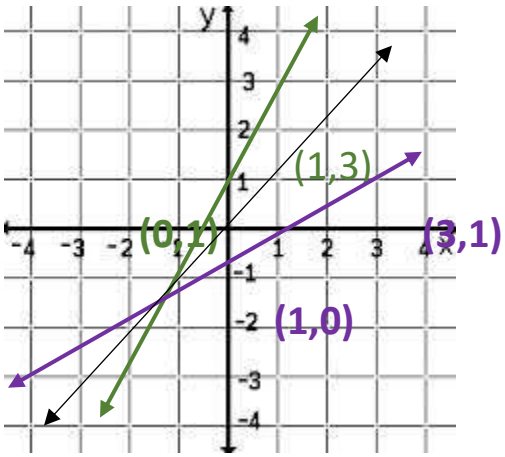


17) The function f(x) = 2x+1 is one-to-one

Find the inverse of f(x) =2x+1

$x = 2y + 1$

$\frac{x-1}{2} = y'$ $\quad f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$



Domain and Range are all reals

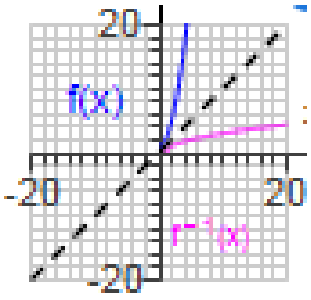
Can't plot fractions on the inverse function

*find points on the **original line** and switch the coordinates for the **inverse line.***

Don't forget to plot the y = x line plot (0,0) and (1,1) to get the line

18) The function $f(x) = x^2 + 1, x \geq 0$ is one-to-one.
(a) Find the inverse of f and check the answer.
(b) Find the domain and the range of f and f^{-1}.
(c) Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

CHOOSE THE GRAPH



X = y^2 + 1
x-1 = y^2
$\sqrt{x-1} = y'$ ***$f^{-1}(x) = \sqrt{x-1}$***

Domain of f is $x \geq 0$
Range of f is $y \geq 1$
Domain of f is $x \geq 1$
Range of f is $y \geq 0$ **the domain and range switch*

19) Find the inverse of $f(x) = \frac{3x}{x+5}$

Switch x and y then solve for y. $x = \frac{3y}{y+5}$ $x(y+5) = 3y$

$xy + 5x = 3y$
$xy - 3y = -5x$

**factor out the y* $y(x-3) = -5x$

$$f^{-1} = \frac{-5x}{x-3}$$

Domain of f is {x|x≠-5} *Range of f is {y|y≠3}*

Domain and Range of function and its inverse are opposite, THEREFORE…

Domain of f^{-1} is {x|x≠3} *Range of f^{-1} is {y|y≠-5}*

20) The domain of a one-to-one function f is $[5,\infty)$, and its range is $[-9,\infty)$. State the domain and the range of f^{-1}.

What is the domain of f^{-1}?

The domain of f^{-1} is $[-9,\infty)$.
(Type your answer in interval notation.)

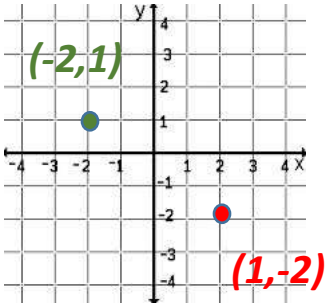
Switch domain and range

What is the range of f^{-1}?

The range of f^{-1} is $[5,\infty)$.
(Type your answer in interval notation.)

21) If f is in quadrant II, what quadrant is f^{-1} in?

Put a point in quadrant II then switch x and y and see what quadrant the inverse coordinate is in.



(1,-2) ***Quadrant IV***

****Quadrants I and III inverse coordinates do not move!***

Ex) ***Find the inverse of*** $f(x) = \frac{7x+9}{4x-7}$

Switch x and y then solve for y. $x = \frac{7y+9}{4y-7}$ $x(4y-7) = 7y+9$

$$4xy - 7x = 7y + 9$$
$$4xy - 7y = 7x + 9$$

**factor out the y* $y(4x-7) = 7x+9$

$$f^{-1} = \frac{7x+9}{4x-7}$$

Domain of f is $\{x|x\neq\frac{7}{4}\}$ *Range of f is* $\{y|y\neq\frac{7}{4}\}$

Domain and Range of function and its inverse are opposite, THEREFORE…

Domain of f^{-1} *is* $\{x|x\neq\frac{7}{4}\}$ *Range of* f^{-1} *is* $\{y|y\neq\frac{7}{4}\}$