5.3 Rational Functions MATH 161 THOMPSON

1) Which of the expressions are equivalent to $\frac{x}{9-x}$?

- [x] A. $\frac{-x}{x-9}$
- [ ] B. $\frac{x}{x-9}$
- [ ] C. $\frac{-x}{9-x}$
- [x] D. $\frac{-x}{-9+x}$

ALL signs change

2) Decide whether each rational expression equals 1, −1, or neither. Answer parts (a) through (d) below.

a. Does the rational expression $\frac{x+1}{x-1}$ equal 1, −1, or neither?

- A. 1
- B. neither
- C. −1

SOME signs change

b. Does the rational expression $\frac{x+1}{1+x}$ equal 1, −1, or neither?

- A. neither
- B. −1
- C. 1

NO signs change

c. Does the rational expression $\frac{x-1}{1-x}$ equal 1, −1, or neither?

- A. neither
- B. −1
- C. 1

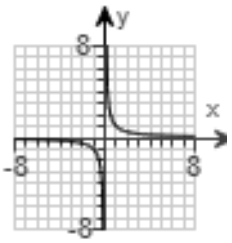
ALL signs change

d. Does the rational expression $\frac{-x-1}{x+1}$ equal 1, −1, or neither?

- A. neither
- B. 1
- C. −1

ALL signs change

3) The domain of the rational function $f(x)=\frac{2}{x}$ is $\{x \mid x \text{ is a real number and } x \neq 0\}$.



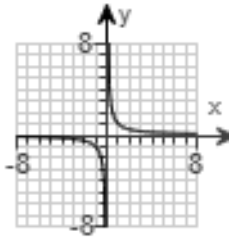
Domain: vertical asymptote

4) The quotient of two polynomial expressions is a rational expression.

Choose the correct answer below.

- [x] True
- [ ] False

5) Graph $y=\frac{1}{x}$.



6) The domain of every rational function is the set of all real numbers.

Choose the correct answer below.

- ○ True
- ✔ False

7) For a rational function R, if the degree of the numerator is less than the degree of the denominator, then R is proper.

8) If a rational function is proper, then y = 0 is a horizontal asymptote.

9) The graph of a rational function may intersect a horizontal asymptote.

Choose the correct answer below.

- ○ False
- ✔ True

10) $F(x) = \dfrac{5x(x-5)}{6x^2-41x-7}$ solve bottom $= \dfrac{\square}{x^2-41x-42}$ $\dfrac{\square}{(x-42)(x+1)}$ divide by 6

Domain: $\{x \mid x \neq 7, -\frac{1}{6}\}$

11) Find the domain of the following rational function.

$H(x) = \dfrac{19x^2+x}{x^2+5}$ Bottom does not factor

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

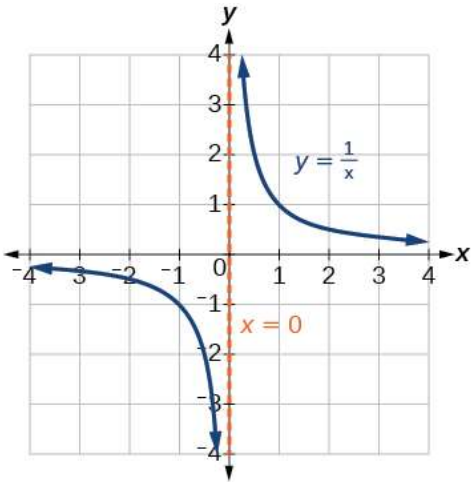
- ○ A. The domain of H(x) is {x | }.
  (Type an inequality. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)
- ✔ B. The domain of H(x) is the set of all real numbers.

GRAPH OF RECIPROCAL FUNCTION

$f(x) = \frac{1}{x}$

Asymptotes are
x = 0 and y = 0

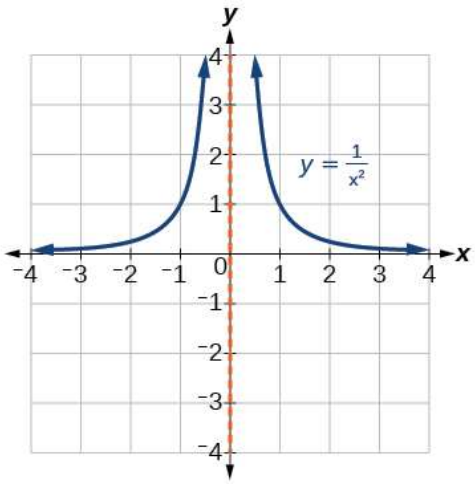
Domain: {x|x≠0} Range: {y|y≠0}



$f(x) = \frac{1}{x^2}$

Asymptotes are
x = 0 and y = 0

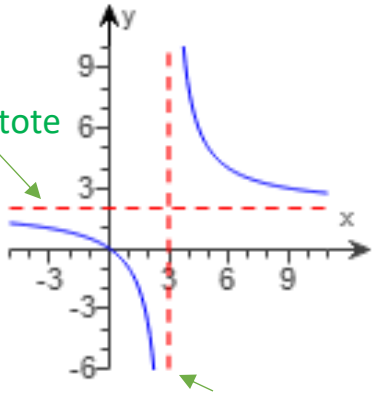
Domain: {x|x≠0} Range: {y|y>0}



12) Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any

a) D: x ≠ 3 R: y ≠ 2
b) x = 0 and y = 0
c) y = 2
d) x = 3
e) No oblique asymptotes

