Rational Equations   MATH 161   THOMPSON
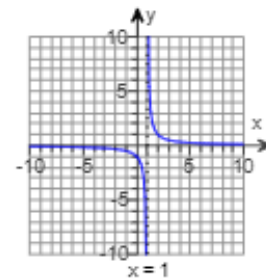
1) The graph of a rational function y = f(x) is given. Use the graph on the right to give the solution set of **(a)** f(x) = 0, **(b)** f(x) < 0, and **(c)** f(x) > 0. Use set notation for part **(a)** and interval notation for parts **(b)** and **(c)**.

x = 1

(a) What is the solution set of f(x) = 0? Select the correct answer below and, if necessary, fill in the answer box to complete your choice.

○ A. The solution set is { ☐ }. (Use a comma to separate answers as needed.)

✔ B. The solution set is the empty set, ∅. *Does not touch the x-axis*

(b) What is the solution set of f(x) < 0? Select the correct answer below and, if necessary, fill in the answer box to complete your choice.

✔ A. The solution set is the interval $(-\infty, 1)$. (Type your answer in interval notation.)

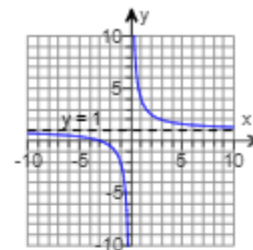○ B. The solution set is the empty set, ∅. *Where the graph is below the x-axis*

(c) What is the solution set of f(x) > 0? Select the correct answer below and, if necessary, fill in the answer box to complete your choice. *Where the graph is above the x-axis*

✔ A. The solution set is the interval $(1, \infty)$. (Type your answer in interval notation.)

2) The graph of a rational function y = f(x) is given. Use the graph to give the solution set of the following.

(a) f(x) = 0
(b) f(x) < 0
(c) f(x) > 0

Do not use a calculator.

(a) What is the solution set of f(x) = 0?

✔ A. The solution set is { -3 }. *Crosses the x-axis at -3*
(Type an integer or a decimal. Use a comma to separate answers as needed.)

○ B. The solution set is the set of all real numbers.
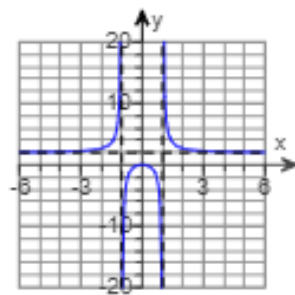
○ C. The solution set is the empty set.

(b) What is the solution set of f(x) < 0?

Where the graph is below the x-axis

✔ A. The solution set is $(-3, 0)$. (Type your answer in interval notation.)

○ B. The solution set is the set of all real numbers.

○ C. The solution set is the empty set.

Where the graph is above the x-axis

(c) What is the solution set of f(x) > 0?

✔ A. The solution set is $(-\infty, -3) \cup (0, \infty)$. (Type your answer in interval notation.)

3) The graph of a rational function y = f(x) is given. Use the graph to give the solution set of the following.

(a) f(x) = 0
(b) f(x) < 0
(c) f(x) > 0

Do not use a calculator.

. . . . .

(a) What is the solution set of f(x) = 0?

✓A. $\{0\}$ (Type an integer or a decimal.)   Crosses the x-axis

○ B. The solution set is the set of all real numbers.

○ C. The solution set is the empty set.

(b) What is the solution set of f(x) < 0?

✓A. $(-1,0)\cup(0,1)$ Below the x-axis: between -1 and 1 not including 0

○ B. $(-\infty,\quad)\cup(\quad,\infty)$

(c) What is the solution set of f(x) > 0?

○ A. $(\quad,0)\cup(0,\quad)$

✓B. $(-\infty,-1)\cup(1,\infty)$ Above x-axis: the left of -1 and right of 1

○ C. The solution set is the set of all real numbers.

○ D. The solution set is the empty set.

4) In some cases, it is possible to solve a rational inequality simply by deciding what sign the numerator and the denominator must have and then using the rules for quotients of positive and negative numbers to determine the solution set. Use similar reasoning to solve the following inequality.

$$\frac{-4}{x^2+2} > 0$$

. . . . .

