

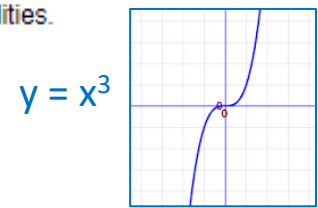
## Polynomial Equations

- 1) Graph the following polynomial function by hand. Then solve the equation and inequalities.

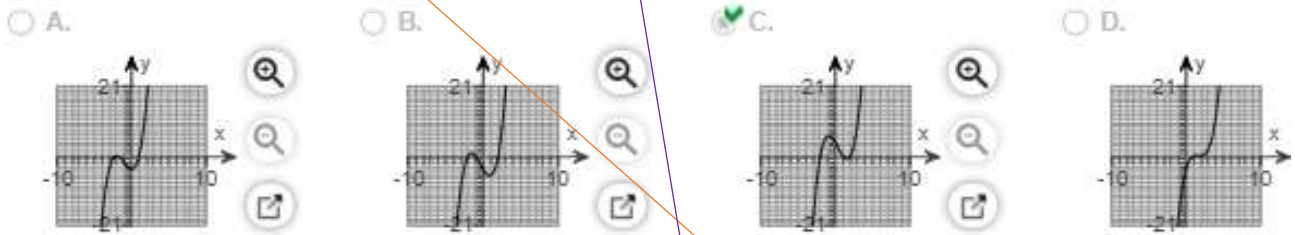
$$P(x) = x^3 - 1x^2 - 4x + 4$$

$$= (x-2)(x-1)(x+2)$$

- (a)  $P(x) = 0$     (b)  $P(x) < 0$     (c)  $P(x) > 0$



open circles <    open circles >



- (a) The solution set for  $P(x) = 0$  is  $\{-2, 1, 2\}$ .  
(Use a comma to separate answers as needed.)

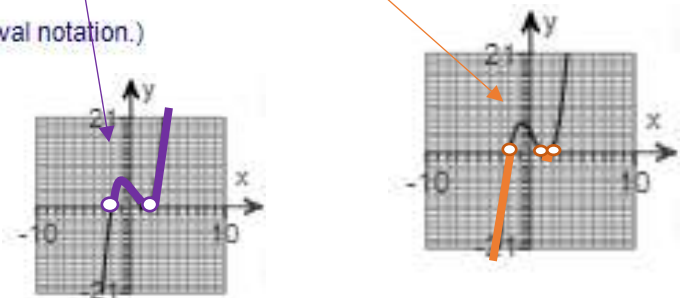
Switch the signs  $(x-2)(x-1)(x+2)$

- (b) The solution set for  $P(x) < 0$  is  $(-\infty, -2) \cup (1, 2)$ .  
(Type your answer in interval notation.)

graph is BELOW the x-axis NOT including zero

- (c) The solution set for  $P(x) > 0$  is  $(-2, 1) \cup (2, \infty)$ .  
(Type your answer in interval notation.)

Where the graph is ABOVE the x-axis  
NOT including zero



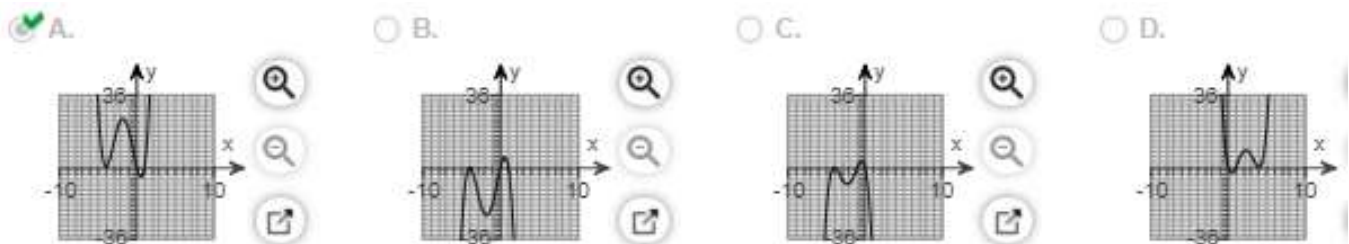
- 2a) Graph the following polynomial function by hand. Then solve the equation and inequalities.

$$P(x) = x^4 + 7x^3 + 8x^2 - 16x$$

$$= x(x-1)(x+4)^2$$

- (a)  $P(x) = 0$     (b)  $P(x) \geq 0$     (c)  $P(x) \leq 0$

closed circles  $\geq$     closed circles  $\leq$



(a) The solution set for  $P(x) = 0$  is  $\{-4, 0, 1\}$ .  
(Use a comma to separate answers as needed.)