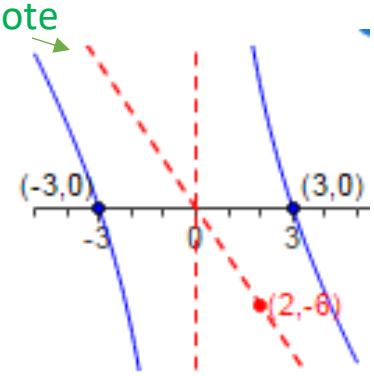
Range:
Horizontal asymptote



domain: vertical asymptote

oblique asymptote

13) Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any



a) D: x≠ 0 R: all real numbers

b) x = -3,3 and no y intercepts

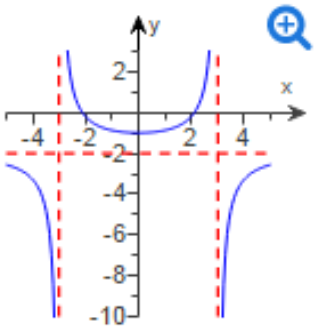
c) no horizontal asymptotes

d) x = 0 (-6/3)

e) Oblique asymptote is equation of the diagonal line y = -3x *(use slope)*

14)

Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any



a) D: x≠ -3,3 R: y < -2, y ≥ -1

b) x = -2, 2 and y = -1

c) y = -2

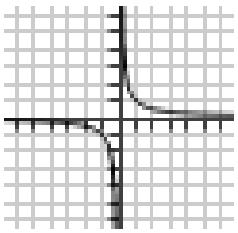
d) x = -3,3

e) no oblique

NOTE* x intercepts may be -1.4, 1-4 if they look between

*GENERAL RULES ON DOMAIN AND RANGE

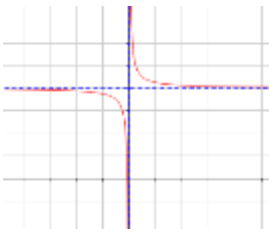
Graph $y = \frac{1}{x}$.



Domain: x≠ 0
Range: y≠ 0

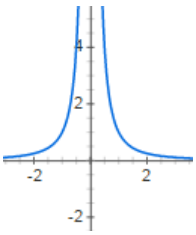
$F(x) = 8 + \frac{1}{x}$,

Up 8 units, the range changes



Domain: x≠ 0
Range: y≠ 8

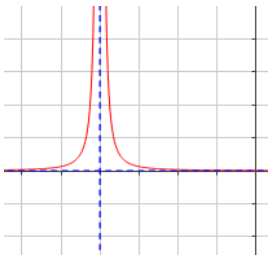
Graph $y = \frac{1}{x^2}$



Domain x≠ 0
Range: y> 0

$F(x) = \frac{1}{(x+8)^2}$

left 8 units, domain changes



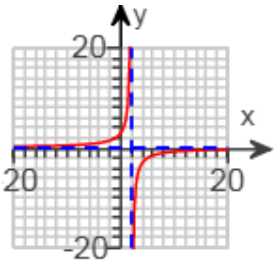
Domain: x≠ -8
Range: y> 0

$H(x) = \frac{-6}{x-2}$

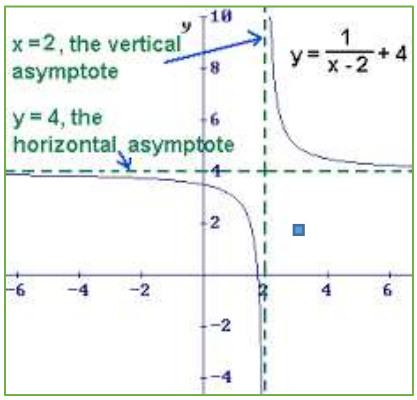
Flips across the x-axis and shifts right 2

Domain: x≠ 2
Range: y≠ 0

Vertical Asymptote x=2
Horizontal Asymptote y=0
No Oblique Asymptote



Asymptote Properties



Vertical asymptote: set denominator =0

***Or look at the horizontal shift**

Horizontal asymptote: y = 0 if $\frac{1}{x}$ (higher x is on bottom)

None if $\frac{x}{1}$ (higher x is on top)

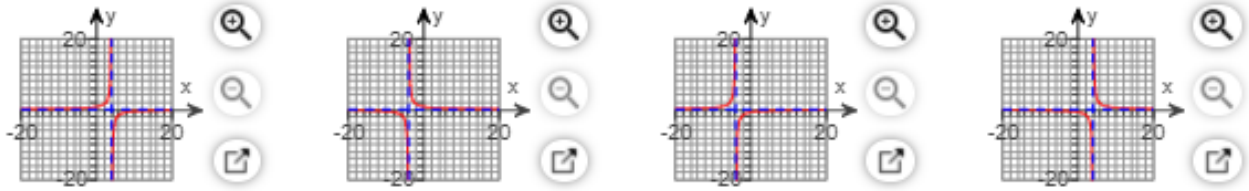
Coefficient if $\frac{3x}{4x}$ y = $\frac{3}{4}$ (x exponents equal)

***Or look at the vertical shift**

15) For the function $H(x) = \frac{-3}{x-4}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

(a) Choose the correct graph below.

A. B. C. D.



(b) What is the domain of the given function? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The domain of the given function is $\{x|x \text{ is a real number, } x > \square\}$.
(Type an integer or a simplified fraction.)

B. The domain of the given function is $\{x|x \text{ is a real number, } x \neq 4\}$. vertical red dotted line
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

What is the range of the given function? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The range of the given function is $\{y|y \text{ is a real number, } y < \square\}$.
(Type an integer or a simplified fraction.)

B. The range of the given function is $\{y|y \text{ is a real number, } y \neq 0\}$. horizontal red dotted line
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

(c) What is/are the vertical asymptote(s)? Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. There is one vertical asymptote. It is $x = 4$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

vertical red dotted line

B. The left vertical asymptote is ____. The right vertical asymptote is ____.
(Type equations. Use integers or fractions for any numbers in the equations.)