Choose the correct answer below.

○ A. The solution set is ___ . (Type your answer in interval notation.)

✓B. The solution set is Ø.

5) In some cases, it is possible to solve a rational inequality simply by deciding what sign the numerator and the denominator must have and then using the rules for quotients of positive and negative numbers to determine the solution set. Use similar reasoning to solve the following inequality.

$$\frac{-x^4-x^2-4}{x^2+3} < 0$$ the coefficient is negative so it is all numbers

. . . . .

Choose the correct answer below.

✓A. The solution set is $(-\infty,\infty)$. (Type your answer in interval notation.)

○ B. The solution set is Ø.

6) Solve the equation.

$$\frac{x}{x+4} = \frac{3}{2}$$

cross multiply $2x = 3x + 12$
                     $-12 = x$

. . . . .

Select the correct choice below and fill in any answer boxes in your choice.

✔ A. The solution set is $\{-12\}$. (Simplify your answer.)

7) Solve the following equation involving rational expressions. Then, identify the equation as an identity, an inconsistent equation, or a conditional equation.

$$\frac{z+7}{z-8} = \frac{10}{-7}$$

$10z - 80 = -7z - 49$

$17z = 31$

. . . . .

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

✔ A. The solution set is $\left\{\frac{31}{17}\right\}$.

(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed.)

○ B. The solution set is $\{z \mid z \neq \boxed{}\}$.
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed.)

○ C. The solution set is the set of all real numbers.

○ D. The solution set is the empty set, Ø.

Classify the equation.

○ an inconsistent equation

✔ a conditional equation

○ an identity

8) Solve the equation.

$$\frac{3}{2x-3} = \frac{2}{x+5}$$

$4x - 6 = 3x + 15$

$x = 21$

. . . . .

Select the correct choice below and fill in any answer boxes in your choice.

✔ A. The solution set is $\{21\}$. (Simplify your answer.)

○ B. There is no solution.

9) Find the real solutions, if any, of the following equation. Use the quadratic formula.

$$\frac{3x}{x-1} + \frac{1}{x} = 4$$

multiply all by x(x-1) $2x^2 + 2x - 6 = 3x^2 - 6x$
                             $0 = x^2 - 5x + 1$

. . . . .

Select the correct choice below and, if necessary, fill in the answer box to complete your answer.

✔ A. The solution set is $\left\{\frac{5 \pm \sqrt{21}}{2}\right\}$.   *quadratic equation*

10) Solve the following equation by factoring.   *multiply all by x* $14x^2 - 45x = 14$

$$14x - 45 = \frac{14}{x}$$

$0 = 14x^2 - 45x - 14$

$0 = (7x + 2)(2x - 7)$

- - - - -

Select the correct choice below and, if necessary, fill in the answer box to complete your choic

✅ **A.**

The solution set is $\left\{ -\dfrac{2}{7}, \dfrac{7}{2} \right\}$.

(Type an integer or a simplified fraction. Use a comma to separate answers as needed

○ **B.** There is no solution.

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11) Solve the equation.

$$\frac{x^2 - 8x + 16}{x + 8} = 0$$ Factor only the TOP to solve $(x-4)(x-4) = 0$

- - - - -

Select the correct choice below and, if necessary, fill in the answer box within your choice.

✅ **A.** The solution is x = 4 . (Use a comma to separate answers as needed.)

○ **B.** There is no real solution.

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12) Solve the equation by making an appropriate substitution.

$$\left(x - \frac{10}{x}\right)^2 - 6\left(x - \frac{10}{x}\right) - 27 = 0$$ u is always middle term $\mathbf{u = x - \dfrac{10}{x}}$

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Make an appropriate substitution and rewrite the equation in quadratic form.

Let u = $x - \dfrac{10}{x}$, then the quadratic equation in u is $u^2 - 6u - 27 = 0$.