Switch the signs  $x(x-1)(x+4)^2$  single  $x=0$

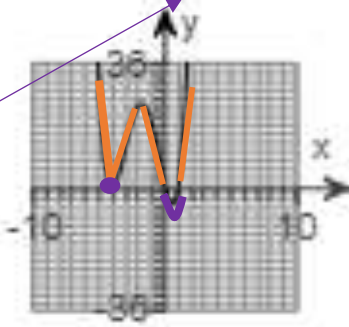
graph is ABOVE the x-axis including zero

(b) The solution set for  $P(x) \geq 0$  is  $(-\infty, 0] \cup [1, \infty)$ .  
(Type your answer in interval notation.)

(c) The solution set for  $P(x) \leq 0$  is  $\{-4\} \cup [0, 1]$ .  
(Type your answer)

graph is BELOW the x-axis including zero  
including the point  $x = 4$

braces  $\{\}$  for  
single number



### USE U SUBSTITUTION FOR X

3)  $x^4 - 10x^2 + 9 = 0$

factor and take half of 1<sup>st</sup> exponent

$$u = x^2 \quad u^2 - 10u + 9 = 0$$

$$(u-9)(u-1) = 0$$

$$u = 9, 1$$

$$x^2 = 9$$

$$x^2 = 1$$

$$x = \sqrt{9}$$

$$x = \sqrt{1}$$

the solution set is  $-3, 3, -1, 1$

4)  $8x^4 - 2x^2 - 1 = 0$

$$u = x^2$$

$$8u^2 - 2u - 1 = 0 \text{ use slide and divide}$$

$$u^2 - 2u - 8 = 0$$

$$(u-4)(u+2) = 0 \text{ divide both by 8}$$

$$u = \frac{1}{2}, -\frac{1}{4} \text{ can't take square root of a negative number}$$

$$x^2 = \frac{1}{2}$$

$$x^2 = -\frac{1}{4}$$

$$x = \sqrt{\frac{1}{2}} \rightarrow \sqrt{\frac{1}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

5)  $x^6 - 26x^3 - 27 = 0$        $u = x^3$        $u^2 - 26u - 27 = 0$  take half of 1<sup>st</sup> exponent  
 $(u-27)(u+1) = 0$  set both equal to zero  
 $u = -1, 27$   
 $x^3 = -1$        $x^3 = 27$   
the solution set is  $-1, 3$

6)  $(x + 2)^2 + 9(x + 2) + 14 = 0$        $u$  is always the middle term  
 $u = x + 2$        $u^2 + 9u + 14 = 0$   
 $(u+7)(u+2) = 0$   
 $u = -7, -2$       PUT X BACK IN FOR U  
 $x + 2 = -7$  and  $x + 2 = -2$   
the solution set is  $-9, -4$

7)  $(x^2 - 2x)^2 - 27(x^2 - 2x) + 72 = 0$        $u$  is always the middle term  
Let  $u = x^2 - 2x$  then the equation in  $u$  is  $u^2 - 27u + 72 = 0$   
 $(u-3)(u-24) = 0$   
 $u = 3, 24$   
 $x^2 - 2x = 3$  and  $x^2 - 2x = 24$   
 $x^2 - 2x - 3 = 0$  and  $x^2 - 2x - 24 = 0$   
 $(x-3)(x+1) = 0$  and  $(x-6)(x+4) = 0$   
the solution set is  $3, -1, 6, -4$

8)  $(2x + 4)^2 + 4(2x + 4) + 4 = 0$        $u$  is always the middle term  
 $u = 2x + 4$  the given equation with correct substitution is  $u^2 + 4u + 4 = 0$   
\*if nothing is in front of the middle term just use  $u$        $(u+2)(u+2) = 0$   
 $u = -2$   
 $2x + 4 = -2$   
the solution set is  $-3$

9)  $(x + 8)^2 + 10(x + 8) + 9 = 0$   $u$  is always the middle term

$u = x + 8$  the given equation with correct substitution is  $u^2 + 10u + 9 = 0$

\*if nothing is in front of the middle term just use  $u$   $(u + 9)(u + 1) = 0$

$$u = -9, -1$$

$$x + 8 = -9 \text{ and } x + 8 = -1$$

the solution set is  $-17, -9$

10)  $2(s + 7)^2 - 13(s + 7) = 7$   $u = s + 7$   $2u^2 - 13u - 7 = 0$  slide and divide