C. There is no vertical asymptote.

What is/are the horizontal asymptote(s)? Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. There is one horizontal asymptote. It is $y = 0$.

horizontal red dotted line

(Type an equation. Use integers or fractions for any numbers in the equation.)

What is/are the oblique asymptote(s)? Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The oblique asymptote with the positive slope is ____ and the oblique asymptote with the negative slope is ____.
(Type equations. Use integers or fractions for any numbers in the equations.)

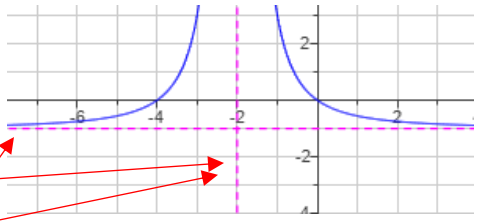
B. There is one oblique asymptote. It is ____.
(Type an equation. Use integers or fractions for any numbers in the equation.)

C. There is no oblique asymptote.

16) For the function $G(x) = -1 + \frac{4}{(x+2)^2}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

C. Vertically stretch the graph of $y = \frac{1}{x^2}$ by a factor of 4, shift it 2 units to the left, and 1 unit down.

Use the shifts and look at the pink asymptotes



The domain of the given function is $\{x|x \neq -2\}$.

There is one vertical asymptote. It is $x = -2$.

The range of the given function is $\{y|y > -1\}$.

y>-1, Graph is above -1

There is one horizontal asymptote. It is $y = -1$.

There is no oblique asymptote.

17) Find the vertical and horizontal asymptotes, if any, for the following rational function.

$$R(x) = \frac{5x}{x+14}$$

Select the correct choice below and fill in any answer boxes within your choice.

Solve the bottom

A. The vertical asymptote(s) is/are x = −14.
(Use a comma to separate answers as needed.)

B. There is no vertical asymptote.

Select the correct choice below and fill in any answer boxes within your choice.

A. The horizontal asymptote(s) is/are y = 5.

Horizontal asymptote: y = 0 if $\frac{1}{x}$ (higher x is on bottom)
None if $\frac{x}{1}$ (higher x is on top)
Coefficient if $\frac{3x}{4x}$ y = $\frac{3}{4}$ (x exponents equal)

18) Find the vertical and horizontal asymptotes, if any, for the following rational function.

$$T(x) = \frac{x^2}{x^4 - 16}$$

$(x^2-4)(x^2+4)$
$(x+2)(x-2)(x^2+4)$

Vertical Asymptote x = -2, 2
Horizontal Asymptote y = 0 *bottom exponent is higher*

19) Find the intercepts of the graph of the equation $y = \frac{x^2 - 16}{x^2 - 9}$.

Select the correct choice below and, if necessary, fill in the an
choice.

A. The intercept(s) is/are $(-4,0),(4,0),\left(0,\frac{16}{9}\right)$.

x-intercepts: solve top by factoring

y-intercept: set x = 0

20) Which type of asymptote will never intersect the graph of a rational function?

Choose the correct answer below.

- oblique
- horizontal
- vertical
- all of these

21) Identify the y-intercept of the graph of $R(x) = \frac{12(x-1)}{(x+1)(x+3)}$.

Choose the correct y-intercept below. Set x = 0

$$\frac{12(0-1)}{(0+1)(0+3)} = \frac{-12}{3} = -4$$

- 1
- 3
- −1
- ✔ −4

22) Follow the steps for graphing a rational function to graph the function $R(x) = \frac{x+2}{x(x+7)}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. R(x) = ☐ (Type your answer in factored form. Do not simplify.)
- ✔ B. R(x) cannot be factored further.

Follow the steps for graphing a rational function to graph the function $R(x) = \frac{x+2}{x(x+7)}$.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\{x|x < \square\}$ (Type an integer or a simplified fraction.)
- B. $\{x|x \geq \square\}$ (Type an integer or a simplified fraction.)
- ✔ C. $\{x|x \neq -7, 0\}$ solve the bottom

Odd exponent crosses
Even exponent touches

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. R(x) = ☐ (Simplify your answer.)
- ✔ B. R(x) is already in lowest terms.

Check multiplicity (exponent) of each asymptote:
If it is odd, it approaches ∞ on one side and -∞ on the other
If it is even, it approaches either ∞ or -∞

The graph has x-intercept(s) −2 and no y-intercept. solve the top for x-intercept, since

The graph will cross the x-axis at x = −2. crosses because exponent on that term is odd

The function has two vertical asymptotes. The leftmost asymptote is x = −7, and the rightmost asymptote is x = 0.

It approaches ∞ on one side of the asymptote(s) at $x = -7, 0$ and $-\infty$ on the other. odd exponent

The function has one horizontal asymptote, $y = 0$. higher x exponent on the bottom y=0

The function has no oblique asymptote.

The graph of R intersects the horizontal or oblique asymptote at $(-2, 0)$ put horizontal asymptote y=0 into the equation

$0 = x - 2$ $x = 2$ write coordinate

The graph of R is above the x-axis on the interval(s) $(-7, -2), (0, \infty)$ and below the x-axis on the interval(s) $(-\infty, -7), (-2, 0)$. Use the + and – intervals you made

Plug in numbers left and right of vertical asymptotes **and** x-intercept

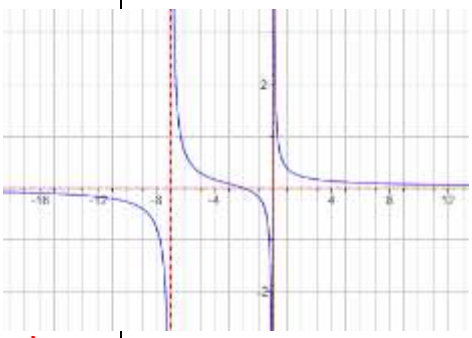
-7 -2 0

-8 -3 -1 1 check + or - intervals

\- + - +

graph using the intervals, crosses

at x = -2 bc it is + left and – right of -2 (the x-intercept)



23) Follow the steps for graphing a rational function to graph the function $R(x) = \frac{5}{x^2 - 36}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $\{x | x \neq -6, 6\}$

$R(x) = \frac{5}{(x-6)(x+6}$ *Does not cross the x-axis

There is no point the graph R intersects the horizontal or oblique asymptotes

The graph has y-intercept $-\frac{5}{36}$ and no x-intercept. y-int set x = 0

no x-intercept x-int: solve the top but no x value

The function has two vertical asymptotes. The leftmost asymptote is $x=-6$, and the rightmost asymptote is $x=6$.