Solve the equation. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

✅ **A.** The solution set is $\{ 10, -1, -5, 2 \}$. $(u-9)(x+3) = 0$ u = 9,-3
(Simplify your answer. Use a comma to separate answers as needed.)

○ **B.** The solution set is the empty set.

$$x - \frac{10}{x} = 9 \qquad\qquad x - \frac{10}{x} = -3$$

multiply all by x $x^2 - 10 = 9x$ $x^2 - 10 = -3x$

$x^2 - 9x - 10 = 0$ $x^2 + 3x - 10 = 0$

$(x-10)(x+1) = 0$ $(x+5)(x-2) = 0$

X = -10,1,-5,2

13) Find the real solutions of the equation.

u is always middle term not including 3 $u = \dfrac{1}{x+1}$

$$\frac{1}{(x+1)^2} = \frac{3}{x+1} + 40$$

$u = \dfrac{1}{x+1}$ $u^2 = 3u + 40$ then $u^2 - 3u - 40 = 0$ $(u-8)(x+5) = 0$

What is the solution set? Select the correct choice below and fill in any answer boxes in your choice.

☑ A. $\left\{ -\dfrac{7}{8}, -\dfrac{6}{5} \right\}$

$\dfrac{1}{x+1} = 8$

$8x + 8 = 1$

$8x = -7$

$\dfrac{1}{x+1} = -5$

$-5x - 5 = 1$

$-5x = -6$

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14) Find the real solutions of the equation.

$u = x^{-1}$ $u^2 + 8x + 15 = 0$ $(u+5)(x+3) = 0$

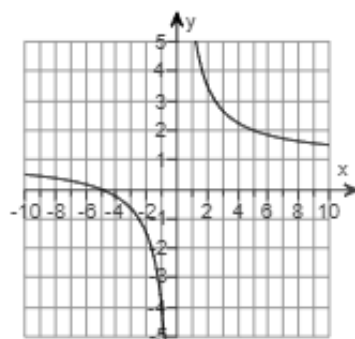$$x^{-2} + 8x^{-1} + 15 = 0$$

$x^{-1} = -5$ $x^{-1} = -3$

Select the correct choice below and, if necessary, fill in the answer box to complete your answer.

☑ A. The solution set is $\left\{ -\dfrac{1}{5}, -\dfrac{1}{3} \right\}$.

make both into fractions

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15) Two possible solutions to the equation $f(x) = 0$ are listed. Use the given graph of $y = f(x)$ to decide which, if any, are extraneous.

$x = -5$ or $x = -4$

Select the correct choice below and, if necessary, fill in the answer box within your choice.

Crosses the x-axis

☑ A. The extraneous solution(s) is/are $x = -4$.
(Use a comma to separate answers as needed. Type integers or fractions.)

◯ B. There is no extraneous solution.

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16) The accompanying graph shows the relationship between levels of a certain gas and time for which permanent brain damage will occur.

a. At 1700 PPM, how long will it take to get permanent brain damage?

b. What level of this gas takes 60 minutes to produce permanent brain damage?

c. What are the horizontal and vertical asymptotes for the curve? Expain what these asymptotes mean in simple terms.
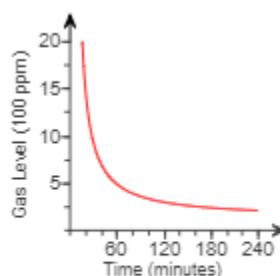
a. At 1700 PPM, it will take about 18 minutes to get permanent brain damage.

b. A gas level of about $\boxed{490}$ PPM will take 60 minutes to produce permanent brain damage.

c. The vertical asymptote is $\boxed{\text{Time}}$ $= \boxed{0}$.

The horizontal asymptote is $\boxed{\text{Gas Level}}$ $= \boxed{0}$.

What do these asymptotes mean in simple terms?

○ A. Only low concentration for a short time will cause permanent brain damage.

○ B. High concentration for a long time or low concentration for a short time will cause permanent brain damage.

✔ C. Low concentration for a long time or high concentration for a short time will cause permanent brain damage.