$$u^2 - 13u - 14 = 0$$

$$(u - 14)(u + 1) = 0 \text{ divide by 2}$$

$$u = 7, -\frac{1}{2}$$

$$s + 7 = 7 \text{ and } s + 7 = -\frac{1}{2}$$

the solution set is  $0, -\frac{15}{2}$

11)  $y = (x + 2)^2 - 11(x + 2) + 28$   $u = x + 2$   $u^2 - 11u + 28 = 0$

$$(u - 7)(u - 4) = 0$$

$$u = 7, 4$$

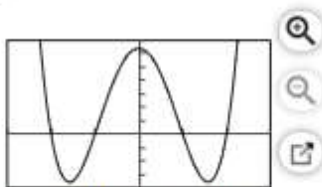
$$x + 2 = 7 \text{ and } x + 2 = 4$$

$$x = 5, 2$$

the x-intercepts are  $5, 2$

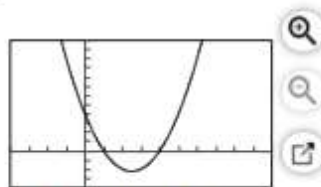
Choose the correct graph below. look at the x-intercepts to find graph

☐ A.



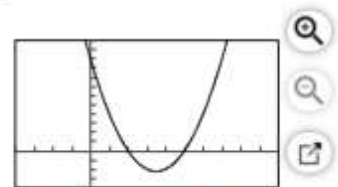
$[-6, 6, 2]$  by  $[-40, 70, 10]$

☐ B.



$[-4, 10, 1]$  by  $[-4, 12, 1]$

☒ C.



$[-4, 10, 1]$  by  $[-4, 12, 1]$

12)  $6(x+2) + 5a(x+2)$  Factor out the  $(x+2)$   
 $(x+2)(6+5a)$

13)  $5x(8x^6+9) - 4(8x^6+9)$  Factor out the  $(8x^6+9)$   
 $(8x^6+9)(5x-4)$

14)  $3x^2 + 4xy + 6x + 8y$  Factor by grouping: take GCF from each highlighted part  
 $x(3x+4y) + 2(3x+4y)$  then factor out the  $(3x+4y)$   
 $(3x+4y)(x+2)$

15)  $3x^2 + 3xy - 7x - 7y$  Factor GCF from each highlighted part  
 $3x(x+y) - 7(x+y)$  then factor out the  $(x+y)$   
 $(x+y)(3x-7)$

16) Watch the section lecture video and answer the question listed below. Note: The counter in the lower right corner of the screen displays the Example number.

As shown in Examples 1-3, what two things have to be true in order to use the zero factor property?

[Click here to watch the video.](#)

Select all that apply.

- ☐ A. Both sides of the equation must be a polynomial.
- ☐ B. One side of the equation must be a polynomial not in factored form.
- ☒ C. One side of the equation must be a factored polynomial.
- ☒ D. One side of the equation must be zero.
- ☐ E. One side of the equation must be one.

17) Find real solutions by factoring  $x^3 - 36x = 0$   
*Real solutions means set each*  $x(x^2-36)=0$  factor out x first  
 $x(x+6)(x-6)=0$  factor difference of two squares  
 $x=0, x+6=0, x-6=0$   
 $0, -6, 6$

18) Find real solutions by factoring  $5x^3 = 2x^2$  move everything to the left  
 $5x^3 - 2x^2 = 0$  Factor out  $x^2$   
 $x^2(5x-2)=0$  set each part =0  
 $\rightarrow 0, \frac{2}{5}$

19) Find real solutions by factoring  $x^3 - 13x^2 + 42x = 0$   
 $x(x^2 - 13x + 42) = 0$  factor out x first then trinomial  
 $x(x-7)(x-6) = 0$  set each part =0  
 $\rightarrow 0, 6, 7$

20) Find real solutions by factoring  $x^3 - x^2 - x + 1 = 0$   
Factor GCF from each highlighted part  $x^2(x-1) - 1(x-1) = 0$   
 $(x^2-1)(x-1) = 0$   
 $(x+1)(x-1)(x-1) = 0$   
do not duplicate answers the solution set is  $-1, 1$

21) Find real solutions by factoring  $x^3 - 8x^2 - 9x + 72 = 0$   
Factor GCF from each highlighted part  $x^2(x-8) - 9(x-8) = 0$   
Factor out the  $(x-8)$   $(x^2-9)(x-8) = 0$   
Factor difference of two squares  $(x+3)(x-3)(x-8) = 0$   
do not duplicate answers the solution set is  $-3, 3, 8$

22) Find real solutions by factoring  $2x^3 + 16 = x^2 + 32x$  move everything to the left  
Factor GCF from each highlighted part  $2x^3 - x^2 - 32x + 16 = 0$   
Factor out the  $(2x-1)$   $x^2(2x-1) - 16(2x-1) = 0$   
 $(x^2-16)(2x-1) = 0$   
Factor difference of two squares  $(x+4)(x-4)(2x-1) = 0$   
 $x+4=0$   $x-4=0$   $2x-1=0$   
the solution set  $-4, 4, \frac{1}{2}$