It approaches ∞ on one side of the asymptote(s) at $x=$ $-6,6$ and $-\infty$ on the other.

green boxes in number 22

The function has one horizontal asymptote, $y=0$.

There is no oblique asymptote.

The graph of R is above the x-axis on the interval(s) $(-\infty,-6),(6,\infty)$ and below the x-axis on the interval(s) $(-6,6)$.

asym and x-intercept

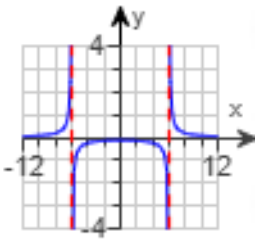
-6 6

-7 0 7

+ - +

graph using intervals

does not cross y asymptote



24) Follow the steps for graphing a rational function to graph the function $R(x)=\dfrac{x(x-5)^2}{(x+6)^3}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x)=$ ☐ (Type your answer in factored form. Do not simplify.)

B. R(x) is already in factored form.

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $\{x|x\neq -6\}$ Solve the bottom

(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. R(x) is already in lowest terms.

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice. x-int: solve the top y-int: set x=0

A. The graph has x-intercept(s) 0,5 and y-intercept 0.

(Simplify your answer. Type integers or fractions. Use a comma to separate answers as needed. Type each answer only once.)

Determine the behavior of the graph of R at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

x-intercept of 0 has exponent of 1 crosses, 5 has exponent of 2 touches

A. The graph will cross the x-axis at x = 0, and touch but not cross the x-axis at x = 5.

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has three vertical asymptotes. The leftmost asymptote is ____, the middle asymptote is ____, and the rightmost asymptote is ____.
(Type equations. Use integers or fractions for any numbers in the equations.)

B. The one vertical asymptote is x = −6.

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

C. It approaches ∞ on one side of the asymptote(s) at x = −6 and −∞ on the other. odd exponent
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The function has two horizontal asymptotes. The top asymptote is ____, and the bottom asymptote is ____.
(Type equations. Use integers or fractions for any numbers in the equations.) $\frac{x^3}{x^3}$

B. The function has one horizontal asymptote, y = 1. coefficient is 1

C. The function has no oblique asymptote.

C. There is no point at which the graph of R intersects the horizontal or oblique asymptote.

Determine the points, if any, at which the graph of R intersects the horizontal or oblique asymptote, if one exists. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Plug in horizontal asymptote y = 1 into the equation:

$1 = \frac{x^3-10x^2+25x}{x^2+x-20}$ cannot be factored so none

Plug in numbers left and right of vertical asymptotes and x-int

-6 0 5

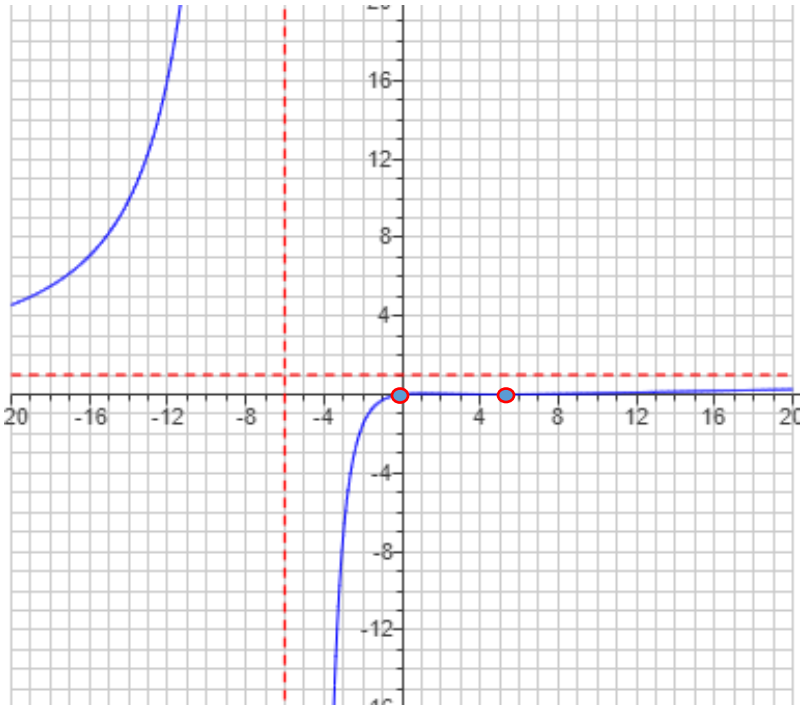
-7 -1 1 6 to check + and - intervals

\+ - + +

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

Crosses at 0 and touches at 5 because of the exponents which makes extra intervals at those points

A. The graph of R is above the x-axis on the interval(s) $(-\infty,-6),(0,5),(5,\infty)$ and below the x-axis on the interval(s) $(-6,0)$.



25) Follow the steps for graphing a rational function to graph the function $R(x)=\dfrac{x^2+5x-36}{x^2-x-12}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of R(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $R(x)=\dfrac{(x+9)(x-4)}{(x+3)(x-4)}$ (Type your answer in factored form. Do not simplify.)

What is the domain of R(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

C. $\{x|x\neq -3,4\}$ only use both x-values in bottom for domain

Write R(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Cancel out common factors (x-4)

A. $R(x)=\dfrac{x+9}{x+3}$ (Simplify your answer.)

Find the intercepts of the graph. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The graph has x-intercept(s) -9 and y-intercept 3.
(Simplify your answers. Type integers or fractions. Use a comma to separate answers as needed. Type each answer only once.)

x-int: solve the top y-int set x=0

Determine the behavior of the graph of R at any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. The graph will cross the x-axis at x = _____ and touch but not cross the x-axis at x = _____.
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

B. The graph will cross the x-axis at x = -9.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

solve the top odd exponent so it crosses

Determine the vertical asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

solve the bottom

C. The function has one vertical asymptote, $x = -3$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

Determine the behavior of the graph on either side of any vertical asymptotes, if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

A. It approaches ∞ on one side of the asymptote(s) at x = _____ and $-\infty$ on the other. It approaches either ∞ or $-\infty$ on both sides of the asymptote(s) at x = _____.
(Type integers or simplified fractions. Use a comma to separate answers as needed. Type each answer only once.)

B. It approaches ∞ on one side of the asymptote(s) at x = -3 and $-\infty$ on the other.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed. Type each answer only once.)

Determine the horizontal asymptote(s), if one exists. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

Exponents top and bottom match so it's the coefficient

A. The function has one horizontal asymptote, $y = 1$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

C. The function has no oblique asymptote.

Determine the points, if any, at which the graph of R intersects the horizontal or oblique asymptote, if one exists. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

C. There is no point at which the graph of R intersects the horizontal or oblique asymptote.

Plug in horizontal asymptote y = 1 into the equation:

$1 = \frac{x+9}{x+3}$ $\quad x + 3 = x + 9$ no solution

Plug in numbers left and right of vertical asymptotes and x-int

-9 -3 4

-10 -4 0 5 to check + and - intervals

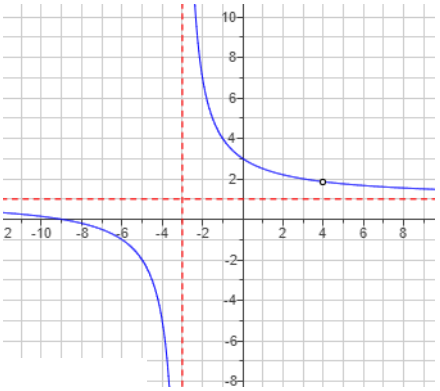
\+ - + +

does not include x=4 because the cancelled term is a whole in the graph,
use intervals to choose graph and the whole makes an extra interval

Use the real zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Select the correct choice below and fill in the answer box(es) to complete your choice.

A. The graph of R is below the x-axis on the interval(s) ____.
(Type your answer in interval notation. Use a comma to separate answers as needed.)

B. The graph of R is above the x-axis on the interval(s) $(-\infty,-9),(-3,4),(4,\infty)$ and below the x-axis on the interval(s) $(-9,-3)$.



26) Follow the steps for graphing a rational function to graph the function $H(x) = \frac{3x-18}{36-x^2}$.

If needed, first write the given function as a single rational expression. Then, factor the numerator and denominator of H(x). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $H(x) = \frac{3(x-6)}{(6-x)(6+x)}$ (Type your answer in factored form. Do not

What is the domain of H(x)? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

B. $\{x|x \neq -6,6\}$

Write H(x) in lowest terms. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $H(x) = \frac{-3}{(x+6)}$ (Simplify your answer.)

C. The graph has y-intercept $-\frac{1}{2}$ and no x-intercept.

D. There is no x-intercept.

A. The function has one vertical asymptote, $x = -6$.
(Type an equation. Use integers or fractions for any numbers in the equation.)

C. It approaches ∞ on one side of the asymptote(s) at $x = -6$ and $-\infty$ on the other.

A. The function has one horizontal asymptote, $y = 0$.

C. There is no oblique asymptote.

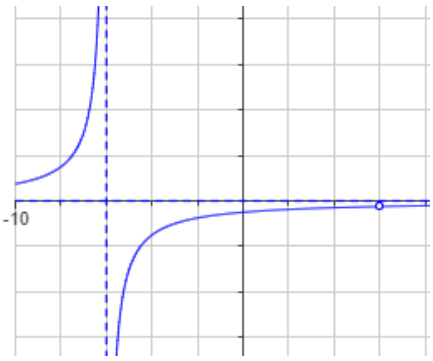
C. There is no point at which the graph of H intersects the horizontal or oblique asymptote.

A. The graph of H is above the x-axis on the interval(s) $(-\infty, -6)$ and below the x-axis on the interval(s) $(-6,6),(6,\infty)$.

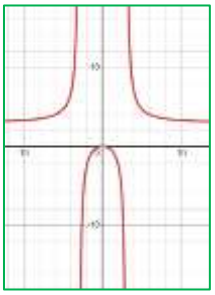
Plug in numbers left and right of vertical asymptotes (-6,6)

-7, 0, 7 to check positive or negative

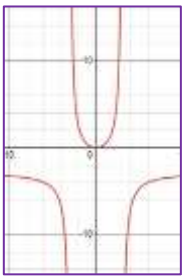
\+ - - does not include x=6 because the cancelled term is a whole in the graph which will make and extra interval



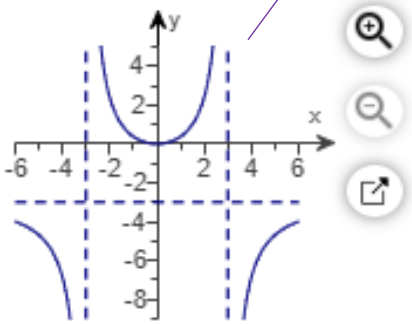
$y = \frac{x^2}{x^2-9}$



$y = -\frac{x^2}{x^2-9}$



27) Which rational function has the given graph?



$X^2 - 9$ on bottom

- A. $R(x) = \frac{3x^2}{x^2 - 9}$
- B. $R(x) = -\frac{3x^2}{x^2 + 9}$
- C. $R(x) = \frac{3x^2}{x^2 + 9}$
- ✔ D. $R(x) = -\frac{3x^2}{x^2 - 9}$

28) Match the rational function with the appropriate description below.

$f(x) = \frac{x - 1}{x + 5}$

Choose the correct attribute of the rational function.

- ✔ A. The vertical asymptote is $x = -5$.

29) Identify the appropriate description for the following rational function.

$f(x) = \frac{1}{x+3}$

- A. The x-intercept is 3.
- B. There is a "hole" in its graph at $x = -3$.
- C. The x-axis is its horizontal asymptote, and the y-axis is its vertical asymptote.
- D. The horizontal asymptote is $y = 3$.
- E. The vertical asymptote is $x = -1$.
- F. The graph has an oblique asymptote.
- G. The y-intercept is 1.
- ✔ H. The x-axis is its horizontal asymptote, and the y-axis is not its vertical asymptote.

30) Match the rational function with the appropriate description below.

$f(x) = \dfrac{x^2 - 16}{x - 4}$ $\dfrac{(x-4)(x+4)}{(x-4)}$

Choose the correct attribute of the rational function.

- A. The vertical asymptote is x = 4.
- B. There is a "hole" in the graph at x = 4.

The term that is canceled leaves a hole

31) Which of the following functions have a graph that does not have a vertical asymptote?

Select all that apply.

- A. $f(x) = \dfrac{3}{x^2}$
- B. $f(x) = \dfrac{1}{x^2 - 2}$
- C. $f(x) = \dfrac{3x + 1}{x - 7}$
- D. $f(x) = \dfrac{1}{x^2 + 2}$

Can not factor

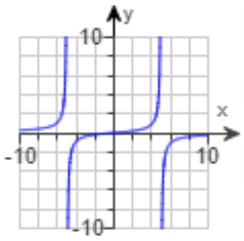
32)

Drag each rational function above to the area that corresponds to the appropriate description. Items can be used only once.

I	II	I	II
The x-intercept is (−8, 0). solve the top	$f(x) = \dfrac{x+8}{x-16}$	There is a hole in the graph at (−5, −10). term cancels	$f(x) = \dfrac{x^2 - 25}{x+5}$
The y-intercept is (0, 3). set x = 0	$f(x) = \dfrac{x+9}{x+3}$	The graph has an oblique asymptote. trinomial on top	$f(x) = \dfrac{x^2 + 2x + 3}{x-4}$
The horizontal asymptote is y = 7. number top, front	$f(x) = \dfrac{7x+4}{x-11}$	The x-axis is its horizontal asymptote and the y-axis is not its vertical asymptote. $x \neq -7$	$f(x) = \dfrac{1}{x+7}$
solve the bottom The vertical asymptote is x = −6.	$f(x) = \dfrac{x+5}{x+6}$	The x-axis is its horizontal asymptote and the y-axis is its vertical asymptote. x^2 on bottom	$f(x) = \dfrac{-6}{x^2}$

33) Graph the rational function.

$$f(x) = \frac{-2x}{x^2 - 25}$$



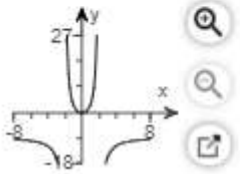
Vertical asymptotes at x = -5,5

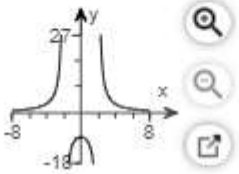
34) Sketch the graph of the rational function.

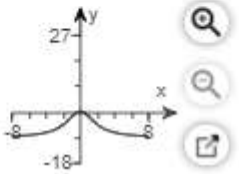
$$f(x) = \frac{9x^2 - 1}{x^2 - 4}$$

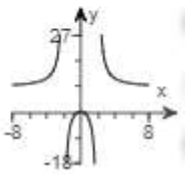
Vertical asymptotes at x = -2,2

Choose the correct graph of the given rational function.

A. 

B. 

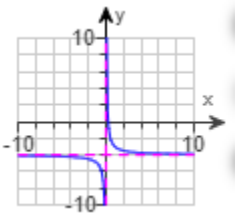
C. 

D. 

EXTRA PROBLEMS:

a) For the function $F(x) = -4 + \frac{1}{x}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

(a) Which of the following transformations is required to graph the given function?



A. Shift the graph of $y = \frac{1}{x}$ down 4 units.

The domain of the given function is $\{x | x \neq 0\}$.

The range of the given function is $\{y | y \neq -4\}$.

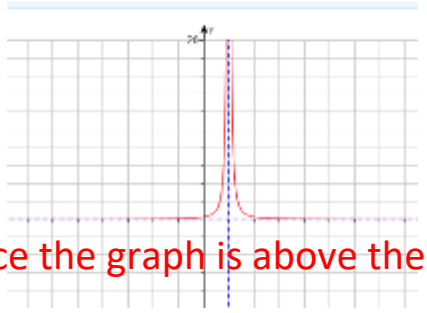
There is one vertical asymptote. It is $x = 0$.

There is one horizontal asymptote. It is $y = -4$.

*Use the graph to answer all of these

b) For the function $F(x) = \frac{1}{(x-2)^2}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

(a) Choose the correct graph below.



The domain of the given function is $\{x|x \text{ is a real number, } x \neq 2\}$

The range of the given function is $\{y|y \text{ is a real number, } y > 0\}$

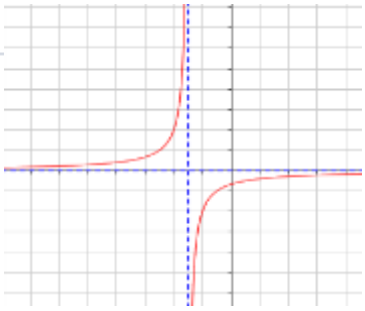
Notice the graph is above the x-axis

There is one vertical asymptote. It is $x = 2$.

There is one horizontal asymptote. It is $y = 0$

There is no oblique asymptote.

c) For the function $H(x) = \frac{-4}{x+3}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.



(a) Choose the correct graph below.

The domain of the given function is $\{x|x \text{ is a real number, } x \neq -3\}$.

The range of the given function is $\{y|y \text{ is a real number, } y \neq 0\}$

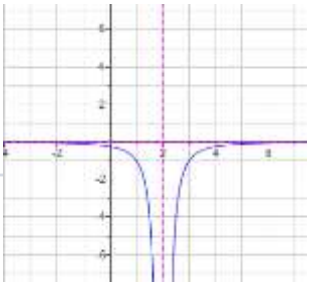
There is one vertical asymptote. It is $x = -3$.

There is one horizontal asymptote. It is $y = 0$.

There is no oblique asymptote.

d) For the function $F(x) = \frac{-1}{x^2 - 4x + 4}$, (a) graph the rational function using transformations, (b)

$(x-2)(x-2)$ is $(x-2)^2$



use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

C. Reflect the graph of $y = \frac{1}{x^2}$ about the x-axis, and then shift it 2 units to the right.

Graph is below 0

The domain of the given function is $\{x|x \neq 2\}$.

The range of the given function is $\{y|y < 0\}$.

There is one vertical asymptote. It is $x = 2$.

There is one horizontal asymptote. It is $y = 0$.

There is no oblique asymptote.

e) $\frac{2x^2-5x-3}{3x^2-8x-3}$ factor $\frac{(2x+1)(x-3)}{(3x+1)(x-3)}$

Vertical asymptote is x = $-\frac{1}{3}$ *solve 3x+1=0*

Horizontal asymptote is y = $\frac{2}{3}$ *x^2 coefficients from original*

No oblique asymptotes

Since we are finding vertical asymptote and not the domain we cross out the common factored terms

f) Find the domain of the following rational function.

$$H(x) = \frac{-3x^2}{(x-5)(x+3)}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The domain of H(x) is $\{x \mid x \neq 5, -3\}$.

g) Find the domain of the following rational function.

$$R(x) = \frac{4\left(x^2-4x-60\right)}{5\left(x^2-100\right)}$$

Select the correct choice below and fill in any answer boxes within your choice.

A. The domain of R(x) is $\{x \mid x \neq -10, 10\}$.

h) Analyze the graph of the function.

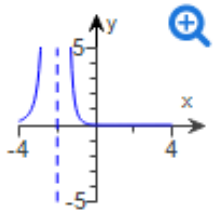
$$R(x) = \frac{x^2 - 25}{x^4 - 16}$$

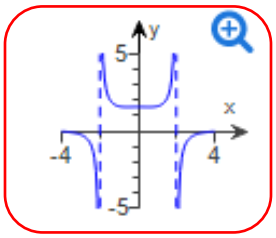
Vertical asymptotes: x = -2,2

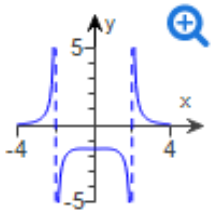
Horizontal asymptote: y = 0

(x+2(x-2)(x^4+4)

CHOOSE GRAPH: ** Plug in zeros for both x values = $\frac{25}{16}$ which is + therefore at x = 0 the graph is positive*







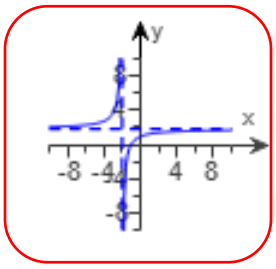
i) Analyze the graph of the function.

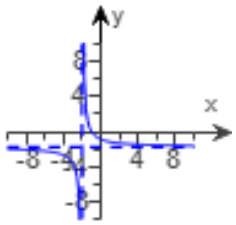
$$R(x) = \frac{9x + 9}{5x + 15}$$

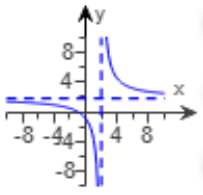
Vertical asymptotes: x = -3

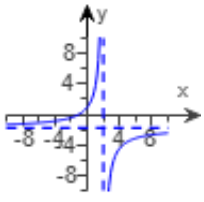
Horizontal asymptote: y = $\frac{9}{5}$

use asymptotes









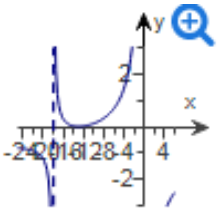
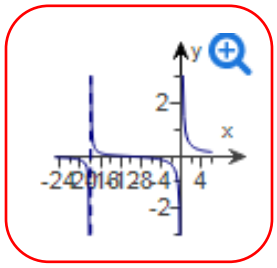
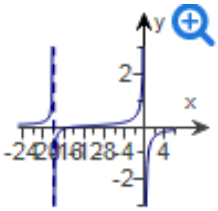
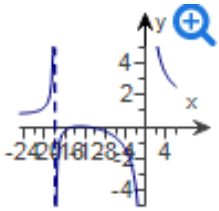
j) Analyze the graph of the function.

$$R(x) = \frac{x + 13}{x(x + 18)}$$

Vertical asymptotes: x = 0,-18

Horizontal asymptote: y = 0

CHOOSE GRAPH: ** consider asymptotes and plug in x=1 because it is the easiest to determine* *and is a positive number*

k) Analyze the graph of the function.

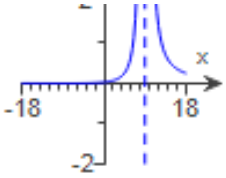
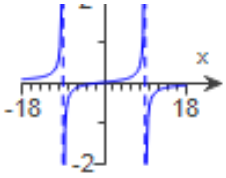
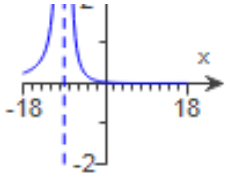
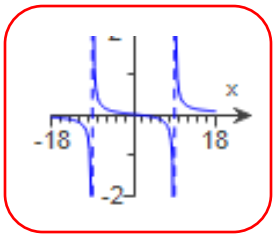
$$R(x) = \frac{x}{x^2 - 81}$$

Vertical asymptotes: x = -9,9

Horizontal asymptote: y = 0

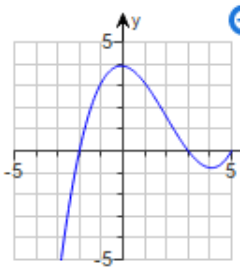
CHOOSE GRAPH: ** since zero is a solution of the top the graph can cross the x-axis at x=0*

plug in x = 1 to check graph, y is negative

l)

Determine whether the graph could be the graph of a polynomial function. If it could be, list the real zeros and state the least degree the polynomial can have.



Select the correct choice below and fill in any answer boxes within your choice.

A. The graph shows a polynomial function. The real zero(s) is/are $-2,3,5$. The least degree the polynomial can have is 3.
(Use a comma to separate answers as needed. Round to the nearest integer as needed.)

B. The graph does not show a polynomial function.

m)

Find the domain of the following rational function.

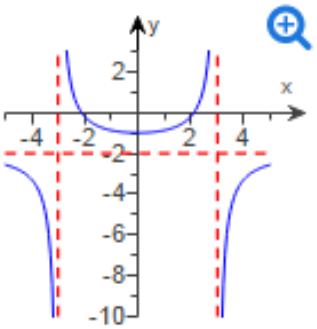
$$F(x) = \frac{4x(x-8)}{2x^2 - 7x - 4}$$

Set denominator = 0 to find domain

Select the correct choice below and fill in any answer boxes within your choice.

A. The domain of F(x) is $\left\{x \mid x \neq -\frac{1}{2}, 4\right\}$.

n) Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any



a) D: $x \neq -3,3$ R: $y < -2, y \geq -1$

b) $x = -1.4, 1.4$ and $y = -1$

c) $y = -2$

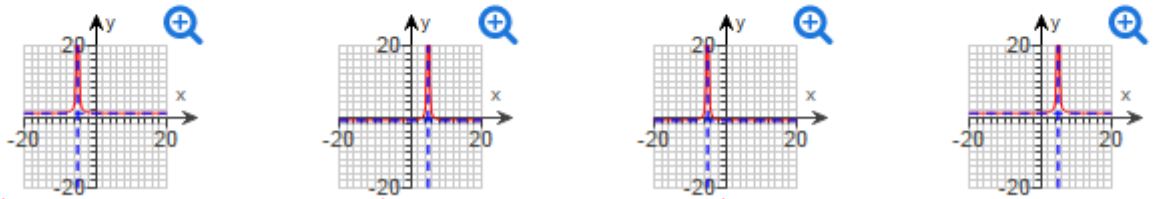
d) $x = -3,3$

e) no oblique

o) For the function $F(x) = -1 + \frac{2}{(x-5)^2}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

(a) Choose the correct graph below.

○ A. ● B. ○ C. ○ D.



b) Domain: x≠ 5 c) Range: y > -1 d) Vertical Asymptote x = 5

e) Horizontal Asymptote y = -1 f) no oblique

Find the vertical, horizontal, and oblique asymptotes, if any, for the following rational function

p) $R(x) = \frac{3x}{x+11}$

Select the correct choice below and fill in any answer boxes within your choice.

★ A. The vertical asymptote(s) is/are x = −11.

Select the correct choice below and fill in any answer boxes within your choice.

★ A. The horizontal asymptote(s) is/are y = 3.

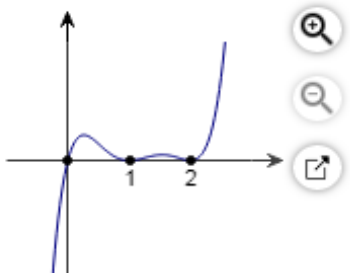
Select the correct choice below and fill in any answer boxes within your choice.

○ A. The oblique asymptote(s) is/are y = .
(Use a comma to separate answers as needed.)

★ B. There is no oblique asymptote.

w) Decide which of the polynomial functions in the list might have the graph below. (More than one answer may be possible.)

(a) $y = -4x(x-1)(x-2)$
(b) $y = x^2(x-1)^2(x-2)$
(c) $y = 3x(x-1)(x-2)$
(d) $y = x(x-1)^2(x-2)^2$
(e) $y = 3x(1-x)(2-x)$
(f) $y = -x(1-x)(x-2)$



Which of the polynomial functions in the list might have the given graph?

d

(Type a, b, c, d, e, or f. Use a comma to separate answers as needed.)

Touches at 1 and 2 therefore look for $(x-1)^2(x-2)^2$